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Research Paper

Micro ψ-Closed Sets in Micro Topological Spaces

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ABSTRACT: *The purpose of this paper is to introduce a new type of sets called Micro ψ-closed sets in Micro topological spaces and derive its properties and the interrelations between Microψ-closed sets with already existing Micro closed sets in Micro topological spaces.*
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KEYWORDS: *Micro closed set, Microαg-closed set, Micro ψ-closed set, Micro semi-$T_{1/3}$-space.*

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I. INTRODUCTION

The notion of approximation and boundary region of a set and concept of rough set theory was originallyproposed by Pawlak[8].Carmel Richard[4] introduced the concept of Nano topology in terms of approximation and boundary region of a subset of anuniverse using an equivalence relation on it.SakkraiveerananChandrasekar [9] introduced the concept of Micro topology which is an extension of Nano topology and he also introduced the concept of Micro pre-open and Micro semi-open sets.Chandrasekar and Swathi [5] introduced Micro α-open sets in Micro topological spaces and derived their properties. Ibrahim [6,7] defined Micro β-open sets and Micro g-closed sets in Micro topological spaces.Anandhi and Balamani [1] introduced the concept of Micro α-generalized closed sets and studied its properties. They[2] have also studied the concept of separation axioms related to Micro α-generalized closed sets in Micro topological spaces. In this paper we have introduced Micro ψ-closed sets in Micro topological spaces. Dependency and independency relations are obtained by comparing the Microψ-closed sets with already existing Micro closed sets. Later we have defined and analyzed Micro semi-$T_{1/3}$-space.

II. PRELIMINARIES

Definition 2.1 [8]Let$U$ be a nonempty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes.Elementbelonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space.Let$X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$.That is, $L_R(X) = \cup_{x\in U}\{R(x): R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.
2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.That is, $U_R(X)= \cup_{x\in U}\{R(x): R(x) \cap X \neq \phi\}$.
3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not$X$ with respect to R and it is denoted by $B_R(X)$.That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2[4]Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then $\tau_R(X)$satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$.
2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
3. The intersection of the elements of any finite sub-collection of $\tau_R(X)$is in $\tau_R(X)$.

That is, $\tau_R(X)$is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets and the complement of a Nano open set is called a Nano closed set.

Definition 2.3[9] Let$(U, \tau_R(X))$ be a Nano topological space. Then $\mu_R(X) = \{N \cup (N' \cap \mu): N, N' \in \tau_R(X)$ and $\mu \notin \tau_R(X)\}$ and $\mu_R(X)$satisfies the following axioms:

(i) $U, \phi \in \mu_R(X)$.

(ii) The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.

(iii) The intersection of the elements of any finite sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then, $\mu_R(X)$is called the Micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called Micro topological space and the elements of$\mu_R(X)$are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.4[9]Let$(U, \tau_R(X), \mu_R(X))$be a Micro topological space and$A \subseteq U$. Then the Micro closure of a set A is denoted by$Mic\text{-}cl(A)$ and is defined as

$Mic\text{-}cl(A) = \cap \{ K: K$ is Micro closed in U and $A \subseteq K\}$.

The Micro interior of a set A is denoted by $Mic\text{-}int(A)$ and is defined as

$Mic\text{-}int(A) = \cup \{K: K$ is Micro open in U and $A \supseteq K\}$.

Definition 2.5 Let $(U, \tau_R(X), \mu_R(X))$be a Micro topological space and $A \subseteq U$. Then A is said to be a

(i) Micro pre-open (briefly Mic-pre-open) set if $A \subseteq Mic\text{-}int(Mic\text{-}cl(A))$.[9]

(ii) Micro semi-open (briefly Mic-semi-open) set if $A \subseteq Mic\text{-}cl(Mic\text{-}int(A))$.[9]

(iii) Microα-open (briefly Mic-α-open) set if $A \subseteq Mic\text{-}int(Mic\text{-}cl(Mic\text{-}int(A)))$.[5]

(iv) Microβ-open(briefly Mic-β-open) set if $A \subseteq Mic\text{-}cl(Mic\text{-}int(Mic\text{-}cl(A)))$.[6]

The complements of the above-mentioned sets are called their respective closed sets.

