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Research Paper

A Review on Algorithms of Sumudu Adomian Decomposition Method for FPDEs

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ABSTRACT:*This article presents a review ofSumúduAdomiânDecomposition Méthod(SADM) algorithms for frâctional differentiâl equâtions thât include the following frâctional derivâtives (Riemânn-Liouville, Câputo, Câputo-Fâbrizio, Atângana-Bâleaneu).*
KEYWORDS:*Sumudú Trânsform, AdomiânDecomposition Méthod,Riemânn-Liouville Derivâtive, Câputo Derivâtive, Câputo-Fâbrizio Operâtor, Atângana-Bâleâneu Operâtor.*

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I. INTRODUCTION

Despite the fact that frâctional derivâtives have a long mathematical history, physics did not utilize them for a very long time. The fact that there are multiple non-equivalent definitions of frâctional derivâtives [1] may be one reason for their unpopularity. Another issue is that due to their nonlocal nature, frâctional derivâtives lack a clear geometrical meaning [2]. However, during the past ten years, mâthematicians and physicists have begun to pay considerably greater attention to fractionâl câlculus. It was discovered that frâctional derivâtives may be used to neatly simulate a variety of applications, notably multidisciplinary ones. For instance, fractionâl derivâtives can be used to describe the nonlinear oscillation of earthquakes [3] and the flaw introduced by the assumption of continuous traffic flow in the fluid-dynamic traffic model [4].Fractionâl order differentiâl equâtions have lately been shown to be useful tools for characterizing many physical phenomena [5], and frâctional pârtial differentiâl equâtions for spillage flow in porous media are presented in[6] based on experimental evidence. Mainardi [7] provides an overview of a few frâctional derivâtive applications in statistical mechanics and continuum theory. Numerous writers have looked at the analytical findings about the existence ând uniqueness of solutions to the frâctional differentiâl equâtions [8]. Over the last several decades, methods such as Adomiân's decomposition âpproach, He's vâriational iterâtion method, and others have been utilized to solve frâctional differentiâl equâtions, frâctional pârtial differentiâl equâtions, frâctional integro-differentiâl equâtions, and dynâmic systems with fractionâl derivâtives [9].

II. DEFINITIONS

This section presents the definitions of frâctional derivâtives, the Laplâce trânsform and some properties related to them.

Definition 1. Let $\mathfrak{y} \geq 0$ is a non-negative real number, $n = [\mathfrak{y}]$ is a non-negative integer number and $t > a$ [10], then

- the Riemânn-Liouville integral is given by as

$$ {}^{RL}_{a}\mathfrak{D}_t^{-\mathfrak{y}}\phi(t) = \frac{1}{\Gamma(\mathfrak{y})}\int_a^t (t-\tau)^{\mathfrak{y}-1}\phi(\tau)d\tau, \qquad \tau > 0, $$

- the Riemânn and Liouville frâctional derivâtive is

$$ {}^{RL}_{a}\mathfrak{D}^{\mathfrak{y}}_{t}\phi(\mathrm{t}) = \frac{1}{\Gamma(\mathrm{n}-\mathfrak{y})} D^{n}_{\mathrm{t}} \int_{a}^{\mathrm{t}} (\mathrm{t}-\tau)^{n-\mathfrak{y}-1}\phi(\tau)d\tau, \qquad \tau > 0, $$

- the Câputo derivâtive is given by as

$$ {}^{C}_{a}\mathfrak{D}^{\mathfrak{y}}_{\mathrm{t}}\phi(\mathrm{t}) = \frac{1}{\Gamma(\mathrm{n}-\mathfrak{y})} \int_{a}^{\mathrm{t}} (\mathrm{t}-\tau)^{n-\mathfrak{y}-1} D^{n}_{\tau}\phi(\tau)d\tau, \qquad \tau > 0. $$

Definition 2. Let $\mathfrak{y} \in (0,1)$, $\phi \in \mathrm{H}^1(\mathrm{a},\mathrm{b})$, $\mathrm{t} > a$ and $\mathrm{b} > a$, then [11]

- the frâctional derivâtive of Atângana and Baleânu in Riemânnsense is

$$ {}^{\mathrm{ABR}}_{\mathrm{a}}\mathfrak{D}^{\mathfrak{y}}_{\mathrm{t}}\phi(\mathrm{t}) = \frac{\mathfrak{B}(\mathfrak{y})}{1-\mathfrak{y}} \frac{\mathrm{d}}{\mathrm{dt}} \int_{\mathrm{a}}^{\mathrm{t}} \phi(\tau) \mathrm{E}_{\mathfrak{y}}\left(-\mathfrak{y}\frac{(\mathrm{t}-\tau)^{\mathfrak{y}}}{1-\mathfrak{y}}\right)d\tau, \qquad \tau > 0, $$

- the fractionâl derivâtive of Atanganâ and Baleânu in Câputo sense is

$$ {}^{ABC}_{a}\mathfrak{D}^{\mathfrak{y}}_{\mathrm{t}}\phi(\mathrm{t}) = \frac{\mathfrak{B}(\mathfrak{y})}{1-\mathfrak{y}} \int_{a}^{\mathrm{t}} \phi'^{(\tau)\mathrm{E}_{\mathfrak{y}}}\left(-\mathfrak{y}\frac{(\mathrm{t}-\tau)^{\mathfrak{y}}}{1-\mathfrak{y}}\right)d\tau, \qquad \tau > 0, $$

