Quest Journals
Journal of Research in Applied Mathematics
Volume 8 ~ Issue 8 (2022) pp: 56-63
ISSN(Online) : 2394-0743 ISSN (Print): 2394-0735
www.questjournals.org

Research Paper

The Application and Analysis of the Lotka-Volterra Model in the Stock Market

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ABSTRACT: *Biological mathematics is a comprehensive discipline that combines biology withmathematics. It is used to study mathematics to solve biological problems.Recently, theLotka-Volterra model has been widely used not only in biology and ecosystems, but alsoin commercial competition and in the prediction of port containers.This articleintroduces the application and analysis of the Lotka-Volterra model in the stock market,finds the nonlinear relationshipamong the multiple factors by studying the backgroundand development status, and describes the relationship using the nonlinear differentialequations.*
KEYWORDS: *Equity Security, Lotka-Volterra Model, Numerical Simulation, Least Square Method*

R*eceived 13 August, 2022;* ***R****evised 26 August, 2022;* ***A****ccepted 28 August, 2022 © The author(s) 2022.* ***P****ublished with open access at* ***www.questjournals.org***

I. INTRODUCTION

Recently, the theoretical studies of the Lotka-Volterra models have mainly focused on the spatial heterogeneity models and the stochastic competition models.Fernandez[1] studied the dynamics of the Lotka-Volterra model for the spatially heterogeneous diffusion, and found that the steady-state solutionof thenonspatial model is linearly unstable, where any steady state of the spatial counterpart is perturbated byit.Heiba[2] has validated the periodic nonuniform Lotka-Volterra model with periodic boundaryconditions by Monte Carlosimulations.Qiu[3] systematically studied the optimal harvestof the stochastic time-delaycompetition Lotka-Volterra model with Livy jumps.Nagatani[4] coupled the Lotka-Volterra model to the CA model, simulating the directional offset of the desert system.Hung[5] proposed an enhanced application of the L-V model with competing components.Marasco[6] proposed a complete non-autonomousLotka-Volterra model for a quantitative study of the interactions between containerports located within the Le Havre-Hamburg range.The fractional Lotka-Volterra model hasrecently become a research hotspot[7-12], with populationsexpanding from 2 to 3, oreven n.

Looking at the stock market trend, we can think that investors regard alcoholic liquor or Moutai, these two stocks, as the two species in the stock market.Of course, most investors determine how to allocate their restricted capital budget to a certain amount of principal for each stock by having the expected investment value of the two stocks.

For this reason, we cannot be sure that the two stocks move in the same direction through a common shock.An economic event can be good news for one stock, and possibly bad news for another stock, and investors can see both stocks as separate portfolio options.This fact shows that the two stocks of Jiuguijiu and Moutai interact as two separate species in the separate liquor market, which also means that, in order to understand the market in detail, it is necessary to clarify the exact relationship between the two stocks.

The rationale of this paper is therefore that the relationship between two stocks is best understood by viewing them as species competing for investor principal.In other words, we see these two stocks as competitors, or as competitive commodities in the single market.This situation can be described by the Lotka-Volterra model.In particular, the Lotka-Volterra equation contains the competition mechanism in the diffusion process of the two stocks.The model incorporates the dynamics of interspecies competition in an ecosystem.It divides the relationships of competitive species, into natural selection, survival of the fittest, and predator-prey interactions under limited space and resources.Few studies have applied the Lotka-Volterra model to apply the stock market.It can be considered as an important contribution of this paper.

II. LOTKA-VOLTERRA MODEL AND THE LEAST SQUARES METHOD

2.1Lotka-Volterra Model

The interaction between the two competitors can be recapitulated by the following Lotka-Volterra model:

$$\begin{cases} \frac{dx(t)}{dt}=x(t)(a_1+b_1x(t)+c_1y(t)) \\ \frac{dy(t)}{dt}=y(t)(a_2+b_2y(t)+c_2x(t)) \end{cases}$$

Let $x(t)=X, y(t)=Y$. Removing the appeal equation in brackets as follows

$$\frac{dX}{dt}=a_1X-b_1X^2-c_1XY$$
$$\frac{dY}{dt}=a_2Y-b_2Y^2-c_2YX$$

and transforming the difference model:

$$\frac{X_{i+1}-X_i}{h}=a_1X_i-b_1X_i^2-c_1X_iY_i \quad (1)$$
$$\frac{Y_{i+1}-Y_i}{h}=a_2Y_i-b_2Y_i^2-c_2Y_iX_i \quad (2)$$

