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Research Paper

Event-triggered prescribed-time bipartite consensus control of nonlinear multi-agent systems

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ABSTRACT: *This paper investigates the event-triggered prescribed-time bipartite consensus of nonlinear multi-agent systems. In order to significantly reduce the communication burden and energy consumption, a novel event-triggered mechanism and triggering condition are proposed. Then, a distributed prescribed-time control protocol is proposed for the discussed nonlinear multi-agent systems to achieve bipartite consensus within the prescribed time based on the event-triggered mechanism. Using the Lyapunov stability theory, the stability of the nonlinear multi-agent systems is proved and the corresponding sufficient conditions are obtained. Moreover, it is shown that the Zeno behavior can be excluded. Simulation results are presented to show the effectiveness of the theoretical results.*

KEYWORDS: *Nonlinear multi-agent systems, Bipartite consensus, Event-triggered, Prescribed-time*

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I. INTRODUCTION

In recent years, the consensus problem in Multi-Agent Systems (MASs) has attracted the attention of scholars because of its extensive applications in artificial intelligence, mobile communication and so on [1-4]. Research on the consensus of MASs principally focuses on the cooperative relationship. However, in practical environments, there are not only cooperative relationships but also competitive relationships among agents [5-7]. Subsequently, bipartite consensus has been discussed by many scholars. Altafini first proposed the concept of bipartite consensus and a signed network, along with the necessary and sufficient conditions to ensure the bipartite consensus of signed networks [8]. An output-feedback-based leader-follower bipartite consensus protocol is designed for a class of high-order Lipschitz nonlinear MASs with unmeasurable states [9]. A finite-time protocol and a fixed-time protocol are developed to solve the bipartite consensus tracking problem [10]. Studies focus on second-order matrix-weighted bipartite consensus on undirected balanced signed networks [11]. A new prescribed-time distributed control protocol for consensus and containment of networked multi-agent systems is proposed, and the article discusses the undirected connection topology and the directed topology [12]. However, the above study of prescribed-time consensus is limited to agents being cooperative, but competitive relationships can also exist among agents. Compared to consensus, bipartite consensus is more widely applicable and more valuable to study.

Traditional research on consensus control of MASs concentrates on achieving asymptotic convergence to the desired state. The convergence time is an important metric for evaluating control performance. To further improve the convergence time, finite-time control has been developed [13-16]. Notably, the finite-time control relies on the initial values of MASs, which limits its practical application in realistic scenarios such as unknown or unavailable initial values of MASs. To solve the problem, fixed-time control is proposed. Fixed-time control guarantees that the settling time function is uniformly bounded regarding the initial values, but depends on the control parameters [17-20]. To solve shortcomings of fixed-time control, prescribed-time control is proposed [21-25]. It is more meaningful to study prescribed-time control of MASs. Prescribed-time control guarantees that the system can achieve convergence at an arbitrarily specified time set by the user. A new prescribed-time control algorithm is presented in the paper.

The communication in the above MASs study is continuous. Traditional continuous consensus control requires adequate communication resources to transmit information among neighboring agents. For the sake of reducing unnecessary control updates and saving resources more effectively, event-triggered mechanism has been broadly introduced into the consensus of MASs, and as attracted the interest of many researchers [26-29]. A consensus protocol with a distributed event-triggered mechanism is developed and the controlled objects are first order [30]. The event-triggered problem in second-order systems is addressed [31]. Additionally, various types of event-triggered mechanisms have been investigated [29,32,33]. Therefore, the study of the combination of prescribed-time control with event triggered mechanisms is more valuable. Compared to these event-triggered mechanism approaches, the proposed study of the combination of event-triggered mechanisms and prescribed-time control is more challenging and has distinct advantages in this paper.

Inspired by the above discussions, and considering that nonlinear systems can more accurately describe the actual systems, the paper comprehensively studies the event-triggered prescribed-time bipartite consensus control of nonlinear multi-agent systems. The remainder of this paper is organized as follows: Section 2 introduces some preliminary definitions, basic properties, and formulates the main problem. Section 3 designs event-triggered mechanism, presents the prescribed-time control protocol, proves the stability of the system, and researches the exclusion of Zeno behavior in detail. Section 4 provides numerical simulations. Conclusions are presented in Section 5.

II. PRELIMINARIES AND PROBLEM FORMULATION

2.1 NOTATION

Let $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices, respectively. The symbol $\otimes$ denotes the Kronecker product. The identity matrix is denoted as $I_m \in \mathbb{R}^{m\times m}$, and 1_m is an *m*-element column vector with each element equal to 1. **0** denotes the vectors or matrices of proper dimensions with all elements being0. $\|\cdot\|$ stands for either the Euclidean vector norm or the spectral norm of a matrix. For a symmetric matrix H, $\lambda_{\max}(H)$ and $\lambda_{\min}(H)$ denote the largest and the smallest values among all its eigenvalues, respectively.

2.2 COMMUNICATION TOPOLOGY

The network topology of the system can be represented by a signed undirected graph $A=(a_{ij})_{n\times n}$, where $\mathrm{V}=\{1,2,...,N\}$ is the node set, E represents the edge set, the pair $(i,j)\in\mathrm{E}$ means that node j can receive information from node i, A pair $(i,j)\in\mathrm{E}$ indicates that node j can receive information from node i. The adjacency matrix $A=(a_{ij})_{n\times n}$ is defined such that $a_{ij}\neq 0$ if $(i,j)\in\mathrm{E}$, otherwise, $a_{ij}=0$. i and j are cooperative if $a_{ij}>0$ and competitive if $a_{ij}<0$. The Laplacian matrix $L=(l_{ij})_{n\times n}$ is denoted as $L=D-A$, where $D=\mathrm{diag}\{d_1,d_2,...,d_n\}$, $d_i=\sum_{j=1}^{n}|a_{ij}|$, that is

$$l_{ij}=\begin{cases}\sum_{j=1}^{n}|a_{ij}|, & i=j,\\ -a_{ij}, & i\neq j.\end{cases}$$

2.3 MODEL DESCRIPTION

Consider a nonlinear second-order MASs consisting of N agents. The communication topology of these agents is represented by an undirected signed graph $\mathrm{G}(A)$. The model of the *i*-th agent is as follows

$$\begin{cases}\dot{x}_i(t)=v_i(t),\\ \dot{v}_i(t)=u_i(t)+f(t,x_i(t),v_i(t)) \quad i=1,2,...,N,\end{cases} \tag{1}$$

where $x_i(t)\in\mathbb{R}^{n\times n}$, $v_i(t)\in\mathbb{R}^{n\times n}$, and $u_i(t)\in\mathbb{R}^{n\times n}$ represent the position, velocity, and control input, respectively. $f(t,x_i(t),v_i(t))\in\mathbb{R}^m$ is a smooth and continuous unknown function, representing the external disturbance of each agent.