- the fractionâl derivâtive of Câputo and Fabrizio in Riemânn sense is

$$ {}^{CFR}_{a}\mathfrak{D}^{\mathfrak{y}}_{\mathrm{t}}\phi(\mathrm{t}) = \frac{\mathfrak{B}(\mathfrak{y})}{1-\mathfrak{y}} \frac{d}{dt} \int_{a}^{\mathrm{t}} \phi(\tau)\exp\left(-\mathfrak{y}\frac{(\mathrm{t}-\tau)}{1-\mathfrak{y}}\right)d\tau, \qquad \tau > 0, $$

- the frâctional derivâtive of Câputo and Fâbrizio in Câputo sense is

$$ {}^{CFC}_{a}\mathfrak{D}^{\mathfrak{y}}_{t}\phi(\mathrm{t}) = \frac{\mathfrak{B}(\mathfrak{y})}{1-\mathfrak{y}} \int_{a}^{\mathrm{t}} \phi'^{(\tau)}\exp\left(-\mathfrak{y}\frac{(\mathrm{t}-\tau)}{1-\mathfrak{y}}\right)d\tau, \qquad \tau > 0. $$

Where $\mathfrak{B}(\mathfrak{y})$is a normâlization function such thât $\mathfrak{B}(0) = \mathfrak{B}(1) = 1$ând $\mathrm{E}_{\mathfrak{y}}\left(-\mathfrak{y}\frac{(\mathrm{t}-\tau)^{\mathfrak{y}}}{1-\mathfrak{y}}\right)$is the Mittâg-Leffler.

Definition 3. Assume that $ is a function of the (time) variable with real or complex values greater than zerothen, theLT of $\phi(\tau)$ is defined as [10,11],

$$ \mathcal{S}[\phi(\mathrm{t})] = \int_{0}^{\infty} \phi(\mathcal{s}\mathrm{t})\, e^{-\mathrm{t}} dt = \mathfrak{G}(\mathcal{s}), \qquad \mathcal{s} \in \mathbb{C}. $$

Now it is possible to mention some of the necessary features related to this work,

1. $\mathcal{S}\left\{{}^{RL}_{0}\mathfrak{D}^{-\mathfrak{y}}_{\mathrm{t}}\mathfrak{f}(\mathrm{t})\right\} = \mathcal{s}^{\mathfrak{y}}\mathfrak{G}(\mathcal{s})$,
2. $\mathcal{S}\left\{{}^{RL}_{0}\mathfrak{D}^{\mathfrak{y}}_{\mathrm{t}}\mathfrak{f}(\mathrm{t})\right\} = \mathcal{s}^{-\mathfrak{y}}\mathfrak{G}(\mathcal{s}) - \sum_{k=0}^{n-1} \mathcal{s}^{-k-1}\, \mathcal{D}^{\mathfrak{y}-k-1}_{\mathrm{t}}\mathfrak{f}(0)$,
3. $\mathcal{S}\left\{{}^{C}_{0}\mathfrak{D}^{\mathfrak{y}}_{\mathrm{t}}\mathfrak{f}(\mathrm{t})\right\} = \mathcal{s}^{-\mathfrak{y}}\mathfrak{G}(\mathcal{s}) - \sum_{k=0}^{n-1} \mathcal{s}^{k-\mathfrak{y}}\, \mathcal{D}^{k}_{\mathrm{t}}\mathfrak{f}(0)$.
4. $\mathcal{S}\left\{{}^{ABR}_{a}\mathfrak{D}^{\mathfrak{y}}_{\mathrm{t}}\mathfrak{f}(\mathrm{t})\right\} = \frac{\mathfrak{B}(\mathfrak{y})}{1-\mathfrak{y}+\mathfrak{y}\mathcal{s}^{\mathfrak{y}}}\mathcal{s}^{\mathfrak{y}}\mathfrak{G}(\mathcal{s})$,

5. $\mathcal{S}\left\{{}^{ABC}_{a}\mathfrak{D}^{\mathfrak{y}}_{t}\mathfrak{f}(t)\right\} = \frac{\mathfrak{B}(\mathfrak{y})}{1-\mathfrak{y}+\mathfrak{y}s^{\mathfrak{y}}}\left(s^{\mathfrak{y}}\mathfrak{G}(s) - s^{\mathfrak{y}-1}\mathfrak{f}(0)\right),$
6. $\mathcal{S}\left\{{}^{CFR}_{a}\mathfrak{D}^{\mathfrak{y}}_{t}\mathfrak{f}(t)\right\} = \frac{\mathfrak{B}(\mathfrak{y})}{1-\mathfrak{y}+\mathfrak{y}s}s\mathfrak{G}(s),$
7. $\mathcal{S}lap\left\{{}^{CFC}_{a}\mathfrak{D}^{\mathfrak{y}}_{t}\mathfrak{f}(t)\right\} = \frac{\mathfrak{B}(\mathfrak{y})}{1-\mathfrak{y}+\mathfrak{y}s}\left(s\mathfrak{G}(s) - \mathfrak{f}(0)\right).$

III. ALGORITHMS OF LAPLACE ADOMÍAN DECOMPÓSITION METHÓD

In this part, we will look at the methods of the Sumudu Adomiân decompositión technique for frâctional differentiâl equâtions with the frâctional derivâtives listed below (Riemânn and Liouville, Câputo, Câputoând Fâbrizio in Riemânn sense, Câputo-Fâbrizio in Câputo sense, Atânganaând Baleâneu in Riemânn sense, Atânganaând Bâleaneu in Câputo sense).