By (1), we see

$$X_{i+1}-X_i=(a_1X_i-b_1X_i^2-c_1X_iY_i)h \quad (3)$$
$$=a_1hX_i-b_1hX_i^2-c_1hX_iY_i$$

Therefore $X_{i+1}=(a_1h+1)\times X_i-b_1hX_i^2-c_1hX_iY_i$ (4)

$$\frac{X_{i+1}}{X_i}=(a_1h+1)-b_1hX_i-c_1hY_i \quad (5)$$

2.2Least Square Method

Least squares is a numerical optimization technique that yields the best matching function by minimizing the sum of error squares. In a unary linear fit, let the known model satisfy $y=ax+b$, and the k group of data is known $(x_1,y_1),(x_2,y_2),\ldots,(x_k,y_k)$. Let $\hat{y}=\hat{a}x+\hat{b}$ be the fitting function, this is, $\varepsilon=y-\hat{y}$ is an error function. Minimizing the sum of error squared $s=\sum_{i=1}^{k}\left(y_i-\hat{y}_i\right)^2$ for parameters $\hat{a},\hat{b}$, this method is called the least squares.

In multiple linear regression, i.e, the model satisfies

$$Y=a_0+a_1X_1+a_2X_2+\ldots+a_nX_n,$$

and known randomly drawn k sample data

$$\{X_{i1},X_{i2},\ldots,\ X_{ik};Y_i\,|\,i=1,2,\ldots k\}.$$

The same set $\hat{y}=\hat{a}_0+\hat{a}_1x_1+\hat{a}_2x_2+\ldots+\hat{a}_nx_n$ is the fitting function, then $\varepsilon=y-\hat{y}$ aserror function.Minimizing the sum of error squares $s=\sum_{i=1}^{k}\left(y_i-\hat{y}_i\right)^2$,seeking parameters $\hat{a}_0,\hat{a}_1,\ldots\hat{a}_n$, s is also expressed as $\min\|AX-B\|_2^2$, where

$$A=\begin{bmatrix}1 & x_{11} & \cdots & x_{1n}\\ 1 & x_{21} & \cdots & x_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ 1 & x_{k1} & \cdots & x_{kn}\end{bmatrix}\ X=\begin{bmatrix}a_0\\ a_1\\ \vdots\\ a_n\end{bmatrix}\ B=\begin{bmatrix}y_0\\ y_1\\ \vdots\\ y_n\end{bmatrix}$$

Using the multivariate differential method and minimizing s, we have

$$\begin{cases} \frac{\partial S}{\partial \hat{a}_0} = \sum_{i=1}^{k}\left(\hat{a}_0 + \hat{a}_1 X_{i1} + \hat{a}_2 X_{i2} + \cdots + \hat{a}_n X_{ik}\right) - \sum Y_i \\ \frac{\partial S}{\partial \hat{a}_1} = \sum_{i=1}^{k}\left(\hat{a}_0 + \hat{a}_1 X_{i1} + \hat{a}_2 X_{i2} + \cdots + \hat{a}_n X_{ik}\right) \times X_{i1} - \sum Y_i X_{i1} \\ \cdots \\ \frac{\partial S}{\partial \hat{a}_n} = \sum_{i=1}^{k}\left(\hat{a}_0 + \hat{a}_1 X_{i1} + \hat{a}_2 X_{i2} + \cdots + \hat{a}_n X_{ik}\right) \times X_{ik} - \sum Y_i X_{ik} \end{cases}$$

Solving this system of linear equations (also called a normal system of equations) $\frac{\partial S}{\partial \hat{a}_j} = 0(j = 0,1,2,...,n)$, we obtain the estimates $\hat{a}_j$ of the $n+1$ parameters $a_j(j = 0,2,\cdots,n)$. If $n=1$, it is a unary linear regression.

III. NUMERICAL SIMULATION

3.1Data Processing

We collected two stocks Jiuguijiu (000799) and Guizhou Maotai (600519) from February 11 to March 24(2022) daily market statistics, including the daily opening price, lowest price, highest, closing price, fluctuation, etc. Here we only discuss the daily closing price, here a total of 30 days of data, given a L-V competition system, study the image of the two stocks, and consider the relationship between the two stocks.