2.4 LEMMAS AND DEFINITIONS

Assumption 1. There exists constant $c_1>0$ and $c_2>0$, such that for all $x,y\in\mathbb{R}$.

$$\|f(x,v,t)-f(y,z,t)\|\leq c_1\|x-y\|+c_2\|v-z\|. \tag{2}$$

Lemma 1. [34] Given a structurally balanced connected graph $G(A)$, we have the following consequent results.

(1) One can always find a matrix $H := \operatorname{diag}(\sigma_1, \sigma_2, ..., \sigma_n)$ with $\sigma_i \in \{1, -1\}$, $i \in V$, ensuring that HAH has all non-negative entries.

(2) With the aid of H, the positive semidefinite matrix $\tilde{L} := HLH$ is obtained, which has a single zero eigenvalue with the corresponding eigenvector 1.

(3) For the vector x which satisfies $1^T x = 0$, it holds that

$$\min_{x \neq 0} \frac{x^T \tilde{L} x}{x^T x} = \lambda_2(\tilde{L}),$$

where $\lambda_2(\tilde{L})$ is the smallest positive eigenvalue of $\tilde{L}$.

(4) L and $\tilde{L}$ are isospectral.

Lemma 2. [35] The following inequality exists

$$\|X\|\|V\| \leq l\|X\|^2 + \frac{1}{4l}\|V\|^2,$$

where X, V are any given vectors with proper dimensions and l is a positive constant.

Lemma 3 . [36] Under Assumption 1, $\tilde{L}$ is a nonsingular M-matrix. Further, a vector $w = (w_1, ..., w_n)^T$ $(w_i \in \mathbb{R}, i = 1, ..., n)$ can be found such that $\tilde{L}w = 1_n$. Denote $W = \operatorname{diag}(1/w_1, ..., 1/w_n)$ and $\Gamma = W\tilde{L} + \tilde{L}^T W$, then W and Γ are positive definite.

To implement prescribed-time control, the following time-varying scaling function $\mu(t)$ defined in [37] will be applied

$$\mu(t) = \begin{cases} \left(\dfrac{T}{t_0 + T - t}\right)^h & t \in [t_0, t_0 + T), \\ 1 & t \in [t_0 + T, \infty), \end{cases} \tag{3}$$

where $h > 0$ and $T > 0$ is a user-specified constant. Then, we construct $\psi(t)$

$$\psi(t) = \begin{cases} \dfrac{\dot{\mu}(t)}{\mu(t)}, & t \in [t_0, t_0 + T), \\ \dfrac{h}{T}, & t \in [t_0 + T, \infty). \end{cases} \tag{4}$$

It is evident that $\mu^{-m}(t)(m > 0)$ is monotonically decreasing during the interval $[0,T)$ with $\mu^{-m}(0) = 1$ and $\lim_{t \to (t_0+T)^-} \mu^{-m}(t) = 0$.

Lemma 4. [38] Consider a system described by

$$\dot{q}(t) = f(q(t), t), \tag{5}$$

where $q(t) \in \mathbb{R}^m$ is the state and $f(\cdot,\cdot)$ is a vector field bounded in time. Let $V(t, q(t))$ with $V(t, 0) = 0$ be the Lyapunov function of (5). For simplicity, we use V to denote $V(t, q(t))$. Which $m_1 \geq 0$, $m_2 > 0$, $(1/h) - m_1 < m_3 < 1/h$ and $\psi(t)$ being defined in (4), then for $t \in [t_0, t_0 + T)$, it yields

$$\dot{V} \leq -m_1 \psi(t) V + m_2 \mu^{-m_3}(t) \quad t \in [t_0, t_0 + T), \tag{6}$$

and

$$\dot{V} \leq -m_1 \psi(t) V \quad t \in [t_0 + T, \infty). \tag{7}$$

Then, it holds that

$$V \leq \left[\exp(-m_1(t - t_0)) V(t_0) + \frac{m_2 T}{h(m_1 + m_3) - 1}\right] \mu^{m_3 - \frac{1}{h}}(t) \quad t \in [t_0, t_0 + T), \tag{8}$$

and

$$V \equiv 0, \quad t \in [t_0 + T, \infty). \tag{9}$$

Definition 1. For the MASs (1), it is said to achieve prescribed-time bipartite consensus if for any user-prescribed time T, there exists

$$\begin{cases} \lim\limits_{t\to t_0+T} \left\| x_i(t) - \sigma_i x_j(t) \right\| = 0, \\ \lim\limits_{t\to t_0+T} \left\| v_i(t) - \sigma_i v_j(t) \right\| = 0, \end{cases}$$

$$\begin{cases} \left\| x_i(t) - \sigma_i x_j(t) \right\| = 0, \forall t \ge t_0 + T, \\ \left\| v_i(t) - \sigma_i v_j(t) \right\| = 0, \forall t \ge t_0 + T, \end{cases}$$

then the prescribed-time bipartite consensus is said to be achieved for the second-order nonlinear MASs (1) at the prescribed-time under the event-triggered mechanism.

III. MAIN RESULTS

This section is divided into two parts. The first part proves that MASs can achieve prescribed-time bipartite consensus by using an event-triggered mechanism, and the second part proves that Zeno behavior can be excluded.

3.1 EVENT-TRIGGERED PRESCRIBED-TIME PROTOCOL

The event-triggered prescribed-time protocol for the *i*-th agent is defined as follows

$$\begin{aligned} u_i(t) = -\psi(t)\Bigg[& k_1\psi(t)\sum_{j=1}^{N}\left|a_{ij}\right|\left(x_i\left(t_k^i\right) - \operatorname{sgn}\left(a_{ij}\right)x_j\left(t_k^i\right)\right) \\ & + k_2\sum_{j=1}^{N}\left|a_{ij}\right|\left(v_i\left(t_k^i\right) - \operatorname{sgn}\left(a_{ij}\right)v_j\left(t_k^i\right)\right)\Bigg] \quad t\in\left[t_k^i, t_{k+1}^i\right), \end{aligned} \tag{10}$$

where k_1, k_2 are positive constant control gains.