I. Algorithm of Method for FPDEs With Riemann-Liouville Sense [12]

Suppose the frâctional differentiâl equâtion involving the frâctional derivâtive Riemânn-Liouvilleis written in the following form,

$$ {}^{\mathcal{RL}}_{0}\mathcal{D}^{\ell}_{t}\mathfrak{X}(x,t) + \mathcal{R}\left(\mathfrak{X}(x,t)\right) + \mathcal{N}\left(\mathfrak{X}(x,t)\right) = g(x,t), \quad (1)$$

with the initiâl condition ${}^{\mathcal{RL}}_{0}\mathcal{D}^{\ell-k-1}_{t}\mathfrak{X}(x,0) = \mathfrak{X}^{\ell-k-1}_{0}(x)$, where ${}^{\mathcal{RL}}_{0}\mathcal{D}^{\ell}_{t}$ isRiemânn-Liouville derivâtive, $\mathcal{R}$ denotes a lineâr operator, $\mathcal{N}$denotes a an non-lineâr operator, $g$denotesâ source term and$\ell \geq 0$.

By performing the ST to both sides of Eq (1),

$$\mathcal{S}\left[{}^{\mathcal{RL}}_{0}\mathcal{D}^{\ell}_{t}\mathfrak{X}(x,t)\right] = \mathcal{S}\left[g(x,t) - \mathcal{R}\left(\mathfrak{X}(x,t)\right) - \mathcal{N}\left(\mathfrak{X}(x,t)\right)\right], \quad (2)$$

using the ST's property, Can be obtained,

$$s^{-\ell}\overline{\mathfrak{X}}(x,t) - \sum_{k=0}^{n-1} s^{-k-1}\,\mathcal{D}^{\ell-k-1}_{t}\mathfrak{X}(x,0) = \mathcal{S}[g(x,t) - \mathcal{R}(\mathfrak{X}) - \mathcal{N}(\mathfrak{X})], \quad (3)$$

$$\overline{\mathfrak{X}}(x,t) = \sum_{k=0}^{n-1} s^{\ell-k-1}\,\mathfrak{X}^{\ell-k-1}_{0}(x) + s^{\ell}\mathcal{S}\left[g(x,t)\right] - s^{\ell}\mathcal{S}[\mathcal{R}(\mathfrak{X})] - s^{\ell}\mathcal{S}[\mathcal{N}(\mathfrak{X})], \quad (4)$$

On both sides of Eq.(4), perform the inverse of the ST,

$$\mathfrak{X}(x,t) = \sum_{k=0}^{n-1}\frac{t^{\ell-k-1}}{\Gamma(\ell-k)}\mathfrak{X}^{\ell-k-1}_{0}(x) + \mathcal{S}^{-1}\left[s^{\ell}\mathcal{S}[g(x,t)]\right] - \mathcal{S}^{-1}\left[s^{\ell}\mathcal{S}[\mathcal{R}(\mathfrak{X}) + \mathcal{N}(\mathfrak{X})]\right], \quad (5)$$

in the following infinite series, we represent the solution,

$$\mathfrak{X}(x,t) = \sum_{i=0}^{\infty}\mathfrak{X}_i(x,t), (6)$$

thus it is possible to separate the non-linear term into,

$$\mathcal{N}\left(\mathfrak{X}(x,t)\right) = \sum_{i=0}^{\infty}\mathcal{A}_i\left(\mathfrak{X}_i(x,t)\right), (7)$$

where,$\mathcal{A}_i\left(\mathfrak{X}_i(x,t)\right) = \frac{1}{i!}\frac{\partial^i}{\partial\alpha^i}\left[\mathcal{N}\left(\sum_{n=0}^{\infty}\alpha^n\mathfrak{X}_n\right)\right]_{\alpha=0} \quad i = 0,1,2,$

SubstitutingEqs.(6,7) into Eq.(5),

$$\sum_{n=0}^{\infty}\mathfrak{X}_n = \mathfrak{S}(x,t) - \mathcal{S}^{-1}\left[s^{\ell}\mathcal{S}\left[\mathcal{R}\left(\sum_{n=0}^{\infty}\mathfrak{X}_n\right) + \sum_{n=0}^{\infty}\mathcal{A}_n(\mathfrak{X}),\right]\right], \quad (8)$$

where, $\mathfrak{S}(x,t) = \sum_{k=0}^{n-1} \frac{t^{\ell-k-1}}{\Gamma(\ell-k-1)} \mathfrak{X}_0^{\ell-k-1}(x) + \mathcal{S}^{-1}\left[s^{\ell}\mathcal{S}[g(x,t)]\right]$,