Definition 2.6 [7]Let$(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro generalized closed(briefly Mic-g-closed) if $Mic\text{-}cl(A) \subseteq L$whenever $A \subseteq L$ and L is Micro openin U.

Definition 2.7 [1]Let$(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of $U$is said to be Micro generalized α-closed (briefly Mic-gα-closed) if $Mic\text{-}\alpha$cl(A)$\subseteq L$, whenever $A \subseteq$L and L is Micro α-open in U.

Definition 2.8 [3]Let$(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of $U$is said to be Micro semi generalized closed (briefly Mic-sg-closed) if $Mic\text{-}scl(A) \subseteq L$,whenever $A \subseteq L$ and L is Mic-semi-open in U.

Definition 2.9 [1]Let$(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of $U$is said to be Microα-generalized closed (briefly $Mic\text{-}\alpha g$-closed) if $Mic\text{-}\alpha cl(A) \subseteq L$, whenever $A \subseteq$L and L is Micro open in U.

Definition 2.10[3]Let$(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of $U$is said to be Micro generalized semi closed (briefly Mic-gs-closed) if $Mic\text{-}scl(A) \subseteq L$,whenever $A \subseteq L$ and L is Micro open in U.

Remark 2.11

1. A is a Micro closed set if and only if $A = Mic\text{-}cl(A)$.[9]
2. Every Micro closed set is Micro semi closed.[9]
3. Every Micro α-closed set is Micro semi closed.[5]

III.MICRO ψ-CLOSED SETS

In this section we introduce Micro ψ-closed sets in Micro topological spaces and derive dependency and independency relations of newly defined sets with already existing Micro closed sets.

Definition 3.1 Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of$U$ is said to be Micro ψ-closed if $Mic\text{-}scl(A) \subseteq L$ whenever $A \subseteq L$ and L is Mic-sg-open in U.

Proposition 3.2 Every Micro closed set in $(U, \tau_R(X), \mu_R(X))$is Micro ψ-closed but not conversely.

Proof: Let A be a Microclosed set then $Mic\text{-}cl(A) = A$. Let $A \subseteq L$ where L is Microsg-open in U. Since every Micro closed set is Micro semi closed, $Mic\text{-}scl(A) \subseteq Mic\text{-}cl(A)$.Thus $Mic\text{-}scl(A) \subseteq A \subseteq L$.Hence $Mic\text{-}scl(A) \subseteq L$. Therefore A is Micro ψ-closed**.**