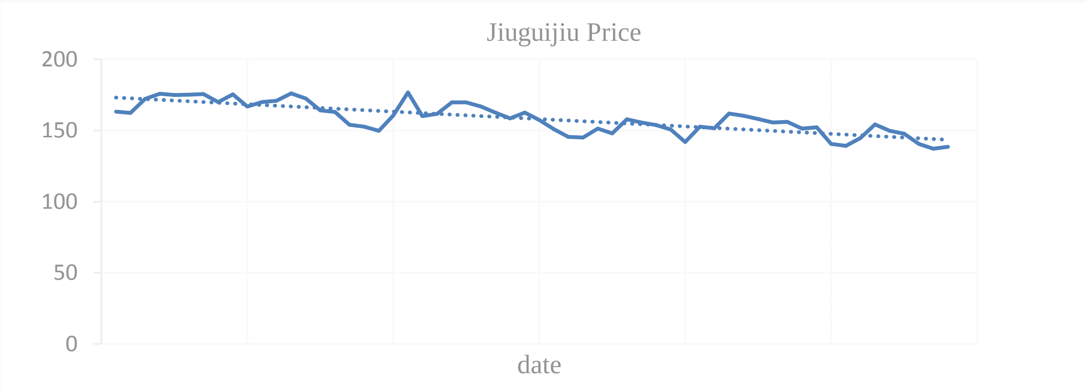

Figure 1: Closing price chart of Jiuguijiu Shares (000799) period from February.11.2022 to March.4.2022

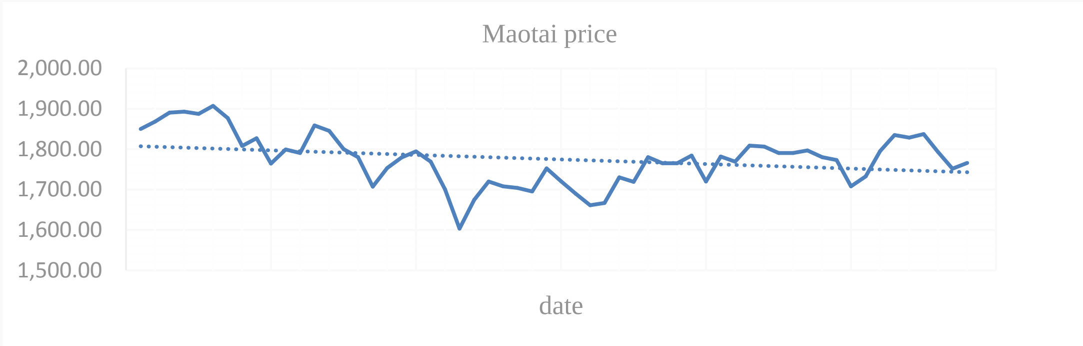

Figure 2: Closing price chart of Moutai Shares (600519) from February.11.2022 to March.24.2022

We discuss the resulting equation (5)

$$\frac{X_{i+1}}{X_i} = (a_1 h + 1) - b_1 h X_i - c_1 h Y_i \quad (5)$$

Among X_{i+1}, X_i are the known quantities,as the closing price of the ith day or $(i+1)th$ day, Y_i as the closing price of Moutai liquor shares on ith day.Value of the unknown parameters $a_1h, b_1h,$ and c_1h arerequired in this paper, here h is the step size, we put $h=1$, equation (5) is:

$$\frac{X_{i+1}}{X_i} = (a_1 + 1) - b_1 X_i - c_1 Y_i \quad (6)$$

Here we can know that the $\frac{X_{i+1}}{X_i}$ is it the ratio of day $(i+1)th$ to the closing price of ith day that can becalculated. We can set it as Y, X_i as X_1, Y_i as X_2.Then the equation (6) can be deformed to:

$$Y = aX_1 + bX_2 + c \quad (7)$$

where $a = (-b_1)$, $b = (-c_1)$, $c = (a_1 + 1)$. Here equation (6) can be applied to a multiplelinear regression in least squares. Using the results obtained above , we have

$$\begin{cases} \sum\left(\hat{a}_0 + \hat{a}_1 X_{i1} + \hat{a}_2 X_{i2}\right) = \sum Y_i \\ \sum\left(\hat{a}_0 + \hat{a}_1 X_{i1} + \hat{a}_2 X_{i2}\right) \times X_{i1} = \sum Y_i X_{i1} \end{cases}$$

This system of linear equations (also known as a normal system of equations) will be calculated to all the parameter values with the help of Python operation(see APPENDIX):

```
intercept 1.2070553254128191
parameter [-0.00259145  0.00012458]
```