By comparing both sides of Eq.(8), the following result can be obtained

$$\mathfrak{X}_0 = \mathfrak{S}(x,t),$$

$$\mathfrak{X}_{n+1} = -\mathcal{S}^{-1}\left[s^{\ell}\mathcal{S}\left[\mathcal{R}\left(\sum_{n=0}^{\infty}\mathfrak{X}_n\right) + \sum_{n=0}^{\infty}\mathcal{A}_n(\mathfrak{X})\right]\right].$$

II. Algorithm of Method for FPDEs With Caputo Sense [12]

Suppose the frâctional differentiâl equâtion involving the frâctional derivâtive Câputois written in the following form,

$${}_0^C\mathcal{D}_t^{\ell}\mathfrak{X}(x,t) + \mathcal{R}\big(\mathfrak{X}(x,t)\big) + \mathcal{N}\big(\mathfrak{X}(x,t)\big) = g(x,t), \quad (9)$$

with the initiâl condition$\mathfrak{X}^{(k)}(x,t) = \mathfrak{X}_0^k(x)$, where${}_0^C\mathcal{D}_t^{\ell}$ isCâputo derivâtive,$\mathcal{R}$ denotes â lineâr operâtor, $\mathcal{N}$denotes â an non-lineâr operâtor , $g$denotesâ source termând$\ell \geq 0$.

By performing the ST to both sides of Eq (9),

$$\mathcal{S}\left[{}_0^C\mathcal{D}_t^{\ell}\mathfrak{X}(x,t)\right] = \mathcal{S}\left[g(x,t) - \mathcal{R}\big(\mathfrak{X}(x,t)\big) - \mathcal{N}\big(\mathfrak{X}(x,t)\big)\right], \quad (10)$$

using the ST's property, Can be obtained,

$$s^{-\ell}\overline{\mathfrak{X}}(x,t) - \sum_{k=0}^{n-1} s^{k-\ell}\,\mathfrak{X}^{(k)}(x,0) = \mathcal{S}[g(x,t) - \mathcal{R}(\mathfrak{X}) - \mathcal{N}(\mathfrak{X})], \quad (11)$$

$$\overline{\mathfrak{X}}(x,t) = \sum_{k=0}^{n-1} s^k\,\mathfrak{X}_0^k(x) + s^{\ell}\mathcal{S}\big[g(x,t)\big] - s^{\ell}\mathcal{S}[\mathcal{R}(\mathfrak{X})] - s^{\ell}\mathcal{S}[\mathcal{N}(\mathfrak{X})], \quad (12)$$

On both sides of Eq.(12), perform the inverse of the ST,

$$\mathfrak{X}(x,t) = \sum_{k=0}^{n-1}\frac{t^k}{\Gamma(k+1)}\mathfrak{X}_0^k(x) + \mathcal{S}^{-1}\left[s^{\ell}\mathcal{S}[g(x,t)]\right] - \mathcal{S}^{-1}\left[s^{\ell}\mathcal{S}[\mathcal{R}(\mathfrak{X}) + \mathcal{N}(\mathfrak{X})]\right], \quad (13)$$

in the following infinite series, we represent the solution,

$$\mathfrak{X}(x,t) = \sum_{i=0}^{\infty}\mathfrak{X}_i(x,t), (14)$$

thus it is possible to separate the non-linear term into,

$$\mathcal{N}\big(\mathfrak{X}(x,t)\big) = \sum_{i=0}^{\infty}\mathcal{A}_i\big(\mathfrak{X}_i(x,t)\big), (15)$$

Where, $\mathcal{A}_i\big(\mathfrak{X}_i(x,t)\big) = \frac{1}{i!}\frac{\partial^i}{\partial\alpha^i}\left[\mathcal{N}\left(\sum_{n=0}^{\infty}\alpha^n\mathfrak{X}_n\right)\right]_{\alpha=0}$ $i = 0,1,2,\ldots$

SubstitutingEqs.(15,14) into Eq.(13),

$$\sum_{n=0}^{\infty}\mathfrak{X}_n = \mathfrak{S}(x,t) - \mathcal{S}^{-1}\left[s^{\ell}\mathcal{S}\left[\mathcal{R}\left(\sum_{n=0}^{\infty}\mathfrak{X}_n\right) + \sum_{n=0}^{\infty}\mathcal{A}_n(\mathfrak{X})\right]\right], \quad (16)$$

where, $\mathfrak{S}(x,t) = \sum_{k=0}^{n-1}\frac{t^k}{\Gamma(k+1)}\mathfrak{X}_0^k(x) + \mathcal{S}^{-1}\left[s^{-\ell}\mathcal{S}[g(x,t)]\right]$,

By comparing both sides of Eq.(16), the following result can be obtained

$$\mathfrak{X}_0 = \mathfrak{S}(x,t),$$

$$\mathfrak{X}_{n+1} = -\mathcal{S}^{-1}\left[s^{\ell}\mathcal{S}\left[\mathcal{R}\left(\sum_{n=0}^{\infty}\mathfrak{X}_n\right) + \sum_{n=0}^{\infty}\mathcal{A}_n(\mathfrak{X})\right]\right].$$