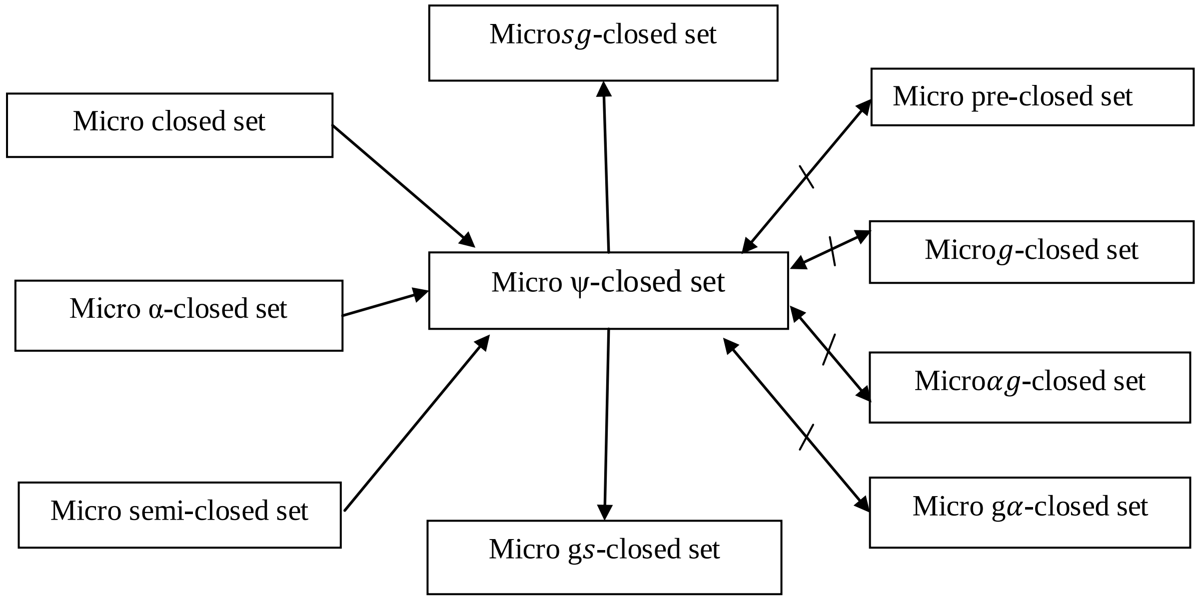
Example 3.3 Let$U = \{a, b, c, d\}$, U/R=$\{\{a\}, \{c\}, \{b, d\}\}$.Let X=$\{b, d\} \subseteq U$. Then $\tau_{\mathbf{R}}$(x)=$\{U, \phi, \{b, d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$.Then $\mu_R(X) = \{U, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$. Micro closed setsare $\phi, \{c\}, \{a, c\}, \{b, c, d\}, U$.Micro ψ-closed sets are $\phi, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, b, c\}, (a, c, d\}, \{b, c, d\}, U$.Here the subset $\{a, b, c\}$ is Micro ψ-closed, but not Micro closed.

Proposition3.**4**Every Micro semi closed set in$(U, \tau_R(X), \mu_R(X))$is Micro ψ- closed but not conversely.

Proof: Let A be a Micro semi closed set and L be any Micro sg- open set containing Ain U. Since A is Micro semi closed, $Mic\text{-}scl(A) = A \subseteq L, Mic\text{-}scl(A) \subseteq L$. Therefore A is Micro ψ- closed.

Example 3.5 Let $U = \{a, b, c, d\}, U/R$=$\{\{c\}, \{d\}, \{a, b\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{d\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U, \phi, \{d\}, \{a, b\}, \{a, b, d\}\}$.Micro semi closed sets are $\phi, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, U$. Micro ψ-closed sets are $\phi, \{c\}, \{d\}, \{a, b\}, \{c, d\}, \{a, b, c\}, U$, Here the subset $\{c\}$ is Micro ψ- closed, but not Micro semi closed.