By denoting the measurement error of agent i as $e_i(t) = \eta_i\left(t_k^i\right) - \eta_i(t)$, where $\eta_i(t) = k_1\delta_{xi}(t) + k_2\delta_{vi}(t)$. In order to eliminate the cross terms in the derivative of the subsequent Lyapunov function, an important state transformation is introduced as $\delta_{xi}(t) = \psi(t)\hat{x}_i(t)$, $\delta_{vi}(t) = \hat{v}_i(t)$. It also yields that

$$\begin{aligned} e_i(t) &= k_1\delta_{xi}\left(t_k^i\right) + k_2\delta_{vi}\left(t_k^i\right) - k_1\delta_{xi}(t) - k_2\delta_{vi}(t) \\ &= \eta_i\left(t_k^i\right) - \eta_i(t). \end{aligned} \tag{11}$$

The triggering instant sequence t_k^i for agent *i* is defined iteratively by

$$t_{k+1}^i = \inf\left\{t > t_k^i : h_i(t) > 0\right\}, \tag{12}$$

where event triggering function

$$h_i(t) = \left\|e_i(t)\right\| - a\left\|\eta_i(t)\right\| - b\mu^{-2c}(t), \tag{13}$$

where $0 < a = \sqrt{\dfrac{\lambda_{\min}(\Gamma)}{\lambda_{\max}(\Gamma)}}\theta < 1$, $b > 0$, $c \ge 0$, and θ is a positive selected parameter. In fact, the event-triggered mechanism (12) reveals that the condition $h_i(t) < 0$ always holds as long as $t \in [t_k^i, t_{k+1}^i)$.

3.2 CONSENSUS ANALYSIS

To carry out the following analysis, some useful intermediate variables are employed. For $i \in \mathrm{V}$, define $\bar{x}_i(t) = \sigma_i x_i(t)$, $\bar{v}_i(t) = \sigma_i v_i(t)$, we get

$$\begin{cases} \dot{\bar{x}}_i(t) = \bar{v}_i(t), \\ \dot{\bar{v}}_i(t) = f\left(t, \bar{x}_i(t), \bar{v}_i(t)\right) + \sigma_i u_i(t). \end{cases} \tag{14}$$

Subsequently, we define the average values of $\phi_x(t)$ and $\phi_v(t)$ as $\phi_x(t) = (1/n)\sum_i \bar{x}_i(t)$ and $\phi_v(t) = (1/n)\sum_i \bar{v}_i(t)$. In addition, we define the $\hat{x}(t) = \bar{x}_i(t) - \phi_x(t)$ and $\hat{v}_i(t) = \bar{v}_i(t) - \phi_v(t)$.

Taking the derivatives of $\delta_{xi}(t)$ and $\delta_{vi}(t)$ along (14) yields

$$\begin{cases} \dot{\delta}_{xi}(t) = \dot{\psi}(t)\delta_{xi}(t) + \psi(t)\delta_{vi}(t), \\ \dot{\delta}_{vi}(t) = f\left(t, \hat{x}_i(t), \hat{v}_i(t)\right) - f\left(t, \phi_x(t), \phi_v(t)\right) - k_1\psi(t)\sum\limits_{i=1}^{N}\tilde{l}_{ij}\delta_{xi}\left(t_k^i\right) - k_2\psi(t)\sum\limits_{i=1}^{N}\tilde{l}_{ij}\delta_{vi}\left(t_k^i\right). \end{cases} \tag{15}$$

Let $\delta_x = \left(\delta_{x1}^T, \delta_{x2}^T, ..., \delta_{xn}^T\right)^T$, $\delta_v = \left(\delta_{v1}^T, \delta_{v2}^T, ..., \delta_{vn}^T\right)^T$, $\hat{e}(t) = \left(\hat{e}_1^T(t), \hat{e}_2^T(t), ..., \hat{e}_n^T(t)\right)^T$, and $F(t, \delta_x, \delta_v) = \left(F^T(t, \delta_{x1}, \delta_{v1}), F^T(t, \delta_{x2}, \delta_{v2}), ..., F^T(t, \delta_{xn}, \delta_{vn})\right)^T$, where the elements are defined as $F(t, \delta_{xi}, \delta_{vi}) = f\left(t, \hat{x}_i(t), \hat{v}_i(t)\right) - f\left(t, \phi_x(t), \phi_v(t)\right)$. By utilizing Kronecker product techniques, (15) can be reformulated into a more compact representation

$$\begin{cases} \dot{\delta}_x(t) = \dot{\psi}(t)\delta_x(t) + \psi(t)\delta_v(t), \\ \dot{\delta}_v(t) = F\left(t, \delta_x(t), \delta_v(t)\right) - \psi(t)\left(\tilde{L} \otimes I_N\right)\left(k_1\delta_x(t) + k_2\delta_v(t) + \hat{e}(t)\right). \end{cases} \tag{16}$$

The sufficient condition for achieving prescribed-time bipartite consensus in MASs described by (1) is provided in the following theorem.

Theorem 1. Suppose that Assumptions 1-2 hold. Given a prescribed time T, the second-order nonlinear MASs described by (1) achieve bipartite consensus within T under the protocol (10), provided that there exist parameters $h \geq 0$, $\beta > 0$, $\gamma > 0$, and $l > 0$ satisfying the following conditions

$$2\beta^2 - \gamma\left(k_1\gamma + k_2\beta\right)\lambda_{\min}(\Gamma)w_{\min} < 0, \tag{17}$$

$$\begin{aligned} &\Lambda_1\lambda_{\min}(\Gamma)w_{\min} + \frac{\beta l}{h} + \frac{T}{h}\Lambda_2 < 0, \\ &\Lambda_4\lambda_{\min}(\Gamma)w_{\min} + \beta + \frac{\beta}{4lh} + \frac{T}{h}\Lambda_5 < 0, \end{aligned} \tag{18}$$

where

$$\begin{aligned} \Lambda_1 &= \frac{k_1\gamma + k_2\beta}{2h} - \frac{k_1\beta}{2} + \beta l + \frac{\beta + \gamma}{4l}\theta^2 k_1^2, \\ \Lambda_2 &= \frac{T}{h}\beta c_x + \Lambda_3, \\ \Lambda_3 &= \frac{T\gamma l c_x + h\beta l c_v}{h}, \\ \Lambda_4 &= -\frac{k_2\gamma}{2} + \gamma l + \frac{(\beta + \gamma)\theta^2 k_2^2}{4l}, \\ \Lambda_5 &= \gamma c_v + \frac{\Lambda_3}{4l^2}. \end{aligned}$$

Proof. Let $\xi = \left[\delta_x^T, \delta_v^T\right]^T$. The Lyapunov function is taken as

$$V(t) = \frac{1}{2}\xi^T\left(\Omega \otimes I_N\right)\xi, \tag{19}$$

where $\Omega = \begin{bmatrix} \frac{k_1\gamma + k_2\beta}{2}\Gamma & \beta W \\ \beta W & \gamma W \end{bmatrix}$, and W is a positive definite matrix defined in Lemma 3.