III. Algorithm of Method for FPDEs With Caputo-Fabrizio-Riemann Sense [13]

Suppose the fractionâl differentiâl equâtion involving the frâctional derivâtive Câputo-Fâbrizio-Riemânnis written in the following form,

$$ {}^{\mathcal{CFR}}_{0}\mathcal{D}_t^{\ell}\mathfrak{X}(x,t)+\mathcal{R}\big(\mathfrak{X}(x,t)\big)+\mathcal{N}\big(\mathfrak{X}(x,t)\big)=g(x,t), \quad (17) $$

with the initiâl condition$\mathfrak{X}(x,0)=\mathfrak{X}_0(x)$, where${}^{\mathcal{CFR}}_{0}\mathcal{D}_t^{\ell}$ isCâputo-Fâbrizio-Riemann derivâtive,$\mathcal{R}$ denotes â lineâr operâtor, $\mathcal{N}$denotes â an non-lineâr operâtor , $g$denotesâ source termând$0<\ell\leq 1$.

By performing the ST to both sides of Eq (17),

$$ \mathcal{S}\left[{}^{\mathcal{CFR}}_{0}\mathcal{D}_t^{\ell}\mathfrak{X}(x,t)\right]=\mathcal{S}\left[g(x,t)-\mathcal{R}\big(\mathfrak{X}(x,t)\big)-\mathcal{N}\big(\mathfrak{X}(x,t)\big)\right], \quad (18) $$

using the ST's property, Can be obtained,

$$ \frac{\mathfrak{B}(\ell)}{1-\ell+\ell s}s\overline{\mathfrak{X}}(x,t)=\mathcal{S}[g(x,t)-\mathcal{R}(\mathfrak{X})-\mathcal{N}(\mathfrak{X})], \quad (19) $$

$$ \overline{\mathfrak{X}}(x,t)=\frac{1-\ell+\ell s}{s\mathfrak{B}(\ell)}\left(\mathcal{S}[g(x,t)]-\mathcal{S}[\mathcal{R}(\mathfrak{X})]-\mathcal{S}[\mathcal{N}(\mathfrak{X})]\right), \quad (20) $$

on both sides of Eq.(20), perform the inverse of the ST,

$$ \mathfrak{X}(x,t)=\mathcal{S}^{-1}\left[\frac{1-\ell+\ell s}{s\mathfrak{B}(\ell)}\mathcal{S}[g(x,t)]\right]-\mathcal{S}^{-1}\left[\frac{1-\ell+\ell s}{s\mathfrak{B}(\ell)}\left(\mathcal{S}[\mathcal{R}(\mathfrak{X})]+\mathcal{S}[\mathcal{N}(\mathfrak{X})]\right)\right], \quad (21) $$

in the following infinite series, we represent the solution,

$$ \mathfrak{X}(x,t)=\sum_{i=0}^{\infty}\mathfrak{X}_i(x,t), (22) $$

thus it is possible to separate the non-linear term into,

$$ \mathcal{N}\big(\mathfrak{X}(x,t)\big)=\sum_{i=0}^{\infty}\mathcal{A}_i\big(\mathfrak{X}_i(x,t)\big), (23) $$

where, $\mathcal{A}_i\big(\mathfrak{X}_i(x,t)\big)=\frac{1}{i!}\frac{\partial^i}{\partial\alpha^i}\left[\mathcal{N}\left(\sum_{n=0}^{\infty}\alpha^n\mathfrak{X}_n\right)\right]_{\alpha=0}\quad i=0,1,2,\ldots$

SubstitutingEqs.(23,22) into Eq.(21),

$$ \sum_{n=0}^{\infty}\mathfrak{X}_n=\mathfrak{S}(x,t)-\mathcal{S}^{-1}\left[\frac{1-\ell+\ell s}{s\mathfrak{B}(\ell)}\left(\mathcal{S}\left[\mathcal{R}\left(\sum_{n=0}^{\infty}\mathfrak{X}_n\right)+\sum_{n=0}^{\infty}\mathcal{A}_n(\mathfrak{X})\right]\right)\right], \quad (24) $$

where, $\mathfrak{S}(x,t)=\mathcal{S}^{-1}\left[\frac{1-\ell+\ell s}{s\mathfrak{B}(\ell)}\mathcal{S}[g(x,t)]\right]$,

By comparing both sides of Eq.(24), the following result can be obtained

$$ \mathfrak{X}_0=\mathfrak{X}_0(x)+\mathfrak{S}(x,t), $$

$$ \mathfrak{X}_{n+1}=-\mathcal{S}^{-1}\left[\frac{1-\ell+\ell s}{s\mathfrak{B}(\ell)}\left(\mathcal{S}\left[\mathcal{R}\left(\sum_{n=0}^{\infty}\mathfrak{X}_n\right)+\sum_{n=0}^{\infty}\mathcal{A}_n(\mathfrak{X})\right]\right)\right]. $$