Proposition 3.6 Every Micro α-closed set in $(U, \tau_R(X), \mu_R(X))$ is Micro ψ-closed but not conversely.
Proof: Let A be a Micro α-closed set and L be any Micro sg-open set containing A in U. Since A is Micro α-closed, $Mic\text{-}\alpha cl(A) = A$. Since every Micro α-closed set is Micro semi closed, $Mic\text{-}scl(A) \subseteq Mic\text{-}\alpha cl(A), Mic\text{-}scl(A) \subseteq A \subseteq L, Mic\text{-}scl(A) \subseteq L$.Therefore A is Micro ψ-closed.
Example 3.7 Let$U = \{a,b,c,d\}$, U/R={{a},{c},{b,d}}.Let X={b,d} $\subseteq U$. Then $\tau_{\mathbf{R}}$(x)=$\{U,\phi,\{b,d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$.Then $\mu_R(X) = \{U,\phi,\{a\},\{b,d\},\{a,b,d\}\}$.Micro α-closed sets are $\phi,\{c\},\{a,c\},\{b,c,d\},U$.Micro ψ-closed sets are $\phi,\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\},U$.Here the subset $\{a,b,c\}$ is Micro ψ-closed, but not Micro α-closed.
Proposition 3.8 Every Micro semi closed set is Micro sg-closed.
Proof: Let A be a Micro semi closed set and L be any Micro semi-open set containing Ain U. Since A is Micro semi closed, $Mic\text{-}scl(A) = A, Mic\text{-}scl(A) = A \subseteq L$. Therefore A is Micro sg- closed.
Proposition 3.9 Every Micro ψ-closed set in $(U, \tau_R(X), \mu_R(X))$ is Micro sg-closed but not conversely.
Proof: Let A be a Micro ψ-closed set and L be any Micro semi open set containing Ain$U$. Sinceevery semi open set is Micro sg-open and A is Micro ψ-closed, $Mic\text{-}scl(A) \subseteq L$. Therefore A is Micro sg-closed.
Example 3.10 Let$U = \{a,b,c,d\}$, U/R={{a,b},{c,d}}.Let X={a,b,c} $\subseteq U$. Then $\tau_{\mathbf{R}}$(x)=$\{U,\phi,\{a,b\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$.Then $\mu_R(X) = \{U,\phi,\{c\},\{a,b\},\{a,b,c\}\}$.Micro ψ-closed sets are $\phi,\{d\},\{a,d\},\{b,d\},\{c,d\},\{a,b,d\},U$.Micro sg-closed sets are $\phi,\{d\},\{a,d\},\{b,d\},\{c,d\},(a,c,d\},\{a,b,d\},\{b,c,d\},U$.Here the subset $\{a,c,d\}$ is Micro sg-closed, but not Micro ψ-closed.
Proposition 3.11 Every Micro ψ-closed set in $(U, \tau_R(X), \mu_R(X))$ is Micro gs-closed but not conversely.
Proof: Let A be a Micro ψ-closed set and L be any Micro open set containing Ain U. Since every Micro open set is Micro sg-open and A is Micro ψ-closed, $Mic\text{-}scl(A) \subseteq L$.Therefore A is Micro gs-closed.
Example 3.12 Let$U = \{a,b,c,d\}$, U/R={{c},{d},{a,b}}.Let X={a,b} $\subseteq U$. Then $\tau_{\mathbf{R}}$(x)=$\{U,\phi,\{a,b\}\}$. Let $\mu = \{d\} \notin \tau_R(X)$.Then $\mu_R(X) = \{U,\phi,\{d\},\{a,b\},\{a,b,d\}\}$.Micro ψ-closed sets are $\phi,\{c\},\{d\},\{a,b\},\{c,d\},\{a,b,c\},U$.Micro gs-closed sets are $\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{b,c,\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\},U$.Herethe subset $\{b,c,d\}$ is Micro gs-closed, but not Micro ψ-closed.
Remark 3.13 The following example shows that Microψ-closed set is independent from Micro pre-closed set.
Example 3.14 Let $U = \{a,b,c,d\}, U/R = \{\{a\},\{b\},\{c,d\}\}$. Let $X = \{c,d\} \subseteq U$. Then $\tau_R(X) = \{U,\phi,\{c,d\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{U,\phi,\{a\},\{c,d\},\{a,c,d\}\}$.Micro pre- closed sets are $\phi,\{b\},\{c\},\{d\},\{a,b\},\{b,c,\},\{b,d\},\{a,b,c\},\{b,c,d\},U$.Micro ψ- closed sets are $\phi,\{b\},\{a,b\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\},\{b,c,d\},U$.Here the subset $\{c\}$ is Micro pre-closed, but not Micro ψ-closed and the subset $\{a,b,d\}$ is Micro ψ-closed, but not Micro pre-closed.
Remark 3.15 The following example shows that Microψ-closed set is independent from Microαg-closed set.
Example 3.16 Let$U = \{a,b,c,d\}$, U/R={{c},{d},{a,b}}.Let X={a,b} $\subseteq U$. Then $\tau_R(X)$=$\{U,\phi,\{a,b\}\}$. Let $\mu = \{d\} \notin \tau_R(X)$.Then $\mu_R(X) = \{U,\phi,\{d\},\{a,b\},\{a,b,d\}\}$.Micro αg-closed sets are$\phi,\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\},U$. Micro ψ-closed sets are $\phi,\{c\},\{d\},\{a,b\},\{c,d\},\{a,b,c\},U$.Here the subset $\{a,c\}$ is Micro αg-closed, but not Microψ-closed and the subset $\{d\}$ is Micro ψ-closed, but not Micro αg-closed.
Remark 3.17 The following example shows that Microψ-closed set is independent from Micro$g$-closed set.
Example 3.18 Let$U = \{a,b,c\}$, $U/R = \{\{c\},\{a,b\}\}$. Let $X = \{a,b\} \subseteq U$.Then $\tau_R(X) = \{U,\phi,\{a,b\}\}$. Let $\mu = \{a\} \notin \tau_R(X)$.Then $\mu_R(X) = \{U,\phi,\{a\},\{a,b\}\}$. Micro g-closed sets are $\phi,\{c\},\{a,c\},\{b,c\},U$. Micro ψ-closed sets are $\phi,\{b\},\{c\},\{b,c\},U$.Here the subset $\{a,c\}$is Micro g-closed, but not Micro ψ-closed and the subset $\{b\}$ is Micro ψ-closed, but not Micro g-closed.
Remark 3.19 The following example shows that Micro ψ-closed set is independent from Micro$g\alpha$-closed set.
Example 3.20 Let$U = \{a,b,c,d\}$, U/R={{c},{d},{a,b}}.Let X={a,b} $\subseteq U$. Then $\tau_R(X)$=$\{U,\phi,\{a,b\}\}$. Let $\mu = \{d\} \notin \tau_R(X)$.Then $\mu_R(X) = \{U,\phi,\{d\},\{a,b\},\{a,b,d\}\}$.Micro $g\alpha$-closed sets are $\phi,\{c\},\{a,c\},\{b,c\},\{c,d\},\{a,b,c\},\{a,c,d\},\{b,c,d\},U$. Micro ψ-closed sets are $\phi,\{c\},\{d\},\{a,b\},\{c,d\},\{a,b,c\},U$.Here the subset $\{b,c\}$ is Micro $g\alpha$-closed, but not Micro ψ-closed and the subset $\{a,b\}$ is Micro ψ -closed but not Micro $g\alpha$-closed.
Remark 3.21 The following diagram shows the dependency and independency relations of Micro ψ-closed set with already existing Microclosed sets.



