The Relationship of Technical Efficiency with Economical or Allocative Efficiency: An Evaluation

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ABSTRACT: The efficiency word became a buzzword not only in economics but also in various areas and in daily life. In spite of its frequent applications, it is a tricky concept. Moreover, there are tricky relationships among efficiency concepts. The fully comprehension of the concept depend on comprehension of the concepts of technical efficiency and allocative efficiency and their relationships. Giving efficiency concepts in graphic analogy, this study aims to supply tools to comprehend the efficiency concept and the relationship between technical and allocative efficiencies. The study carries out an important discussion on this relationship and reaches conclusions over the evaluation. The study reaches that this very essence issue does not change by addition of the other related concepts naming total efficiency, slackness, cost efficiency and revenue efficiency.

Keywords: Allocative Efficiency, Economical Efficiency, Efficiency, Technical Efficiency,

I. INTRODUCTION

Efficiency is a buzzword today not in just economic areas but also in many different areas. Many of these usages refer different concepts. However, the usage of the efficiency word in economics is in rather technical manner. There are intricate relationships between the technical efficiency and economical or allocative efficiency. Understanding the differences and differentiating points are deeply important to understand the general economic notion of the concept.

The purpose of this article to set mathematical definitions for the mentioned efficiency terms first and using graphical analogy then to raise a discussion on the relationship of the different efficiency concepts.

Economy can be defined around the production activity of goods and services. Taking this production function as the starting point, we can scrutinize the efficiency concepts on the production possibilities curve representing the production frontier and on the isoquant curves. In this manner the production function and production frontier is explained in second section. Third section is allocated for the conceptual analysis; in this section the technical and allocative efficiency concepts, which are important for thorough understanding, are analyzed. In fourth section the other related concepts such as total efficiency, product efficiency and cost efficiency are situated around this relationship between the technical and allocative efficiencies. Fifth section concludes the evaluation.

II. PRODUCTION FUNCTION

Defining the efficiency through the functional form representing production activity we face the development of the efficiency concepts in economics. We can represent production activities by parametric or non-parametric functions. Mathematical programing can be used for non-parametric functions and econometric estimations can be used for the parametric functions. In this paper, mainly parametric functions are being taken account to develop related econometric methods. In addition non-parametric functions are being included just to hold the coherence of analysis.

A variety of functional forms can be used to represent the production activities in economic endeavor such as linear, log-linear, Cobb-Douglas (log-log), translog, CES, Zellner-Revankar general function or non-linear functions. If closed functions are used, a production function which has the inputs and outputs as components can be used to represent the production activity. On the other hand, if we include the output prices to the closed function we can reach a revenue function, and if we include the output prices we can reach to a cost function.
In this way starting from the different representation we can conclude to different concepts such as revenue efficiency, cost efficiency, profit efficiency and scale efficiency.

As a main method in this study, the production function having its inputs and outputs as components is used to represent the production activity. As we elaborate in the next section, when we apply to efficiency analysis to the production functions technical efficiency concept is coming into the agenda. Thus the technical efficiency concept can also be called as production efficiency in literature. Some of the most important functional forms can be used in efficiency measuring given below:

Linear production function is given in Equation 1.

\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u \]  
(Eq. 1)

Cobb-Douglas (log-log) production function is given in Equation 2.

\[ \ln(y) = \beta_0 + \beta_1 \ln(x_1) + \beta_2 \ln(x_2) + u \]  
(Eq. 2)

Translog production function is given in Equation 3.

\[ \ln(y) = \beta_0 + \beta_1 \ln(x_1) + \beta_2 \ln(x_2) + \beta_3 \ln(x_1)^2 + \beta_4 \ln(x_2)^2 + \beta_5 \ln(x_1x_2) + u \]  
(Eq. 3)

Cobb-Douglas function is used prevalently as in the estimation of efficiency levels thanks to some advantages as mathematical representation. Thus after the seminal paper of Farrell (1957) in which non-parametric framework is used for efficiency measurements calls a parametric approach, Aigner and Chu (1968) following Farrell used the parametric function given in Equation 4:

\[ y = f(x) . e^{-u} \]  
(Eq. 4)

Putting the Equation 4 in linear notation we reach Equation 5.

\[ \ln y = X B - u \]  
(Eq. 5)

\[ u: \text{non-negative stochastic residual} \]

III. EFFICIENCY AND ITS MEASUREMENTS

Efficiency can be defined, in a general sense, as a ratio to approach to the optimum level. Economic activities are generally represented by functions. Therefore, the efficiency rate can be defined as the ratio of observed level to the optimum levels in the functionally represented activities.