Here,the intercept retains 2 decimal places, and the parameter retains 4 decimal places. $c = (a_1 + 1) = 1.21$, $a = (-b_1) = -0.0026$, $b = (-c_1) = 0.00012$.The solution equation of the equation (7) is then obtained:

$$Y = -0.0026 * X_1 + 0.00012 X_2 + 1.21 \quad (8)$$

Here X_1 is the closing price of Jiuguijiu ith day. Here X_2 is the closing price of MaoTaijiu ith day. Hence we get the Jiuguijiu closing price comparison map

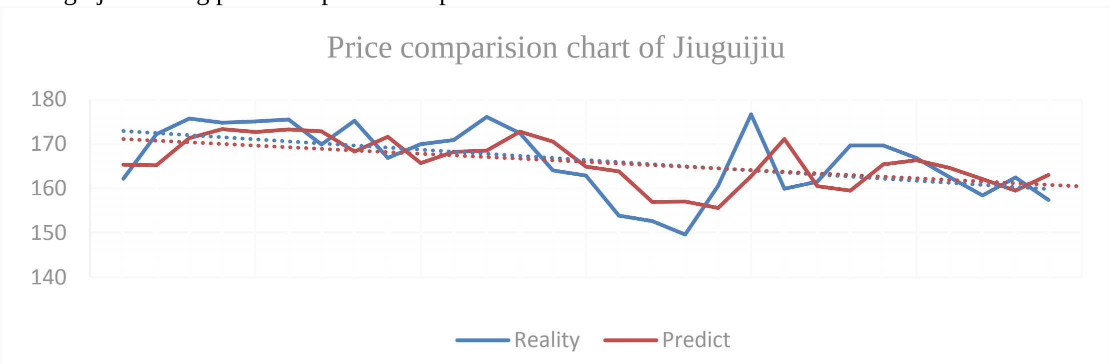

Figure 3: Actual and forecast closing price chart of Jiuguijiu Shares (000799) during the period from February 11.2022 to March.24.2022

Similarly, the closing price comparison chart of Moutai liquor is obtained:

```
intercept 1.1928724113237332
parameter [-5.84422551e-05 -5.48999399e-04]
```

Here, the intercept retains 2 decimal places, and the parameter retains 4 decimal places. $c = (a_2 + 1) = 1.19$, $a = (-b_2) = -0.000058$, $b = (-c_2) = -0.000549$.The solution equation of the equation (7) is then obtained:

$$Y_1 = -0.000058 * X_1 - 0.000549 X_2 + 1.19 \quad (9)$$

Here X_1 is the closing price of Maotaijiu ith day. Here X_2 is the closing price of Jiuguijiu ith day. Hence we get moutai liquor closing price comparison chart

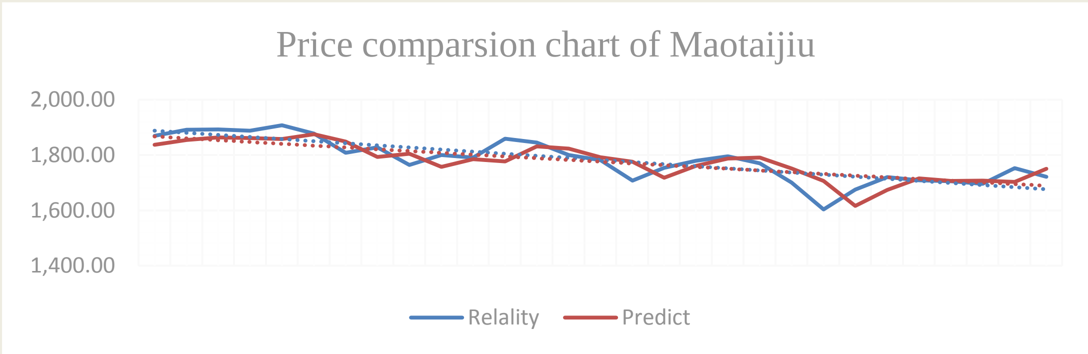

Figure 4: Actual and forecast closing price chart of Moutai Liquor Shares (600519) between February.11.2022 and March.24.2022

3.2 Error Analysis

Because the stock market fluctuates greatly, it may fluctuate greatly at any time because of some new policies or other reasons. Moreover, the stock price has the characteristics of high noise, nonlinearity and being easily influenced by policies,so it is difficult for this paper to achieve 100% simulation results nearby. Only the simulation results can be used for qualitative analysis, but not quantitative analysis. However, the prediction error rate of this paper is relatively small between five points. By comparing the data, researchers can compare the two stocks and further standardize their investment behavior.