IV. PROPERTIES OF MICRO ψ-CLOSED SETS

In this section we derive the fundamental properties of Micro ψ-closed sets.

Theorem 4.1 If A is a Micro ψ-closed set of $(U, \tau_R(X), \mu_R(X))$ such that $A \subseteq B \subseteq Mic\text{-}scl(A)$,then B is also a Micro ψ-closed set of $(U, \tau_R(X), \mu_R(X))$.

Proof: Suppose that$A$ is a Micro ψ-closed set.Let L be a $Mic\text{-}sg$-open set such that $B \subseteq L$. Then $A \subseteq L$. Since A is Micro ψ-closed, $Mic\text{-}scl(A) \subseteq L$. Now $Mic\text{-}scl(B) \subseteq Mic\text{-}scl(Mic\text{-}scl(A)) = Mic\text{-}scl(A) \subseteq L$. Therefore B is also a Micro ψ- closed set of $(U, \tau_R(X), \mu_R(X))$.

Theorem 4.2 If A is both Micro sg-open and Micro ψ-closed set in $(U, \tau_R(X), \mu_R(X))$, then A is Micro semi closed in $(U, \tau_R(X), \mu_R(X))$.

Proof: Let A be Micro sg-open and Micro ψ-closed set in $(U, \tau_R(X), \mu_R(X))$ then by the definition of Micro ψ-closed set, $Mic\text{-}scl(A) \subseteq A$. Always $A \subseteq Mic\text{-}scl(A)$. Therefore $Mic\text{-}scl(A) = A$. Hence A is Micro semi closed.