When we take the efficiency in the account as an economical term, we need to address to efficiency components, its relationships between the other economical concepts, its measuring and the approach methods to these issues. It is possible to handle the efficiency concept in the classification of technical efficiency and allocative efficiency; and in this classification using the input-oriented approach and output-oriented approach is possible. In such an analysis we can start with presenting the production function on which the efficient and inefficient situations will be determined.

Using a production function as given in Figure 1 efficiency cases can be shown as in Figure 2. A full efficient agent will be on the frontier of production function as of point A in Figure 1. If there are some factors causing inefficiency, the production level will be under the production frontier as of point B in Figure 2.

![Figure 1. Representation of an anonymous production function](image-url)
The definition we give above actually the efficiency of a production function with single input and single output. Efficiency has the various components as technical efficiency and allocative efficiency. Because the technical efficiency term is used generally in the performance measuring studies, the efficiency term can be used instead of the technical efficiency term.

Theoretical background and the components of the efficiency are inspected in the next sub section. A classification of the efficiency concepts is carried out first by Farrell by classifying overall efficiency to the technical and allocative efficiency (the last one is called as price efficiency by Farrell). Later, Knox Lovell and Schmidt added the slackness concept to this analysis. Moreover, the input-oriented and output-oriented approaches are used in the literature so they are both included in this paper as well.

3.1 Technical Efficiency
Koopmans (1951, p.61) formally defines technical efficiency as: “If a producer needs to decrease one of the outputs or increase one of the inputs in order to increase its output, the situation is technical efficient”. Similarly it can also be defined as “If a producer needs to increase one of the inputs or decrease one of the outputs in order to decrease its input, the situation is technical efficient” (Kumbhakar and Lovell, 2000, p.43; Fried et al., 1993). As can be seen in the definition technical efficiency can be analyzed by using input-oriented or output-oriented approaches.

Input-oriented approach:
Debreu (1951) and Farrell (1957) developed a measurement for the technical efficiency. This measurement (DFI) defined as “the maximum possible reduction in inputs when the output is given”. The input set the producers use and the output set they obtain with these inputs are given as below (Kumbhakar and Lovell, 2000, p.18-49):

Input : \( x = (x_1, x_2, ..., x_n) \in \mathbb{R}^n_+ \)

Output : \( y = (y_1, y_2, ..., y_m) \in \mathbb{R}^m_+ \)

Production technology can be represented by the input:

\[
L(y) = \{x : (y, x) \text{ member of the feasible set}\}
\]

Having completed the necessary pre definitions as above, Debreu-Farrel input-oriented technical efficiency can be formally defined as in Equation 6.

\[
DFI_{I}(y, x) = \min\{\lambda : \lambda x \in L(y)\} \quad \text{(Eq. 6)}
\]

Definition can be explained in Figure 3:
Shephard (1953; 1970) developed a similar method. Shepard’s input distance function is given in Equation 7.

$$D_I(y, x) = \max \{ \lambda : (x / \lambda) \in L(y) \}$$ (Eq. 7)

It can be clearly set that Shephard and Debreu-Farrel input distance functions are opposite of each other. This relationship is given in Equation 8:

$$DF_I(y, x) = \frac{1}{D_I(y, x)}$$ (Eq. 8)

Shephard input distance function ($D_I$) can be shown in the same way that $DF_I$ function is shown in Figure 3. Debreu-Farrel and Sheppard input distance functions actually reach the same point by using different paths. The results coming out can be compared as following: For example, if a input combination represented by A point in Figure 3 is multiplied by a real number such as 0.7 it drops on the $L(y)$ isoquant without changing its output level, it shows we can decrease the inputs by a factor 0.7 or to its 70% levels. In this case we can give the input efficiency as 70%. An important point here the 0.7 number here should be maximum possible number to reach the frontier of same level production. A lower factor like 0.65 here prevents to reach production frontier. On the other hand we can evaluate the Shepard’s measurement in a similar way. Input combinations in point A can be divided by a factor 1.43 so that the input combination set fall to $L(y)$ isoquant without changing output amount. Therefore inputs here can be decreased by the factor of 1.43 here and the values greater than 1 can be used as a measurement of the inefficiency.

