Upper Mantle Conductivity determined from the Solar Quiet Day Ionospheric Currents in the Equatorial and Low Latitudes of Africa

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Abstract
The mantle electrical conductivity-depth profile of equatorial and low latitudes of Africa have been determined using solar quiet day ionospheric current (Sq). The magnetometer data obtained for 2008 from geomagnetic stations installed across Africa by magnetic data acquisition set (MAGDAS) were employed in this study. Ilorin (ILR) was used to represent the equatorial stations while Hermanus (HER) was used to represent the low latitude stations. Gauss spherical harmonic analysis (SHA) method was used to separate the internal and external field contributions to Sq current system. The result depicted that the conductivity in both the equatorial and low latitudes of Africa had a downward increase with a high conductivity region spotted between 100 km and 200 km. This high conductivity region agreed with the global seismic low velocity region, the asthenosphere. The electrical conductivity depth profiles of the low latitude of Africa have a sharp characteristics rise in electrical conductivity towards 400 to 600km depth. A peak was observed from 448.71km with average value 0.301 sm$^{-1}$. This characteristics was not conspicuous in the low latitude of Africa. The conductivity at the upper mantle obtained in the equatorial latitude of Africa is seen to be 2.98 times higher than that obtained both in the low latitude of Africa.

I. INTRODUCTION

A fluctuating electric current flowing in the Earth’s atmosphere causes corresponding electric currents to flow in the conducting Earth below the source current. The magnitude, direction, and depth of penetration of the induced currents are determined by the characteristics of the source currents as well as the distribution of electrically conducting materials in the Earth. At the Earth’s surface observatories, magnetometers measure the composite of external (source) and internal (induced) field components from the currents. Separating these currents into their individual parts using Spherical Harmonic Analysis (SHA) or other integral methods, the amplitudes and phase relationships were shown to be useful in determining the conductivity of the deep earth [1]. The depth of penetration of the induced current to the deep earth depends on the period of variation of the source current and the distribution of electrically conducting materials in the region of the earth begin investigated [2].

[3] used selected geomagnetic field records to establish the 1990 quiet-day current system (Sq) for Australia. They also determined the Earth’s deep electrical conductivity using the sq current system. [4] used magnetic data obtained from a chain of ten magnetotelluric stations installed in the African sector during the international equatorial electojet year (IEEY) to establish the 1993 quiet day current system (Sq) for West Africa and to determine the Earth’s upper mantle electrical conductivity in the region.

The aim of this work is to separate the quiet-day field variations obtained in the mid and high equitorial region of Europe into their external and internal field contributions and then to use the paired external and internal coefficient of the SHA to determine the earth’s upper mantle conductivity for the region.

Data Source
The average hourly geomagnetic data used in this study were obtained from geomagnetic stations established in parts of the region (Ilorin (8.5°N, 4.68°E), Lagos (6.4°N, 3.27°E), Addis Ababa (9.04°N, 38.77°E) and Hermanus (34.34°S, 19.24°E)) by magnetic data acquisition set (MAGDAS). Japan for the year 2008.
Upper Mantle Conductivity determined from the Solar Quiet Day Ionospheric Currents in the...

II. METHOD OF ANALYSIS

The method employed in this work involves the Spherical Harmonic Analysis (SHA) devised by [5] in solving the magnetic potential function $V$. It was [6] who showed that the potential has two parts: the external (source) and internal (induced) parts of the potential function. He expressed the magnetic potential of the Sq field, $V$ measured from the daily mean values at universal time, $T$ comprises of both the internal (induced) current and the external source current as a sum of spherical harmonics as

$$V_n^m = C + a \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \left( \frac{m e_n}{a_n} \left( \frac{r}{a} \right)^n + \frac{m i_n}{a_n} \left( \frac{r}{a} \right)^{n+1} \right) \cos(m \phi) + \left( \frac{m e_m}{b_m} \left( \frac{r}{a} \right)^n + \frac{m i_m}{b_m} \left( \frac{r}{a} \right)^{n+1} \right) \sin(m \phi) \right) \int_{\phi}^{\theta} p_n(\phi) \, d\phi \tag{1}$$