Remark 4.3 The intersection of any two Micro ψ-closed sets in $(U, \tau_R(X), \mu_R(X))$ need not to be a Micro ψ-closed set in $(U, \tau_R(X), \mu_R(X))$ as seen from the following example.

Example 4.4 Let$U = \{a, b, c, d\}$, U/R=$\{\{c\}, \{d\}, \{a, b\}\}$.Let X=$\{c\} \subseteq U$. Then $\tau_R(X)$=$\{U, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$.Then$\mu_R(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, U\}$. Micro ψ- closed sets are $\phi, \{a\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, U$.Herethe subsets $\{a, c\}$ and $\{c, d\}$ are Micro ψ-closed sets but their intersection $\{a, c\} \cap \{c, d\} = \{c\}$ is not Micro ψ-closed.

Theorem 4.5 Let A be a Microψ- closed set of$(U, \tau_R(X), \mu_R(X))$.Then $Mic\text{-}scl(A) - A$ contains no non empty micro closed set.

Proof: Suppose that A is a Micro ψ- closed set of$(U, \tau_R(X), \mu_R(X))$. Let F be a Micro closed subset of $Mic\text{-}scl(A) - A$.Then F^c is Micro open and hence Micro sg-open such that $A \subseteq F^c$. Since A is a Micro ψ-closed set,$Mic\text{-}scl(A) \subseteq F^c$.Thus $F \subseteq (\text{Mic-}scl(A))^c$.Therefore $F \subseteq (\text{Mic-}scl(A))^c \cap Mic\text{-}scl(A) = \phi$. Hence $F = \phi$.

Theorem 4.6 Let A be a Micro ψ- closed set of$(U, \tau_R(X), \mu_R(X))$ if and only if $Mic\text{-}scl(A) - A$ does not contain any non empty Micro sg-closed set.

Proof: **Necessity:** Suppose that A is a Micro ψ- closed set of$(U, \tau_R(X), \mu_R(X))$. Let F be a Micro sg-closed set such that $F \subseteq Mic\text{-}scl(A) - A$.Then$A \subseteq F^c$. Since A is a Micro ψ-closed set and $F^c$is Micro sg-open, then $Mic\text{-}scl(A) \subseteq F^c$. This implies $F \subseteq (Mic\text{-}scl(A))^c$.So $F \subseteq (Mic\text{-}scl(A))^c \cap (Mic\text{-}scl(A) - A) \subseteq (Mic\text{-}scl(A))^c \cap Mic\text{-}scl(A) = \phi$. Therefore $F = \phi$.

Sufficiency: Suppose that $Mic\text{-}scl(A) - A$ contains no non empty Micro sg-closed set. Let $A \subseteq H$ and H be Micro sg-open. If $Mic\text{-}scl(A)$ is not a subset of H then $Mic\text{-}scl(A) \cap H^C$ is a non empty Micro sg-closed subset of $Mic\text{-}scl(A) - A$, which is a contradiction. Therefore $Mic\text{-}scl(A) \subseteq H$ and hence A is Micro ψ-closed.

Theorem 4.7 If $Mic\text{-}$scl$(\{x\}) \cap A \neq \phi$ holds for every $x \in Mic\text{-}scl(A)$, then $Mic\text{-}scl(A) - A$ does not contain a non-empty Micro semi closed set.

Proof: Suppose there exists a non-empty Micro semi closed set F such that $F \subseteq Mic\text{-}scl(A) - A$. Let $\in F$, then $x \in Mic\text{-}scl(A)$. It follows that $F \cap A = [Mic\text{-}scl(A) - A] \cap A \supseteq Mic\text{-}scl(\{x\}) \cap A \neq \phi$. Hence$F \cap A \neq \phi$, which is a contradiction. Thus $F = \phi$.

Theorem 4.8 Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. Then, for each $x \in U$, either $\{x\}$ is Micro sg-open or $U - \{x\}$ is Micro ψ-closed.