First of all, we need to ensure the validity of $V(t)$. According to Schur's Complement Lemma, $\Omega > 0$ if the followings hold: (1) $\gamma W > 0$, (2) $\frac{1}{2}\left(k_1\gamma + k_2\beta\right)\Gamma\gamma W - \beta W\beta W > 0$, (1) of them is obviously true. Moreover, if the inequality in condition (17) is satisfied, then (2) holds. Thus, $V(t)$ is valid under the constraint (18).

Next, we prove that the bipartite consensus for MASs (1) can be achieved in the prescribed time T. Consider the time interval $[t_0, t_0 + T)$. In this case $\dot{\psi}(t) = \psi(t)/h$. Calculating the derivative of V given in (19) along (16), one has

$$\begin{aligned} \dot{V}(t) = &\frac{k_1\gamma + k_2\beta}{2h}\psi(t)\delta_x^T\left(\Gamma \otimes I_N\right)\delta_x + \beta\psi(t)\delta_v^T\left(W \otimes I_N\right)\delta_v \\ &+ \frac{k_1\gamma + k_2\beta}{2}\psi(t)\delta_x^T\left(\Gamma \otimes I_N\right)\delta_v + \frac{\beta}{h}\psi(t)\delta_v^T\left(W \otimes I_N\right)\delta_x \\ &- \psi(t)\left(\beta\delta_x^T + \gamma\delta_v^T\right)\left(W \otimes I_N\right)\left(\tilde{L} \otimes I_N\right)\left(k_1\delta_x + k_2\delta_v + e(t)\right) \\ &+ \left(\beta\delta_x^T + \gamma\delta_v^T\right)\left(W \otimes I_N\right)F\left(t, \delta_x(t), \delta_v(t)\right). \end{aligned} \tag{20}$$

Based on Assumption 1, Lemma 2 and the fact $\frac{1}{\psi(t)} \le \frac{T}{h}$, one has

$$\begin{aligned}
&\left(\beta\delta_x^T + \gamma\delta_v^T\right)\left(W \otimes I_N\right) F\left(t, \delta_x(t), \delta_v(t)\right) \\
&= \sum_{i=1}^{N} \frac{1}{w_i}\left(\beta\delta_x^T + \gamma\delta_v^T\right)\left[f\left(t, \hat{x}_i(t), \hat{v}_i(t)\right) - f\left(t, \phi_x(t), \phi_v(t)\right)\right] \\
&\le \sum_{i=1}^{N} \frac{1}{w_i}\left[\frac{\beta c_x}{\psi(t)}\left\|\delta_x\right\|^2 + \gamma c_v\left\|\delta_v\right\|^2 + \left(\frac{\gamma c_v}{\psi(t)} + \beta c_v\right)\left\|\delta_x\right\|\left\|\delta_v\right\| \right] \\
&\le \left(\frac{T}{h}\beta c_x + \Lambda_3\right)\delta_x^T\left(W \otimes I_N\right)\delta_x + \left(\gamma c_v + \frac{\Lambda_3}{4l^2}\right)\delta_v^T\left(W \otimes I_N\right)\delta_v.
\end{aligned} \tag{21}$$

By simple calculations, we obtain

$$\begin{aligned}
&\left(\beta\delta_x^T + \gamma\delta_v^T\right)\left(W \otimes I_N\right)\left(\tilde{L} \otimes I_N\right)\left(k_1\delta_x + k_2\delta_v\right) \\
&= k_1\beta\delta_x^T\left[\frac{1}{2}\left(W\tilde{L} + \tilde{L}^T W\right) \otimes I_N\right]\delta_x + k_2\gamma\delta_v^T\left[\frac{1}{2}\left(W\tilde{L} + \tilde{L}^T W\right) \otimes I_N\right]\delta_v \\
&+ \frac{k_1\gamma + k_2\beta}{2}\delta_x^T\left[\frac{1}{2}\left(W\tilde{L} + \tilde{L}^T W\right) \otimes I_N\right]\delta_v \\
&= \frac{k_1\beta}{2}\delta_x^T\left(\Gamma \otimes I_N\right)\delta_x + \frac{k_2\gamma}{2}\delta_v^T\left(\Gamma \otimes I_N\right)\delta_v + \frac{k_1\gamma + k_2\beta}{2}\delta_x^T\left(\Gamma \otimes I_N\right)\delta_v,
\end{aligned} \tag{22}$$

and

$$\begin{aligned}
\delta_x^T\left(\Gamma \otimes I_N\right)\delta_x &\ge \lambda_{\min}(\Gamma)\sum_{i=1}^{N}\left\|\delta_{xi}\right\|^2 \\
&\ge \lambda_{\min}(\Gamma) w_{\min}\sum_{i=1}^{N}\frac{1}{w_i}\left\|\delta_{xi}\right\|^2 \\
&= \lambda_{\min}(\Gamma) w_{\min}\delta_x^T\left(W \otimes I_N\right)\delta_x.
\end{aligned} \tag{23}$$

Similarly, one gets

$$\delta_v^T\left(\Gamma \otimes I_N\right)\delta_v \ge \lambda_{\min}(\Gamma) w_{\min}\delta_v^T\left(W \otimes I_N\right)\delta_v, \tag{24}$$

with the help of Lemma 1, we get

$$\begin{aligned}
\delta_v^T\left(W \otimes I_N\right)\delta_x &= \sum_{i=1}^{N}\frac{1}{w_i}\delta_{vi}^T\delta_{xi} \\
&\le \sum_{i=1}^{N}\frac{1}{w_i}\left(l\left\|\delta_{xi}\right\|^2 + \frac{1}{4l}\left\|\delta_{vi}\right\|^2\right) \\
&= l\delta_x^T\left(W \otimes I_N\right)\delta_x + \frac{1}{4l}\delta_v^T\left(W \otimes I_N\right)\delta_v.
\end{aligned} \tag{25}$$