**Output-oriented approach:**

Same logic is valid for the output-oriented approach as well. Output-oriented $DF_I$ can be defined as “the maximum possible increase in outputs when the input is given”. The input set and the output set are given again as below:

Input : $x = (x_1, x_2, ..., x_n) \in \mathbb{R}^n$

Output : $y = (y_1, y_2, ..., y_m) \in \mathbb{R}^m$

Production technology can be represented by the output:

$$P(x) = \{ y : (x, y) \text{ member of the feasible set} \}$$

After given the necessary definitions of input set, output set and production frontier as above, Debreu-Farrel output-oriented technical efficiency can be formally defined as in Equation 9.

$$DF_O(x, y) = \max \{ \lambda : \lambda y \in P(x) \}$$ (Eq. 9)

The meaning of the DF technical efficiency measurement can be explained in Figure 3 by using graphical analogy:

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Shepard’s output oriented distance function is given in Equation 10.

\[ D_O(x, y) = \max \{ \lambda : (y / \lambda) \in P(x) \} \]  

(Eq. 10)

The same relation between the Shepard and Debreu-Farrel technical efficiency measurements are valid for output-oriented technical efficiency measurements as in input-oriented technical efficiency measurements. To name explicitly, Shepard and Debreu-Farrel output-oriented distance functions are equal to reverse of each other as given Equation 11.

\[ DF_O(x, y) = \frac{1}{D_O(x, y)} \]  

(Eq. 11)

\( D_I \) can be shown similarly on the graphic (Figure 5):

Debreu-Farrel and Sheppard output-oriented distance functions can be compared in a similar way as done for input-oriented functions: For example, if an output set as in point A in Figure 4 (same as Figure 5) can be multiplied by 1.43 to reach production frontier A, it shows we have opportunity to improve the output level by factor 1.43 without changing input set. In this case the values greater than 1 indicate the existence of inefficiency. The critical point here is, the number 1.43 here should be the maximum number in current production and technological set so a greater number such as 1.50 here would refer impossibility with the current input and production set. Shepard output-oriented distance measurement can be evaluated parallel here: output vector of OA here can be divided by 0.7 so that it can reach the production frontier \( P(x) \) without change the input set. Therefore the values under 1 indicate an inefficiency situation.

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Figure 4. DFO-Debreu-Farrel output-oriented technical efficiency measurement 
(Source: Kumbhakar and Lovell, 2000, pp.47-9)

Figure 5. \( D_O \) - Sheppard’s output distance function 
(Source: Kumbhakar and Lovell, 2000, p.31)
3.2 **Allocative Efficiency**

Allocative efficiency called as price efficiency and defined as the measurement of the success in the selection of the input set among the optimal input set by Farrell (1957). Starting from this definition Forsund, Lovell and Schmidt (1980), develop the following formulation for the allocative efficiency:

Production plan: \((Y^0, X^0)\)

\[
\frac{f_i(X^0)}{f_j(X^0)} = \frac{w_i}{w_j} \tag{Eq. 12}
\]

\(w_j\): input price of \(X_j\)
\(f_j\): marginal product of \(X_j\) input

Equation 12 represent a relationship in which the prices of the inputs \(i\) and \(j\) belongs to \(X^0\) input set must be equal to price ratio of the marginal outputs of that input. Therefore, among the input combinations that can give same output level on isoquant curve, the input combination that in parallel with the market price ratio and its output level is called as allocative efficient. This definition is in conformity with Pareto efficiency definition (see Nicholson, 1998, p.502). Such a definition related with the optimum usage of the production sources, having been found related with the general description of the economy, have been used time to time as economic efficiency instead of allocative efficiency (for example see Lee, 2012; Battese and Coelli, 1991, p.2).