Where $C$, $\theta$, $a$, $r$ and $\phi$ denote a constant of integration, the geomagnetic colatitude, the earth’s radius and the local time of the observatory respectively. $a_n^m, b_n^m, a_n^{mi}, b_n^{mi}$ are Legendre polynomial coefficients, $e$ and $i$ represent the external and internal values, respectively. $p_n^m$ are Legendre polynomials and are functions of colatitude $\theta$ only. The integers, $n$ and $m$ are called degree and order respectively. Following [7], the equivalent current function, $J(\phi)$ in Ampere’s for an hour of the day, $\phi/15$ (the longitude divided by $15^\circ$) is obtained from:

$$J = \sum_{m=1}^{4} \sum_{n=1}^{12} \{ U_n^m \cos(m \phi) + V_n^m \sin(m \phi) \} P_n^m \tag{2}$$

With 4 for the maximum value of $m$, and 12 for the maximum value of $n$. For the external current representation, we have:

$$U_n^m = - \left( \frac{5}{2} \right) \left( \frac{n}{n+1} \right)^{n+1} a_n^m \left( \frac{R}{a} \right)^n \tag{3}$$

$$V_n^m = - \left( \frac{5}{2} \right) \left( \frac{n}{n+1} \right)^{n+1} a_n^{mi} \left( \frac{R}{a} \right)^n \tag{4}$$

And the internal current representation, we have:

$$U_n^m = \left( \frac{5}{2} \right) \left( \frac{n}{n+1} \right)^{n+1} b_n^m \left( \frac{R}{a} \right)^n \tag{5}$$

$$V_n^m = \left( \frac{5}{2} \right) \left( \frac{n}{n+1} \right)^{n+1} b_n^{mi} \left( \frac{R}{a} \right)^n \tag{6}$$

Where, $R$ is the radius of the Earth in kilometers.

The value of $a$ is the radius of a sphere whose surface is located where a current could flow to give the fields described at the Earth’s surface by the SHA, hence the name “Equivalent Current”. It is believed that the dynamo current sources is in the ionospheric E-region (near 100km altitude). Because there is other evidence that the dynamo current source is in the E-region ionosphere, near 100km altitude, the value of $a \approx R$ and the ratio $\left( \frac{R}{a} \right)$ may be omitted from the current computations [8].

The transfer equations necessary for obtaining conductivity versus depth profile from the separated external and internal SHA is given in [9] as:

$$C_n^m = z - ip \tag{7}$$

a complex number in which the real ($z$) and imaginary ($-ip$) parts are given by:
Upper Mantle Conductivity determined from the Solar Quiet Day Ionospheric Currents in the ..

\[ z = \frac{R}{n(n+1)} \left( A_n^m \left[ na_n^{me} - (n+1)a_n^{mi} \right] + b_n^m \left[ nb_n^{me} - (n+1)b_n^{mi} \right] \right) \frac{(A_n^m)^2 + (B_n^m)^2}{(A_n^m)^2 + (B_n^m)^2} \] (8)

\[ p = \frac{R}{n(n+1)} \left( A_n^m \left[ nb_n^{me} - (n+1)b_n^{mi} \right] - B_n^m \left[ na_n^{me} - (n+1)a_n^{mi} \right] \right) \frac{(A_n^m)^2 + (B_n^m)^2}{(A_n^m)^2 + (B_n^m)^2} \] (9)

Where \( R \) is the Earth’s radius in km, \( z \) and \( p \) are given in km and the coefficient sums are also given by:

\[ a_n^{me} + a_n^{mi} = A_n^m \quad \text{and} \quad b_n^{me} + b_n^{mi} = B_n^m \] (10)

For each \( n, m \) sets of coefficients the depth in km to the uniform substitute layer is given by:

\[ d_n^m = z - p \] (11)

And a substitute layer conductivity in Sm\(^{-1}\) given by:

\[ \sigma_n^m = 5.4 \times 10^4 \frac{m}{(\pi p)^2} \] (12)

The data processing followed the steps shown below in Fig. 2.

Select Magnetically Quiet days

Obtain field record of \( H, D, Z \) and their monthly mean values for five selected quietest days.

Carry out a spectral (Fourier) analysis for each component and obtain spectral analysis (4 harmonics) of each Fourier (sine and cosine) coefficients.

Carry out spherical harmonic analysis with degree \( n = 12 \), order \( m = 4 \) obtaining the Legendre Polynomial external and internal coefficients.

Determine conductivity versus depth values for each day/month and for each pair of \( m \) and \( n \).

Regression fits for depth versus conductivity profile.