From the trigger condition (13), one gets

$$\begin{aligned}
e^T\left(\Gamma \otimes I_N\right)e &\le \lambda_{\max}(\Gamma) e^T e \\
&\le 2\lambda_{\max}(\Gamma)\left[\alpha^2\left(k_1^2\left\|\delta_x\right\|^2 + k_2^2\left\|\delta_v\right\|^2\right) + b^2\mu^{-4c}(t)\right] \\
&\le 2\frac{\lambda_{\max}(\Gamma)a^2}{\lambda_{\min}(\Gamma)}\left[k_1^2\delta_x^T\left(\Gamma \otimes I_N\right)\delta_x + k_2^2\delta_v^T\left(\Gamma \otimes I_N\right)\delta_v + b^2\mu^{-4c}(t)\right] \\
&\le \theta^2 k_1^2\delta_x^T\left(\Gamma \otimes I_N\right)\delta_x + \theta^2 k_2^2\delta_v^T\left(\Gamma \otimes I_N\right)\delta_v + \theta^2 b^2\mu^{-4c}(t).
\end{aligned} \tag{26}$$

In this way, one can obtain

$$
\begin{aligned}
&-\left(\beta\delta_x^T+\gamma\delta_v^T\right)\left(\Gamma\otimes I_N\right)e(t)\\
&=-\beta\delta_x^T\left(\Gamma\otimes I_N\right)e(t)-\gamma\delta_v^T\left(\Gamma\otimes I_N\right)e(t)\\
&\le \beta l\delta_x^T\left(\Gamma\otimes I_N\right)\delta_x+\gamma l\delta_v^T\left(\Gamma\otimes I_N\right)\delta_v+\frac{\beta+\gamma}{4l}e^T(t)\left(\Gamma\otimes I_N\right)e(t)\\
&\le\left(\beta l+\frac{(\beta+\gamma)\theta^2k_1^2}{4l}\right)\delta_x^T\left(\Gamma\otimes I_N\right)\delta_x+\left(\gamma l+\frac{(\beta+\gamma)\theta^2k_2^2}{4l}\right)\delta_v^T\left(\Gamma\otimes I_N\right)\delta_v+\theta^2b^2\mu^{-4c}(t).
\end{aligned}
\tag{27}
$$

Combining (20)-(27) results in

$$
\begin{aligned}
\dot V(t)&\le\xi^T(t)\left(R\otimes I_N\right)\xi(t)+\theta^2b^2\mu^{\frac{1}{h}-4c}(t)\\
&\le\lambda_{\min}(R)\xi^T(t)\xi(t)+\theta^2b^2\psi(t)\mu^{\frac{1}{h}-4c}(t),
\end{aligned}
$$

where $R=\begin{bmatrix}\Lambda_1W & 0\\ 0 & \Lambda_4W\end{bmatrix}$, Then, according to Lemma 4, it can be deduced that

$$
V(t)<\left[V(t_0)+\frac{\theta^2b^2T}{h\left(-\frac{\lambda_{\min}(S)}{\lambda_{\max}(\Omega)}+4c-\frac{1}{h}\right)-1}\right]\mu^{\frac{2}{h}-4c}(t).
$$

Above all, we have

$$
\lambda_{\min}(\Omega)\xi^T(t)\xi(t)\le V(t)\le\left[V(t_0)+\frac{\theta^2b^2T}{h\left(-\frac{\lambda_{\min}(S)}{\lambda_{\max}(\Omega)}+4c-\frac{1}{h}\right)-1}\right]\mu^{\frac{2}{h}-4c}(t).
\tag{28}
$$

Hence

$$
\|\xi(t)\|\le\sqrt{\frac{1}{\lambda_{\min}(\Omega)}\left[V(t_0)+\frac{\theta^2b^2T}{h\left(-\frac{\lambda_{\min}(S)}{\lambda_{\max}(\Omega)}+4c-\frac{1}{h}\right)-1}\right]}\mu^{\frac{1}{h}-2c}(t).
\tag{29}
$$

Therefore, $\|\xi(t)\|\to 0$ when $t\to t_0+T$ and $\|\xi(t)\|\equiv 0$ when $t\in[t_0+T,\infty)$, that is $\lim_{t\to t_0+T}\|x_i(t)-\sigma_i x_j(t)\|=0$ and $\lim_{t\to t_0+T}\|x_i(t)-\sigma_i x_j(t)\|=0,(i=1,2,...,N)$, so the proof is completed.

3.3 ZENO BEHAVIOR ANALYSIS

Theorem 2. Using event-triggered function (13) and control protocol (10) to achieve bipartite consensus of system (1), the Zeno behavior is excluded.

Proof.

$$
\begin{aligned}
D^+\left(\|\hat e(t)\|\right)&=\left\|\dot{\hat e}(t)\right\|=\left\|-\dot\eta(t)\right\|\\
&=\left\|k_1\dot\delta_x(t)+k_2\dot\delta_v(t)\right\|\\
&\le\left\|\left(k_1\dot\psi(t)+k_2c_x\right)\delta_x(t)\right\|+\left\|\left(k_1\psi(t)+k_2c_v\right)\delta_v(t)\right\|+\left\|\Xi(t_k^i)\right\|.
\end{aligned}
$$

Let $\Delta=\max\left\{\left(k_1\dot\psi(t)+k_2c_x\right),\left(k_1\psi(t)+k_2c_v\right)\right\}$, then

$$
\begin{aligned}
\left\|\dot{\hat e}(t)\right\|&\le\Delta\left(\|\delta_x(t)\|+\|\delta_v(t)\|\right)+\left\|\Xi(t_k^i)\right\|\\
&\le 2\Delta\|\xi(t)\|+\left\|\Xi(t_k^i)\right\|.
\end{aligned}
$$

According to (17), we know that

$$\|\xi(t)\| \leq \sqrt{\frac{1}{\lambda_{\min}(\Omega)}\left[V(t_0)+\frac{\theta^2 b^2 T}{h\left(-\frac{\lambda_{\min}(S)}{\lambda_{\max}(\Omega)}+4c-\frac{1}{h}\right)-1}\right]}\mu^{\frac{1}{h}-2c}(t). \tag{30}$$

Hence

$$\begin{aligned} D^{+}\left(\|\hat{e}(t)\|\right) &= \|\dot{\hat{e}}(t)\| \\ &\leq 2\Delta\sqrt{\frac{1}{\lambda_{\min}(\Omega)}\left[V(t_0)+\frac{\theta^2 b^2 T}{h\left(-\frac{\lambda_{\min}(S)}{\lambda_{\max}(\Omega)}+4c-\frac{1}{h}\right)-1}\right]}\mu^{\frac{1}{h}-2c}(t)+\left\|\Xi\left(t_k^i\right)\right\|. \end{aligned} \tag{31}$$