Allocative efficiency can be more analyzed graphically, as in Figure 6, using the definitions graphical discussions taking palece in Farrell (1957), C.P. Timmer (1971), Anandalingan and Kulatilaka (1987), Fried, Lovell and Schmidt (1993). Allocative efficiency is explained on input-input map, therefore as in input-oriented approach, below in Figure 6:

![Figure 6 Allocative efficiency in input-input map](image)

The points \(X_1\) and \(X_2\) in Figure 6 have both have the same relative prices with the market prices, in other words they are allocative efficient. The difference between them is the point \(X_2\) is technical inefficient because of its higher level use of inputs. Moving from the analysis made here we can bring the technical efficiency and allocative efficiency into scrutiny in the same figure as Figure 7.

![Figure 7. Allocative and technical efficiencies in input-input map](image)
We can think that the curve AB in Figure 7 as the frontier of the all firms in the market, i.e. the production frontier in the market, and that the line DD as relative price line in the market. Q' and Q are both technical efficient but Q is allocative efficient. P has some technical inefficiency and allocative inefficiency. The measurement of inefficiency of P (or efficiency of it) can be given the situation that is both technical and allocative efficient:

$$Eff_p = \frac{OR}{OP}$$

This efficiency definition defined for the point P in Figure 7 can be separated into 2 components: \(\frac{OQ}{OP}\) amount of this inefficiency comes from technical inefficiency and \(\frac{OR}{OQ}\) amount of that inefficiency refers a situation that even the technical efficient point would have an inefficiency causing from the inefficient allocation. The same analysis can be done for the output-oriented approach:

![Figure 8. Allocative efficiency output-output map](image)

The points A and B allocate its production output set in exactly the same way, and because this allocation ratio is equal to the ratio of the market prices both A and B points are output-allocative efficient here. The only difference between the points A and B here is point A here is technically inefficient for its lower production output level.

![Figure 9. The components of the inefficiency of point P in output-output map](image)

In Figure 9 the frontier of the firms belongs to a market in which 2 goods, \(y_1\) and \(y_2\), are produced and the relative price lines of the Q, Q' and P firms. In this situation we can decompose the efficiency of P, overall efficiency as Farrel defined, into its components:
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\[ \text{Eff}_p = \frac{\text{OP}}{\text{OR}} \] is the efficiency ratio. There is inefficiency because \( \frac{\text{OP}}{\text{OR}} < 1 \). The amount of this inefficiency is caused from technical inefficiency, the amount of this inefficiency is that the amount of the allocative inefficiency even if the technical efficient situation would have, i.e. the amount represents the component of allocation inefficiency.

The point \( X_1 \) in input-oriented approach and \( B \) in output-oriented approach represent the cost efficient and revenue efficient situations respectively. These descriptions can be generalized after discussing the slackness term in the next subsection.

3.3 Slackness

Production function can be, as stated in previous sections, parametrical or non-parametrical functions. When we have non-parametrical production we have different type of functions as represented below.

![Some of the non-parametrical production function graphic representations](image)

If we have such a position as given in Figure 10 the slackness is also one of the components that cause inefficiency. In other words it is one of the components of the efficiency measurement. In Figure 11 the points \( A \) and \( B \) are on the production frontier. However, at point \( A \) more amount of input \( L \) is used than is used at point \( B \) but this does not cause a production increase. The unproductive excessive amount on the non-parametric frontier is called as slackness.

![The slackness situation on the production frontier](image)
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To give an example from the real life, the developing countries, such as Turkey, having experienced a transformation from agricultural labor force to industrial labor force. In many cases agriculture sector has an unproductive excessive employment, i.e. implicit unemployment. Especially the traditional family structure assumes the land used for agriculture is a family business and the excessive unproductive labor force basically shares the total earned income.

In application, however, the slackness is not easy to be discriminated. Because of this practical difficulty it is generally combined into technical efficiency or allocative efficiency.