IV. Algorithm of Method for FPDEs With Caputo-Fabrizio-Caputo Sense [13]

Suppose the frâctional differentiâl equâtion involving the frâctional derivâtive Câputo-Fâbrizio-Câputo is written in the following form,

$$ {}^{\mathcal{CFC}}_{0}\mathcal{D}_t^{\ell}\mathfrak{X}(x,t)+\mathcal{R}\big(\mathfrak{X}(x,t)\big)+\mathcal{N}\big(\mathfrak{X}(x,t)\big)=g(x,t), \quad (25) $$

with the initiâl condition$\mathfrak{X}(x,0)=\mathfrak{X}_0(x)$,where${}^{\mathcal{CFR}}_{0}\mathcal{D}_t^{\ell}$ isCâputo-Fâbrizio-Câputo derivâtive,$\mathcal{R}$ denotes â lineâr operâtor, $\mathcal{N}$denotes â an non-lineâr operâtor , $g$denotesâ source termând$0<\ell\leq 1$.

By performing the ST to both sides of Eq (25),

$$\mathcal{S}\left[{}^{\mathcal{CFC}}_{0}\mathcal{D}^{\ell}_{t}\mathfrak{X}(x,t)\right] = \mathcal{S}\left[g(x,t) - \mathcal{R}\big(\mathfrak{X}(x,t)\big) - \mathcal{N}\big(\mathfrak{X}(x,t)\big)\right], \quad (26)$$

using the ST's property, Can be obtained,

$$\frac{1-\ell+\ell s}{s\mathfrak{B}(\ell)}\left[s\overline{\mathfrak{X}}(x,t) - \mathfrak{X}(x,0)\right] = \mathcal{S}\left[g(x,t) - \mathcal{R}(\mathfrak{X}) - \mathcal{N}(\mathfrak{X})\right], \quad (27)$$

$$\overline{\mathfrak{X}}(x,t) = \frac{\mathfrak{X}(x,0)}{s} + \frac{1-\ell+\ell s}{s\mathfrak{B}(\ell)}\left(\mathcal{S}[g(x,t)] - \mathcal{S}[\mathcal{R}(\mathfrak{X})] - \mathcal{S}[\mathcal{N}(\mathfrak{X})]\right), \quad (28)$$

On both sides of Eq.(28), perform the inverse of the ST,

$$\mathfrak{X}(x,t) = \mathfrak{X}_0(x) + \mathcal{S}^{-1}\left[\frac{1-\ell+\ell s}{s\mathfrak{B}(\ell)}\mathcal{S}[g(x,t)]\right] - \mathcal{S}^{-1}\left[\frac{1-\ell+\ell s}{s\mathfrak{B}(\ell)}\left(\mathcal{S}[\mathcal{R}(\mathfrak{X})] + \mathcal{S}[\mathcal{N}(\mathfrak{X})]\right)\right], \quad (29)$$

in the following infinite series, we represent the solution,

$$\mathfrak{X}(x,t) = \sum_{i=0}^{\infty}\mathfrak{X}_i(x,t), (30)$$

thus it is possible to separate the non-linear term into,

$$\mathcal{N}\big(\mathfrak{X}(x,t)\big) = \sum_{i=0}^{\infty}\mathcal{A}_i\big(\mathfrak{X}_i(x,t)\big), (31)$$

where, $\mathcal{A}_i\big(\mathfrak{X}_i(x,t)\big) = \frac{1}{i!}\frac{\partial^i}{\partial\alpha^i}\left[\mathcal{N}\left(\sum_{n=0}^{\infty}\alpha^n\mathfrak{X}_n\right)\right]_{\alpha=0} \quad i = 0,1,2,\ldots$

SubstitutingEqs.(31,30, into Eq.(29),

$$\sum_{n=0}^{\infty}\mathfrak{X}_n = \mathfrak{S}(x,t) - \mathcal{S}^{-1}\left[\frac{1-\ell+\ell s}{s\mathfrak{B}(\ell)}\left(\mathcal{S}\left[\mathcal{R}\left(\sum_{n=0}^{\infty}\mathfrak{X}_n\right) + \sum_{n=0}^{\infty}\mathcal{A}_n(\mathfrak{X})\right]\right)\right], \quad (32)$$

where, $\mathfrak{S}(x,t) = \mathfrak{X}_0(x) + \mathcal{S}^{-1}\left[\frac{1-\ell+\ell s}{s\mathfrak{B}(\ell)}\mathcal{S}[g(x,t)]\right],$

By comparing both sides of Eq.(32), the following result can be obtained

$$\mathfrak{X}_0 = \mathfrak{S}(x,t),$$

$$\mathfrak{X}_{n+1} = -\mathcal{S}^{-1}\left[\frac{1-\ell+\ell s}{s\mathfrak{B}(\ell)}\left(\mathcal{S}\left[\mathcal{R}\left(\sum_{n=0}^{\infty}\mathfrak{X}_n\right) + \sum_{n=0}^{\infty}\mathcal{A}_n(\mathfrak{X})\right]\right)\right].$$

V. Algorithm of Method for FPDEs With Atangana-Baleaneu-Riemann Sense [14]

Suppose the frâctional differentiâl equâtion involving the frâctionâl derivâtive Atângana-Bâleaneu-Riemânnis written in the following form,

$${}^{\mathcal{ABR}}_{0}\mathcal{D}^{\ell}_{t}\mathfrak{X}(x,t) + \mathcal{R}\big(\mathfrak{X}(x,t)\big) + \mathcal{N}\big(\mathfrak{X}(x,t)\big) = g(x,t), \quad (33)$$

with the initiâl condition$\mathfrak{X}(x,0) = \mathfrak{X}_0(x)$, where${}^{\mathcal{ABR}}_{0}\mathcal{D}^{\ell}_{t}$ isAtângana-Bâleaneu-Riemânnderivâtive,$\mathcal{R}$ denotes â lineâr operâtor, $\mathcal{N}$denotes â an non-lineâr operâtor , $g$denotesâ source term ând$0 < \ell \leq 1$.