Fig 2: Data Processing chart

III. RESULTS AND DISCUSSION

![Conductivity vs Depth Profile for ILR](image.png)

Figure 3 displays the Electrical conductivity-depth profile of the upper mantle and transition zone based on the Arican solar quiet day variation. The station Ilorin (ILR) is used to represent the equatorial latitude stations, while Hermanus (HER) is used to represent the low latitude stations. The small squares represent the conductivity-depth computation results, while the solid line is the regression fitted values.
The scatter points in the plots were more concentrated within the crust down to about 800 km. At greater depths beyond 800km, the density of the scatter tends to reduce. The observed pattern of the scatter plots distribution could be as a result of the variability of source current location, magnetic field contributions produced by sources other than solar quiet time field conditions (such as lunar, Equatorial Electrojet, polar cap, etc) error from SHA fitting, and error from field measurements. A polynomial trend line of order 3 was fitted to the data points in order to get an average values from the many scattered points. There is a rise in conductivity values from $0.079\text{sm}^{-1}$ at a depth of 106.97km to about $0.12\text{sm}^{-1}$ at 200.44km in ILR, and $0.037\text{sm}^{-1}$ at 115.66 to 0.691$\text{sm}^{-1}$ at 217.87km in HER and this correspond to the global low velocity region, the asthenosphere [9] [10]. [11] observed a rise in conductivity within this region and noted that seismology has revealed some features of the upper mantle, such as the low velocity region between about 60 and 200 km depths and the large regional variations in primary (P) and secondary (S) velocity structure.

The shape of the conductivity depth profile shows the upper mantle can be viewed as a stack of inhomogeneous layer with downward increased in conductivity. This result is in agreement with the global model which shows a sleep rise in conductivity from about 300km – 700km. [12], [13], [14], [15], [16], [17], [18], [3], [4], [5], [6], [7]. There seemed to be some evidence of discontinuities near 100–200 km, 200–400 km, 400–600 km and 600–900 km and these locations are near phase change depths identified on seismic records by [2],[3] [5] worked in the West African sub region and got a conductivity profile which rose rapidly from $0.037\text{sm}^{-1}$ at a depth of 100km to $0.095\text{sm}^{-1}$ at 205km. The profile then rose steadily till it reached $0.15\text{sm}^{-1}$ at 476km near the base of the upper mantle, $0.2\text{sm}^{-1}$ at 880km and $0.22\text{sm}^{-1}$ at 1200km at the lower mantle. The general correspondence observed in this work between high conductivity zone and low velocity zone, the asthenosphere is in agreement with the global result of [11]. The main feature of the conductivity depth distribution in the mantle revealed by global studies is a downward increase of conductivity between depths of 300 and 1000km [1] which is also in line with the result of this work. The high conductivity values observed in this work is in line with the work of [10] who noted high conductivity values in West African and Asian regions. Also, [4] worked in the South African region and found high conductivity values between about 150 and 350km and a general increase thereafter which agrees with the result of this work. [12] worked in the Himalayan region and obtained conductivity values of $0.06\text{sm}^{-1}$ from 50km to approximately 350km depth with alternating relative maxima near 125 and 275km interspersed by relative minima near 210 and 330km.

Having compared our results with data obtained in other regions of the world, we therefore infer from our work that the most conductive layer of the Earth can be obtained with two different regions viz. between the depths of about 229km to 470km and layers beyond 1250km depth in the lower mantle.

**IV. CONCLUSIONS**

The application of the solar quiet day ionosphere current has enabled us to determine the conductivity depth structure of the upper mantle in the European region. The following deductions can be made from the results:

1. The electrical conductivity increases downwards in agreement with the global models, thereby attaining its maximum values in the lower mantle.

2. The most conductive layer of the Earth can be obtained with two different regions viz. between the depths of about 229km to 470km and layers beyond 1250km depth in the lower mantle.
Upper Mantle Conductivity determined from the Solar Quiet Day Ionospheric Currents in the.. 

3. The electrical conductivity depth profiles have a characteristics rise in electrical conductivity towards 400 to 600km depth. This region corresponds to the mantle transition zone, which is part of the Earth’s mantle and located between the lower mantle and the upper mantle between depths of 410 and 660km.

4. The rise in conductivity around the region between 100 – 200 km depth coincides with the low velocity seismic layer revealed by previous studies.

5. There seemed to be some evidence of discontinuities near 100–200 km, 200–400 km, 400–600 km and 600–900 km and these locations are near phase change depths identified on seismic records.

REFERENCES


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