It follows from $\hat{e}_i\left(t_k^i\right)=0$ that

$$\begin{aligned} \|\hat{e}_i(t)\| &\leq \int_{t_k^i}^{t} 2\Delta\sqrt{\frac{1}{\lambda_{\min}(\Omega)}\left[V(t_0)+\frac{\theta^2 b^2 T}{h\left(-\frac{\lambda_{\min}(S)}{\lambda_{\max}(\Omega)}+4c-\frac{1}{h}\right)-1}\right]}\mu^{\frac{1}{h}-2c}(\tau)\,d\tau \\ &\quad+\int_{t_k^i}^{t}\left\|\Xi\left(t_k^i\right)\right\|(\tau)\,d\tau \\ &\leq -\frac{2\Delta\sqrt{\frac{1}{\lambda_{\min}(\Omega)}\left[V(t_0)+\frac{\theta^2 b^2 T}{h\left(-\frac{\lambda_{\min}(S)}{\lambda_{\max}(\Omega)}+4c-\frac{1}{h}\right)-1}\right]}\left(T+t_0\right)^{1-2hc}}{2hb} \\ &\quad\times\left(\left(T+t_0-t\right)^{2hc}-\left(T+t_0-t_k^i\right)^{2hc}\right)+\left\|\Xi\left(t_k^i\right)\right\|\left(t-t_k^i\right). \end{aligned} \tag{32}$$

At trigger instant t_{k+1}^i, it follows

$$\begin{aligned} \left\|e_i\left(t_{k+1}^i\right)\right\| &\leq -\frac{2\Delta\sqrt{\frac{1}{\lambda_{\min}(\Omega)}\left[V(t_0)+\frac{\theta^2 b^2 T}{h\left(-\frac{\lambda_{\min}(S)}{\lambda_{\max}(\Omega)}+4c-\frac{1}{h}\right)-1}\right]}\left(T+t_0\right)^{1-2hc}}{2hc} \\ &\quad\times\left(\left(T+t_0-t_{k+1}^i\right)^{2hc}-\left(T+t_0-t_k^i\right)^{2hc}\right)+\left\|\Xi\left(t_k^i\right)\right\|\left(t_{k+1}^i-t_k^i\right). \end{aligned} \tag{33}$$

According to (13), at trigger instant t_{k+1}^i,

$$\left\|e_i\left(t_{k+1}^i\right)\right\| \geq a\left\|\eta_i\left(t_{k+1}^i\right)\right\|+b\mu^{-2c}\left(t_{k+1}^i\right). \tag{34}$$

Combined with (33) and (34), it follows

$$\begin{aligned} a\mu^{-2b}\left(t_{k+1}^i\right) &\leq -\frac{2\Delta\sqrt{\frac{1}{\lambda_{\min}(\Omega)}\left[V(t_0)+\frac{\theta^2 b^2 T}{h\left(-\frac{\lambda_{\min}(S)}{\lambda_{\max}(\Omega)}+4c-\frac{1}{h}\right)-1}\right]}\left(T+t_0\right)^{1-2hc}}{2hc} \\ &\quad\times\left(\left(T+t_0-t_{k+1}^i\right)^{2hc}-\left(T+t_0-t_k^i\right)^{2hc}\right)+\left\|\Xi\left(t_k^i\right)\right\|\left(t_{k+1}^i-t_k^i\right). \end{aligned} \tag{35}$$

Next, by using reduction to absurdity, it will be proved that the trigger interval $t_{k+1}^i - t_k^i \neq 0$. Assume that $t_{k+1}^i - t_k^i = 0$. Then, from (35), it follows that $a = 0$, which leads to a contradiction. Therefore, the hypothesis is not valid, which means that Zeno behavior is excluded. This completes the proof.

IV. NUMERICAL RESULTS

In this section, examples are provided to demonstrate the theoretical results. The network topology is assumed as in Figure 1, apparently, the network is structurally balanced with $V_1 = \{1,2,3\}$ and $V_2 = \{3,4,5\}$.

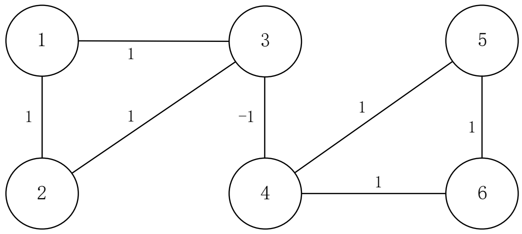

Figure 1: The communication topology.

Consider MASs with the topological relationship of six agents shown in Figure 1. The dynamics are described by **Error! Reference source not found.**. We set $k_1 = 3$, $k_2 = 4$, $h = 2$, and $f(t, x_i(t), v_i(t)) = 0.2\sin(x_i(t))v_i(t)$. Substitute the above parameters into the MASs **Error! Reference source not found.**, the control protocol **Error! Reference source not found.**, and the time scale function (3). Moreover, we set the initial position states $x(0) = [5,\ -2,\ 4,\ 1,\ -5,\ 3]$, and the initial velocity states $v(0) = [-1,\ 2,\ -4,\ 4,\ 5,\ -6]$ for the protocol in **Error! Reference source not found.** to achieve of control objectives.

Setting the parameter $a = 0.51$, $b = [0.7,\ 0.6,\ 0.5,\ 0.5,\ 0.7,\ 0.7]$ and $c = 0.5$ in the event-triggering function, the event-triggered function becomes

$$h_i(t) = \|e_i(t)\| - 0.5\|\eta_i(t)\| - b\mu^{-1}(t) \quad i = 1,2,3,\ldots,6.$$

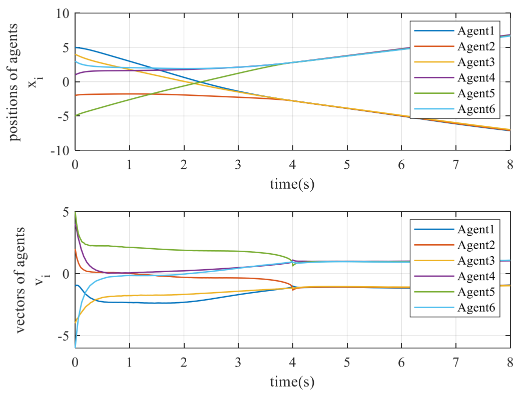

Figure 2: State trajectories of each agent (T=4s).