It is not easy to place slackness even in the theory. It can be proposed that the point A in Figure 12 is on production frontier so it is technical efficient. The allocative efficiency of A is \( \frac{OD}{OA} \), the \( \frac{OC}{OA} \) being the amount of it caused from slackness inefficiency. On the other hand, there is an unproductive \( x_2 \) amount so there exist an inefficiency directly embedded to the frontier. In this perspective the inefficiency caused by slackness must be located into the technical efficiency. In this case, comparing with the point B, the technical efficiency of A will be \( \frac{OC}{OA} \) and the allocative efficiency will be \( \frac{OD}{OC} \).

![Figure 12. The inefficient components relative to total efficient E point on the non-parametric production frontier](image)

If A in Figure 12 compared relative to extended EB line instead of compared with point B having no slackness it would cause different results as it is shown in Figure 12. The linear extension of EB process would aim to indicate the amount of the slackness inefficiency caused from the elbowing at points B and E. The same analysis can be applied to other type of the production functions given in Figure 11 or it can be applied to output-oriented approach as well.

4. OVERALL EFFICIENCY AND EVALUATING THE EFFICIENCY TYPES AS ITS COMPONENTS

So far in the analyses the definitions of technical efficiency and allocative efficiency have been made and evaluated by the help of graphical analogy, in addition the slackness for the non-parametric functions. Using same methodology we can analyze these concepts altogether in a single frame. This kind of analysis is being conducted for parametric functions in Figure 13 and Figure 14 for the input-oriented and output-oriented approaches respectively. It must be noted because the parametric functions are used the slackness is irrelevant here. For the input-oriented approach:
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**Figure 13. The components of efficiency relative to total efficient case in input-input map**

\[
\text{TE}_p = \frac{OQ}{OP}; \text{ Technical Efficiency}
\]

\[
\text{AE}_p = \frac{OR}{OP}; \text{ Allocative Efficiency}
\]

At the point P in Figure 13 a production is performed at the level of [AB] isoquant. The input combination can be decreased until point Q on the isoquant without changing its production output level. Consequently there is inefficiency by the ratio of OQ/OP and this ratio is given as measurement of the technical efficiency as above. Nevertheless, at the technical efficient Q point, there is allocative inefficiency by ratio of OR/OQ, which is caused from allocation of inputs different than the rate of market prices. We can formulate of an overall efficiency at point P in Eq. 13 relative the point Q' that fulfills the both technical and allocative efficiency.

\[
\text{Eff}_p = \text{TE}_p \cdot \text{AE}_p
\]

\[
\text{Eff}_p = \frac{OQ}{OP} \cdot \frac{OR}{OQ}; \text{ Overall Efficiency}
\]

Same analysis can be applied for output-oriented approach in Figure 14:

\[
\text{TE}_p = \frac{OP}{OQ}; \text{ Technical Efficiency}
\]

\[
\text{AE}_p = \frac{OQ}{OR}; \text{ Allocative Efficiency}
\]

In the analysis of Figure 14 the point P represents the current production situation. Here it is possible to increase the production up to point Q without changing the input set. The same output ratio is valid at point Q. Consequently OP/OQ can be given as the measurement of the technical efficiency as above. However on the production possibility curve, at point Q, we have still an inefficiency causing from the allocation of outputs differently than the market does. This measurement of the allocative efficiency can be given as OQ/OR, relative to the overall efficient point Q'. The overall efficiency is the multiplication of the technical and allocative efficiency as given in Eq. 13 once again. It must be underlined here that the analyses for input-oriented and output-oriented approach show the overall efficiency measurements does not effect from inclusion of cost and revenue efficiency dimensions or simply from inclusion of prices. Here we can evaluate that the use of overall/total efficiency concept is useful to underline the difference in efficiency components, especially if we think the short cut use of allocative efficiency as economic efficiency or as just efficiency.

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**Figure 14. The components of efficiency relative to total efficient case in output-output map**

\[ Eff_p = TEp \cdot AEp \]  
(Eq. 13)

\[ Eff_p = \frac{OP \cdot OQ}{OQ \cdot OR} \] ; Overall Efficiency

\[ Eff_p = \frac{OP}{OR} \]

It must be noted here that in the input-oriented and output-oriented analyses conducted in Figure 13 and Figure 14 the prices of inputs and outputs can be added in analysis to reach the cost efficiency and revenue efficiency. Therefore the cost and revenue efficiency can be given on the same input-input and output-output maps.