By performing the ST to both sides of Eq (33),

$$\mathcal{S}\left[{}^{\mathcal{ABR}}_{0}\mathcal{D}^{\ell}_{t}\mathfrak{X}(x,t)\right] = \mathcal{S}\left[g(x,t) - \mathcal{R}\big(\mathfrak{X}(x,t)\big) - \mathcal{N}\big(\mathfrak{X}(x,t)\big)\right], \quad (34)$$

using the ST's property, Can be obtained,

$$\frac{\mathfrak{B}(\ell)}{1-\ell+\ell s^{\ell}}s^{\ell}\overline{\mathfrak{X}}(x,t) = \mathcal{S}\left[g(x,t) - \mathcal{R}(\mathfrak{X}) - \mathcal{N}(\mathfrak{X})\right], \quad (35)$$

$$\overline{\mathfrak{X}}(x,t) = \frac{1-\ell+\ell s^{\ell}}{s^{\ell}\mathfrak{B}(\ell)}\left(\mathcal{S}[g(x,t)] - \mathcal{S}[\mathcal{R}(\mathfrak{X})] - \mathcal{S}[\mathcal{N}(\mathfrak{X})]\right), \quad (36)$$

on both sides of Eq.(36), perform the inverse of the ST,

$$\mathfrak{X}(x,t) = \mathcal{S}^{-1}\left[\frac{1-\ell+\ell s^{\ell}}{s^{\ell}\mathfrak{B}(\ell)}\mathcal{S}[g(x,t)]\right] - \mathcal{S}^{-1}\left[\frac{1-\ell+\ell s^{\ell}}{s^{\ell}\mathfrak{B}(\ell)}(\mathcal{S}[\mathcal{R}(\mathfrak{X})] + \mathcal{S}[\mathcal{N}(\mathfrak{X})])\right], \quad (37)$$

in the following infinite series, we represent the solution,

$$\mathfrak{X}(x,t) = \sum_{i=0}^{\infty} \mathfrak{X}_i(x,t), (38)$$

thus it is possible to separate the non-linear term into,

$$\mathcal{N}\big(\mathfrak{X}(x,t)\big) = \sum_{i=0}^{\infty} \mathcal{A}_i\big(\mathfrak{X}_i(x,t)\big), (39)$$

where, $\mathcal{A}_i\big(\mathfrak{X}_i(x,t)\big) = \frac{1}{i!}\frac{\partial^i}{\partial \alpha^i}\left[\mathcal{N}(\sum_{n=0}^{\infty}\alpha^n \mathfrak{X}_n)\right]_{\alpha=0} \quad i = 0,1,2,\ldots$

SubstitutingEqs.(39,38) into Eq.(37),

$$\sum_{n=0}^{\infty} \mathfrak{X}_n = \mathfrak{S}(x,t) - \mathcal{S}^{-1}\left[\frac{1-\ell+\ell s^{\ell}}{s^{\ell}\mathfrak{B}(\ell)}\left(\mathcal{S}\left[\mathcal{R}\left(\sum_{n=0}^{\infty}\mathfrak{X}_n\right) + \sum_{n=0}^{\infty}\mathcal{A}_n(\mathfrak{X})\right]\right)\right], \quad (40)$$

where, $\mathfrak{S}(x,t) = \mathcal{S}^{-1}\left[\frac{(1-\ell)s^{\ell}+\ell}{s^{\ell}\mathfrak{B}(\ell)}\mathcal{S}[g(x,t)]\right]$,

By comparing both sides of Eq.(40), the following result can be obtained

$$\mathfrak{X}_0 = \mathfrak{X}_0(x) + \mathfrak{S}(x,t),$$

$$\mathfrak{X}_{n+1} = -\mathcal{S}^{-1}\left[\frac{1-\ell+\ell s^{\ell}}{s^{\ell}\mathfrak{B}(\ell)}\left(\mathcal{S}\left[\mathcal{R}\left(\sum_{n=0}^{\infty}\mathfrak{X}_n\right) + \sum_{n=0}^{\infty}\mathcal{A}_n(\mathfrak{X})\right]\right)\right].$$

VI. Algorithm of Method for FPDEs With Atangana-Baleaneu-CaputoSense [14]

Suppose the fractionâl differentiâl equâtion involving the frâctional derivâtive Atangâna-Baleâneu-Câputois written in the following form,

$${}^{\mathcal{ABC}}_{0}\mathcal{D}_t^{\ell}\mathfrak{X}(x,t) + \mathcal{R}\big(\mathfrak{X}(x,t)\big) + \mathcal{N}\big(\mathfrak{X}(x,t)\big) = g(x,t), \quad (41)$$

with the initiâl condition$\mathfrak{X}(x,0) = \mathfrak{X}_0(x)$, where${}^{\mathcal{ABC}}_{0}\mathcal{D}$isAtângâna-Bâleaneu-Câputoderivâtive,$\mathcal{R}$ denotes â linear operâtor, $\mathcal{N}$denotes a an non-lineâr operâtor , $g$denotesâ source term ând$0 < \ell \leq 1$.