3.3Interpretation of Result

Under different parameters, the competition and cooperation relations between the two competitors are shown in Table 1:

Table 1
Competition between the various characters

c_1	c_2	Competitor relationship
>0	>0	mutualism
>0	$=0$	commensalism
>0	<0	Species 1 feeds on species 2
<0	>0	Species 2 feeds on species 1
<0	$=0$	amensalism
<0	<0	compete

Through the analysis of the above alcoholic liquor and Maotai liquor parameters, the Jiuguijiu shares (000799) $c_1>0$. And Maotaijiu shares (600519) parameters $c_2<0$.According to the chart in the above table, this paper can conclude that the stocks held between February.11.2022 and March.24.2022 are more competitive, can occupy more resources, and are more popular with investors.

The table is the closing price of Jiujiu and Moutai Shares at [13] from February.11.2022 to March 24.2022

date	Jiuguijiu/settlement (yuan)	Maotai/settlement(yuan)
2022-02-11	163. 15	1, 849. 97
2022-02-14	162. 2	1, 868. 67
2022-02-15	172. 2	1, 890. 62
2022-02-16	175. 76	1, 892. 55
2022-02-17	174. 8	1, 886. 99
2022-02-18	175. 12	1, 907. 00

2022-02-21	175.53	1,876.99
2022-02-22	169.87	1,807.87
2022-02-23	175.25	1,827.01
2022-02-24	166.85	1,764.11
2022-02-25	170	1,799.06
2022-02-28	170.89	1,790.40
2022-03-01	176.08	1,858.48
2022-03-02	172.45	1,844.88
2022-03-03	164.08	1,800.00
2022-03-04	162.92	1,780.50
2022-03-07	153.91	1,707.00
2022-03-08	152.69	1,753.20
2022-03-09	149.7	1,779.18
2022-03-10	160.6	1,794.43
2022-03-11	176.66	1,769.01
2022-03-14	160	1,700.00
2022-03-15	161.57	1,603.00
2022-03-16	169.7	1,674.95
2022-03-17	169.69	1,720.15
2022-03-18	166.9	1,707.79
2022-03-21	162.55	1,704.30
2022-03-22	158.45	1,695.00
2022-03-23	162.49	1,752.19
2022-03-24	157.46	1,720.93

IV. CONCLUSION

This paper is just a special case given Lotka-Volterra competition model, the research way is also applicable to the future stock price prediction, make people have a general cognition of stock, to judge their own investment behavior, of course, this prediction method is used for short-term and qualitative research, long time prediction also need to be considered. The current common application of Lotka-Volterra model is used to study the relationship between populations in nature. This paper, this model is relatively novel for stock use, and it can certainly be applied to other aspects.For example, by studying the relationship of the market, or the development of the research industry and the network group competition, it can be seen that the Lotka-Volterra model is widely used, and has great significance in biology, economy, mathematics and other aspects.

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APPENDIX: RELATED OPERATION CODES

```
import pandas
from sklearn.preprocessing import MinMaxScaler
import numpy as np
from sklearn import model_selection
from sklearn import linear_model
from sklearn.linear_model import LogisticRegression,LinearRegression
from collections import Counter
from sklearn.datasets import make_classification
from collections import Counter
from sklearn.metrics import accuracy_score
from sklearn import metrics
from sklearn.metrics import cohen_kappa_score
import time
from sklearn.metrics import r2_score,mean_absolute_error,mean_squared_error
from sklearn.metrics import roc_auc_score
def LR(path1,path2):

data = pandas.read_excel(path1)
X = data.drop(["put"], axis=1)
   y = data.put
   X_train, X_test, y_train, y_test = model_selection.train_test_split(X, y, test_size=0.25, random_state=1234)

sklearn_logistic =LinearRegression()
#sklearn_logistic.fit(X_train, y_train)
sklearn_logistic.fit(X, y)

print("intercept",sklearn_logistic.intercept_)
   print("parameter ",sklearn_logistic.coef_)
   path3 =path2+"-"+"Forecast results"

datas = pandas.read_excel(path2)

X1 = datas.drop(["put"], axis=1)
   y = datas.put
#print(y)
predict = pandas.DataFrame(sklearn_logistic.predict(X1))
   print("forecast",predict)
   predict.to_csv(path3 + '.csv', index=True, header=True)

if __name__ == '__main__':
   t1=time.time()
   path1="an.xls"
```

```
path2="an.xls"
LR(path1,path2)
   t2=time.time()
   print("run time:",t2-t1)
```