Proof: Let $x \in U$ and $\{x\}$ is not Micro sg-closed in U. Then $U - \{x\}$ is not Micro sg-open in U. Hence U is the only Micro sg-open set containing $U - \{x\}$. i.e. $U - \{x\} \subseteq U$. Hence Mic-scl$(U - \{x\}) \subseteq U$. Thus $U - \{x\}$ is Micro ψ-closed.

Theorem 4.9 Let A be a Micro ψ-closed set of $(U, \tau_R(X), \mu_R(X))$. Then A is Micro semi-closed if and only if $Mic\text{-}scl(A) - A$ is Micro sg-closed.

Proof: **Necessity:** Let A be a Micro semi closed subset of $(U, \tau_R(X), \mu_R(X))$, then $Mic\text{-}scl(A) = A$ and therefore $Mic\text{-}scl(A) - A = \phi$ which is a Micro sg-closed set.

Sufficiency: Let $Mic\text{-}scl(A) - A$ be a Micro sg-closed set. Since A is Micro ψ-closed by Theorem 4.6, $Mic\text{-}scl(A) - A$ does not contain any non empty Micro sg-closed set which implies $Mic\text{-}scl(A) - A = \phi$. (i.e) $Mic\text{-}scl(A) = A$. Hence A is Micro semi-closed.

Definition 4.10 A Micro topological space $(U, \tau_R(X), \mu_R(X))$ is said to be a Micro semi-$T_{1/3}$ -space (briefly Mic-semi-$T_{1/3}$-space) if every Micro ψ-closed subset of $(U, \tau_R(X), \mu_R(X))$ is Micro semi-closed in $(U, \tau_R(X), \mu_R(X))$.

Example4.11 Let $U = \{a, b, c\}$, $U/R = \{\{a, b, c\}\}$, Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{a, b\}\}$. Let $\mu = \{c\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{c\}, \{a, b\}, U\}$. Micro semi-closed sets are $\phi, \{c\}, \{a, b\}, U$. Micro ψ-closed sets are $\phi, \{c\}, \{a, b\}, U$. Therefore U is a Micro semi- $T_{1/3}$-space.

Theorem 4.12 For a Micro topological space $(U, \tau_R(X), \mu_R(X))$, the following conditions are equivalent:

1. $(U, \tau_R(X), \mu_R(X))$ is a Micro semi-$T_{1/3}$ -space.
2. Every $\{x\}$ is either Micro sg-closed or Micro semi-open.

Proof: 1⇒2 Let $x \in U$. Suppose that $\{x\}$ is not Micro sg-closed set of U. Then $U - \{x\}$ is not a Micro sg open set. Hence U is only Micro sg open set containing $U - \{x\}$. So $U - \{x\}$ is Micro ψ-closed set in U. Since $(U, \tau_R(X), \mu_R(X))$ is Micro semi-$T_{1/3}$ space, $U - \{x\}$ is Micro semi-closed in U. Hence $\{x\}$ is Micro semi open in $(U, \tau_R(X), \mu_R(X))$.

2⇒1 Let A be Micro ψ-closed set of $(U, \tau_R(X), \mu_R(X))$ and $x \in Mic\text{-}scl(A)$. We show that $x \in A$ for the following cases.

Case(1): Assume that $\{x\}$ is Micro sg-closed and $x \notin A$. Then $Mic\text{-}scl(A) - A$ contains a non-empty Micro sg-closed $\{x\}$. This contradicts Theorem 4.6 as A is a Micro ψ-closed set. Therefore $x \in A$.

Case(2): Assume that $\{x\}$ is Micro semi open set. Then $U - \{x\}$ is Micro semi-closed. If $x \notin A$, then $A \subseteq U - \{x\}$. Since $x \in Mic\text{-}scl(A)$, we have $x \in U - \{x\}$, which is a contradiction. Hence $x \in A$.

V. CONCLUSION

The study of Micro ψ-closed sets in Micro topological spaces have been initiated in this article and its basic properties have been analyzed. FurtherMicro ψ-continuous maps and Micro ψ-irresolute maps in Micro topological spaces can be continued as a future work.

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