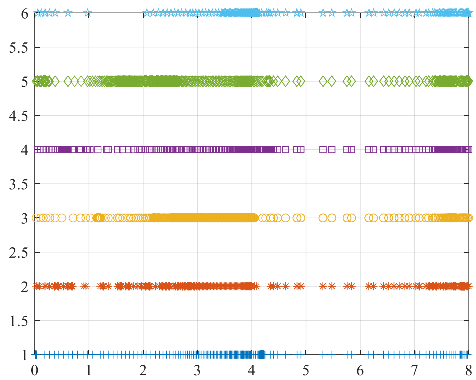

Figure 3: Interevent intervals of agents (T=4s).

Under the above-mentioned initial conditions, the state trajectories of the MASs for $T = 4s$ and $T = 5s$ are shown in Figure 2 and Figure 4, respectively. From Figure 2, it can be seen that the MASs **Error! Reference source not found.** achieve bipartite consensus within $T = 4s$, and the results in Figure 4 show that bipartite consensus can also be achieved within $T = 5s$. The simulation step size set in this paper is 0.01, and the total simulation time is 8s, so the total number of iterations is 800. Furthermore, by comparing the results in Figure 2 and Figure 4, it can be concluded that the designed protocol is able to drive the MASs to achieve bipartite consensus within a prescribed time regardless of the initial state of the MASs.

Figure 3 and Figure 5 show the inter-event intervals of each agent, i.e., the number of triggers for 6 agents. During the simulation time of 8s, the event-triggered mechanism greatly reduces the communication burden and excludes Zeno behavior.

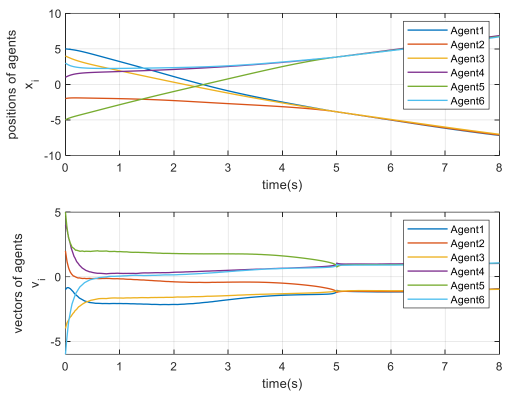

Figure 4: State trajectories of each agent (T=5s).

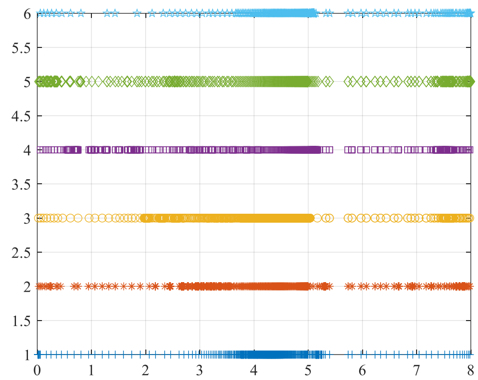

Figure 5: Interevent intervals of agents (T=5s).

V. CONCLUSION

This paper investigated the event-triggered prescribed-time bipartite consensus control problem of nonlinear MASs. A novel event-triggered mechanism was designed for each agent, which requires sampled states of both the agent and its neighbors. Based on the designed event-triggered mechanism, a prescribed-time control protocol was proposed to achieve bipartite consensus within the prescribed time. The stability of the nonlinear MASs was analyzed in detail, and the corresponding sufficient conditions were established. Additionally, it was proved that Zeno phenomenon can be excluded. Numerical simulations results have been provided to confirm and illustrate the theoretical results.

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REFERENCES

[1] W. Liu, S. Zhou, Y. Qi, and X. Wu, “Leaderless consensus of multi-agent systems with lipschitz nonlinear dynamics and switching topologies,” *Neurocomputing*, vol. 173, pp. 1322–1329, Jan. 2016.

[2] M. Lu, J. Wu, X. Zhan, T. Han, and H. Yan, “Consensus of second-order heterogeneous multi-agent systems with and without input saturation,” *ISA Transactions*, vol. 126, pp. 14–20, Jul. 2022.

[3] H. Geng, H. Wu, J. Miao, S. Hou, and Z. Chen, “Consensus of heterogeneous multi-agent systems under directed topology,” *IEEE Access*, vol. 10, pp. 5936–5943, 2022.

[4] C. Gao, D. Zhao, J. Li, and H. Lin, “Private bipartite consensus control for multi-agent systems: A hierarchical differential privacy scheme,” *Information Fusion*, vol. 105, p. 102259, May 2024.

[5] G. Wen, H. Wang, X. Yu, and W. Yu, “Bipartite tracking consensus of linear multi-agent systems with a dynamic leader,” *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 65, no. 9, pp. 1204–1208, Sep. 2018.

[6] H. Xu, C. Liu, Y. Lv, and J. Zhou, “Adaptive bipartite consensus of second-order multi-agent systems with bounded disturbances,” *IEEE Access*, vol. 8, pp. 186441–186447, 2020.

[7] J. Li, X. Yang, and Y. Li, “Distributed global adaptive bipartite consensus of multi-agent systems with signed communication topology structure,” *Engineering Applications of Artificial Intelligence*, vol. 124, p. 106514, Sep. 2023.

[8] C. Altafini, “Consensus problems on networks with antagonistic interactions,” *IEEE Transactions on Automatic Control*, vol. 58, no. 4, pp. 935–946, Apr. 2013.

[9] K. Li, C. Hua, X. You, and C. K. Ahn, “Output feedback predefined-time bipartite consensus control for high-order nonlinear multiagent systems,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 68, no. 7, pp. 3069–3078, Jul. 2021.

[10] M. Zhao, C. Peng, and E. Tian, “Finite-time and fixed-time bipartite consensus tracking of multi-agent systems with weighted antagonistic interactions,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 68, no. 1, pp. 426–433, Jan. 2021.

[11] S. Miao and H. Su, “Bipartite consensus for second-order multiagent systems with matrix-weighted signed network,” *IEEE Transactions on Cybernetics*, vol. 52, no. 12, pp. 13038–13047, Dec. 2022.

[12] J. Ke, J. Zeng, and Z. Duan, “Observer-based prescribed-time consensus control for heterogeneous multi-agent systems under directed graphs,” *International Journal of Robust and Nonlinear Control*, vol. 33, no. 2, pp. 872–898, 2023.

[13] W. Zou, P. Shi, Z. Xiang, and Y. Shi, “Finite-time consensus of second-order switched nonlinear multi-agent systems,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 31, no. 5, pp. 1757–1762, May 2020.