- **CE**- cost efficiency: the rate of minimum cost to observed cost
- **RE**- revenue efficiency: the rate of observed revenue to maximum revenue

In efficiency measurements, because the prices are multiplied both in the nominator and in the denominator it does not change the results. Overall efficiency is given still by the multiplication of technical and allocative efficiency and because they include the prices this time it equals the cost efficiency at the same time as given in Eq. 13 below.

\[ Eff = CE = TE \cdot AE \]  
(Eq. 14)

Similarly we can include the prices into output-oriented analysis. Again having the prices in nominator and denominator as same factor it does not change the results. Therefore the overall efficiency of point P in Figure 14 can be calculated by multiplying of technical and allocative efficiencies as given Eq. 15 below.

\[ Eff = RE = TE \cdot AE \]  
(Eq. 15)

In conclusion, it can be evaluated that the cost efficiency and revenue efficiency are not necessary to be analyzed differently because they are all included in the total efficiency measurement.

**V. EVALUATION AND CONCLUSION**

Efficiency concept has elaborate variations despite its recently achieved buzzword status. Understanding the efficiency concept depends on a good understanding of the concepts of technical efficiency and allocative or economic efficiency and the difference between them.

Technical efficiency can be defined around the concept of the reaching the maximum output. Economical activities are generally defined around a production activity; therefore technical efficiency can be easily defined through a production function. Allocative efficiency measures the degree of conformity of inputs or outputs with their relative market prices.

According to the relationship of these two efficiency concepts concretized by the help of the graphical analogy it becomes possible to define a total/overall efficiency concept. Actually the overall or total efficient situation can be summarized as the situation that is technical and also allocative efficient at the same time. The efficiency measurement can be summarized as the decomposition of the inefficient components relative to the overall/total efficient situation. In the result of this decomposition it can be shown that the multiplication of technical efficiency and allocative efficiency gives overall efficiency. Moreover it has been shown that this
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analysis did not change in price included input-oriented and output-oriented maps, which means the overall efficiency analysis will include the cost and revenue efficiency analysis indirectly.

Efficiency concept has widespread usage and many times is used as replaceable with the allocative or economic efficiency. On the other hand technical efficiency being defined through the production function is suitable for the performance measurements and serves as a theoretical base for performance measuring. This suitable structure of technical efficiency leads its prevalent use in performance measuring. Allocative efficiency, on the other hand, is important despite its practical measurement difficulty because it serves an important comparison relative to the optimum situation representing market valuation via market prices. Most efficiency concepts are used as replaceable with short usage of efficiency. However, we can conclude here that the allocative efficiency concept is the most suitable for the short usage as “efficiency” because of its referring the optimum situation.

Applications might be difficult to be differentiated in terms of efficiency concepts. For example, the efficiency of an internal combustion engine which is normally has low rates can be increased by using the waste heat of engine to heat inside the car, however does not affect the allocative efficiency. Similarly, if a new technology increase the output in terms of movement units but is more expensive than the customer would volunteer to pay, it increase the technical efficiency yet decrease the economic efficiency of the internal combustion engine.

Even more ambiguous examples exist in practice. For example the discussion on state interference versus liberty including wide range effects would have ambiguity to pin down. If we include the social consequences it will be even vuger. In one hand we can propose the state regulations can increase the technical efficiency, yet liberality or even democracy can cause some technical inefficiency. On the contrary, democracy and freedom of speech (as one of the liberal rights) can increase the efficiency of informing society and increasing information and therefore rising precisions in decisions can increase the allocative efficiency (despite the decreasing technical efficiency).

However complex the examples in life would be, the scientific method aims to simplify, classify and discriminate them. The prevalent economic models are simply a part of this toolbox. The graphical analogy widely applied in our study is evaluated as useful and effective in simplifying the relationships between the efficiency concepts. Moreover, so long as it can achieve that, it can serve as a tool or an analytic base to solve out the complexity belongs the life applications.

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