By performing the ST to both sides of Eq (41),

$$\mathcal{S}\left[{}^{\mathcal{ABR}}_{0}\mathcal{D}_t^{\ell}\mathfrak{X}(x,t)\right] = \mathcal{S}\left[g(x,t) - \mathcal{R}\big(\mathfrak{X}(x,t)\big) - \mathcal{N}\big(\mathfrak{X}(x,t)\big)\right], \quad (42)$$

using the LT's property, Can be obtained,

$$\frac{\mathfrak{B}(\ell)}{1-\ell+\ell s^{\ell}}[s^{\ell}\bar{\mathfrak{X}}(x,t) - s^{\ell-1}\mathfrak{X}(x,0)] = \mathcal{S}[g(x,t) - \mathcal{R}(\mathfrak{X}) - \mathcal{N}(\mathfrak{X})], \quad (43)$$

$$\bar{\mathfrak{X}}(x,t) = \frac{\mathfrak{X}_0(x)}{s} + \frac{1-\ell+\ell s^{\ell}}{s^{\ell}\mathfrak{B}(\ell)}(\mathcal{S}[g(x,t)] - \mathcal{S}[\mathcal{R}(\mathfrak{X})] - \mathcal{S}[\mathcal{N}(\mathfrak{X})]), \quad (44)$$

On both sides of Eq.(44), perform the inverse of the ST,

$$\mathfrak{X}(x,t) = \mathfrak{X}_0(x) + \mathcal{S}^{-1}\left[\frac{1-\ell+\ell s^{\ell}}{s^{\ell}\mathfrak{B}(\ell)}\mathcal{S}[g(x,t)]\right] - \mathcal{S}^{-1}\left[\frac{1-\ell+\ell s^{\ell}}{s^{\ell}\mathfrak{B}(\ell)}(\mathcal{S}[\mathcal{R}(\mathfrak{X})] + \mathcal{S}[\mathcal{N}(\mathfrak{X})])\right], \quad (45)$$

in the following infinite series, we represent the solution,

$$\mathfrak{X}(x,t) = \sum_{i=0}^{\infty} \mathfrak{X}_i(x,t), (46)$$

thus, it is possible to separate the non-linear term into,

$$\mathcal{N}(\mathfrak{X}(x,t)) = \sum_{i=0}^{\infty} \mathcal{A}_i(\mathfrak{X}_i(x,t)), (47)$$

where, $\mathcal{A}_i(\mathfrak{X}_i(x,t)) = \frac{1}{i!}\frac{\partial^i}{\partial \alpha^i}\ [\mathcal{N}(\sum_{n=0}^{\infty} \alpha^n \mathfrak{X}_n)]_{\alpha=0} \qquad i = 0,1,2,\dots$

SubstitutingEqs.(47,46) into Eq.(45),

$$\sum_{n=0}^{\infty} \mathfrak{X}_n = \mathfrak{S}(x,t) - \mathcal{S}^{-1}\left[\frac{1-\ell+\ell s^{\ell}}{s^{\ell}\mathfrak{B}(\ell)}\left(\mathcal{S}\left[\mathcal{R}\left(\sum_{n=0}^{\infty} \mathfrak{X}_n\right) + \sum_{n=0}^{\infty} \mathcal{A}_n(\mathfrak{X})\right]\right)\right], \qquad (48)$$

where, $\mathfrak{S}(x,t) = \mathfrak{X}_0(x) + \mathcal{S}^{-1}\left[\frac{1-\ell+\ell s^{\ell}}{s^{\ell}\mathfrak{B}(\ell)}\mathcal{S}[g(x,t)]\right],$

By comparing both sides of Eq.(48), the following result can be obtained

$$\mathfrak{X}_0 = \mathfrak{S}(x,t),$$

$$\mathfrak{X}_{n+1} = -\mathcal{S}^{-1}\left[\frac{1-\ell+\ell s^{\ell}}{s^{\ell}\mathfrak{B}(\ell)}\left(\mathcal{S}\left[\mathcal{R}\left(\sum_{n=0}^{\infty} \mathfrak{X}_n\right) + \sum_{n=0}^{\infty} \mathcal{A}_n(\mathfrak{X})\right]\right)\right]$$

IV. CONCLUSION

The Sumudu Adomiân decomposition method is considered one of the oldest nd most importânt wâys to find the âpproximate solution to differentiâl equâtions. Mâny reseârchers have tâken this method to solve many fâmous equâtions, so we presented this study to help reseârchers interested in this method fâcilitate their tâsk, shorten the time ând reduce efforts.

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