[14] P. Li, X. Wu, X. Chen, and J. Qiu, “Distributed adaptive finite-time tracking for multi-agent systems and its application,” *Neurocomputing*, vol. 481, pp. 46–54, Apr. 2022.

[15] Y. Yin, F. Wang, Z. Liu, and Z. Chen, “Finite-time leader-following consensus of multiagent systems with actuator faults and input saturation,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 52, no. 5, pp. 3314–3325, May 2022.

[16] Q. Wang, Y. Hua, X. Dong, P. Shu, J. Lü, and Z. Ren, “Finite-time time-varying formation tracking for heterogeneous nonlinear multiagent systems using adaptive output regulation,” *IEEE Transactions on Cybernetics*, vol. 54, no. 4, pp. 2460–2471, Apr. 2024.

[17] A. Polyakov, “Nonlinear feedback design for fixed-time stabilization of linear control systems,” *IEEE Transactions on Automatic Control*, vol. 57, no. 8, pp. 2106–2110, Aug. 2012.

[18] C. Wang, C.-L. Liu, and F. Liu, “Fixed-time consensus tracking of heterogeneous multi-agent systems,” in *2019 Chinese Automation Congress (CAC)*, Hangzhou, China: IEEE, Nov. 2019, pp. 984–989. Accessed: Mar. 19, 2025. [Online]. Available: https://ieeexplore.ieee.org/document/8997210/

[19] W. Zou, K. Qian, and Z. Xiang, "Fixed-Time Consensus for a Class of Heterogeneous Nonlinear Multiagent Systems," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 67, no. 7, pp. 1279–1283, Jul. 2020.
[20] W. Mi, L. Luo, and S. Zhong, "Fixed-time consensus tracking for multi-agent systems with a nonholomonic dynamics," *IEEE Transactions on Automatic Control*, vol. 68, no. 2, pp. 1161–1168, Feb. 2023.
[21] X. Gong, Y. Cui, J. Shen, Z. Shu, and T. Huang, "Distributed Prescribed-Time Interval Bipartite Consensus of Multi-Agent Systems on Directed Graphs: Theory and Experiment," *IEEE Transactions on Network Science and Engineering*, vol. 8, no. 1, pp. 613–624, Jan. 2021.
[22] Y. Ren, W. Zhou, Z. Li, L. Liu, and Y. Sun, "Prescribed-time cluster lag consensus control for second-order non-linear leader-following multiagent systems," *ISA Transactions*, vol. 109, pp. 49–60, Mar. 2021.
[23] K. Zhang, B. Zhou, X. Yang, and G. Duan, "Prescribed-time leader-following consensus of linear multi-agent systems by bounded linear time-varying protocols," *Science China Information Sciences*, vol. 67, no. 1, p. 112201, Jan. 2024.
[24] Z. Zhang and Z. Fu, "Prescribed-time leader-follower consensus of multi-agent systems by event-triggered linear time-varying control," 2025.
[25] C. Chen, Y. Han, S. Zhu, and Z. Zeng, "Prescribed-time cooperative output regulation of heterogeneous multiagent systems," *IEEE Transactions on Industrial Informatics*, vol. 20, no. 2, pp. 2432–2443, Feb. 2024.
[26] M. Zhao, C. Peng, W. He, and Y. Song, "Event-Triggered Communication for Leader-Following Consensus of Second-Order Multiagent Systems," *IEEE Transactions on Cybernetics*, vol. 48, no. 6, pp. 1888–1897, Jun. 2018.
[27] Y. Zhou and G. Zhang, "Distributed finite-time lag-consensus for second-order nonlinear multi-agent systems with disturbances," in *2018 13th World Congress on Intelligent Control and Automation (WCICA)*, Changsha, China: IEEE, Jul. 2018, pp. 1594–1599. Accessed: Mar. 19, 2025. [Online]. Available: https://ieeexplore.ieee.org/document/8630616/
[28] X. Jin, "Nonrepetitive leader–follower formation tracking for multiagent systems with LOS range and angle constraints using iterative learning control," *IEEE Transactions on Cybernetics*, vol. 49, no. 5, pp. 1748–1758, May 2019.
[29] J. Sun, J. Zhang, H. Zhang, and R. Zhang, "Adaptive event-triggered control approach to the cooperative output regulation of heterogeneous multiagent systems under digraphs," *IEEE Transactions on Cybernetics*, vol. 53, no. 5, pp. 3388–3395, May 2023.
[30] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 5, pp. 1291–1297, May 2012.
[31] M. Zhao, C. Peng, W. He, and Y. Song, "Event-Triggered Communication for Leader-Following Consensus of Second-Order Multiagent Systems," *IEEE Transactions on Cybernetics*, vol. 48, no. 6, pp. 1888–1897, Jun. 2018.
[32] B. Cheng, X. Wang, and Z. Li, "Event-triggered consensus of homogeneous and heterogeneous multiagent systems with jointly connected switching topologies," *IEEE Transactions on Cybernetics*, vol. 52, no. 7, pp. 7225–7225, Jul. 2022.
[33] K. Dong, G.-H. Yang, and H. Wang, "Estimator-based event-triggered output synchronization for heterogeneous multi-agent systems under denial-of-service attacks and actuator faults," *Information Sciences*, vol. 657, p. 119955, Feb. 2024.
[34] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.
[35] Y. Zhou and G. Zhang, "Distributed finite-time lag-consensus for second-order nonlinear multi-agent systems with disturbances," in *2018 13th World Congress on Intelligent Control and Automation (WCICA)*, Changsha, China: IEEE, Jul. 2018, pp. 1594–1599. Accessed: Jul. 03, 2024. [Online]. Available: https://ieeexplore.ieee.org/document/8630616/
[36] Z. Qu, J. Wang, and R. A. Hull, "Cooperative control of dynamical systems with application to autonomous vehicles," *IEEE Transactions on Automatic Control*, vol. 53, no. 4, pp. 894–911, May 2008.
[37] X. Chen, H. Yu, and F. Hao, "Prescribed-time event-triggered bipartite consensus of multiagent systems," *IEEE Transactions on Cybernetics*, vol. 52, no. 4, pp. 2589–2598, Apr. 2022.
[38] X. Chen, H. Yu, and F. Hao, "Prescribed-Time Event-Triggered Bipartite Consensus of Multiagent Systems," *IEEE Transactions on Cybernetics*, vol. 52, no. 4, pp. 2589–2598, Apr. 2022.