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Research Paper

Influence of LTNE on Double Diffusive Convection in a Saturated Porous Layer with Soret Effect

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ABSTRACT: *Double diffusive convection in a fluid-saturated porous layer, heated from below and cooled from above, is investigated under the influence of local thermal non-equilibrium. Both linear and nonlinear stability analyses are employed to study the system behavior. The Darcy model, including the time derivative term, is used for the momentum equation. A two-field model is adopted, where the fluid and solid phases are allowed to have separate temperature fields in the energy equation. Analytical expressions for the onset of stationary, and oscillatory, convection are derived. The results indicate that even a small interphase heat transfer coefficient significantly affects the stability characteristics of the system. A competition between thermal and solutal diffusion processes determines whether convection occurs in stationary, oscillatory, or finite-amplitude modes. The influence of key parameters solute Rayleigh number, porosity-modified conductivity ratio, Lewis number, diffusivity ratio (τ), Darcy number, Prandtl number, and Soret parameter on convection and heat and mass transfer is analyzed and presented graphically.*

KEYWORDS: *Double-diffusive convection (DDC), Porous media, Internal heat source, Solute Rayleigh number, Soret parameter, LTNE model, Heat and mass transfer, Rayleigh number, linear stability analysis*

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I. INTRODUCTION

Double diffusive convection in porous media has attracted considerable attention due to its importance in both fundamental research and industrial applications. These include high-quality crystal growth, liquefied gas storage, moisture migration in fibrous insulation, contaminant transport in saturated soils, and the solidification of magma. Comprehensive reviews on this topic are available in the works of Ingham and Pop [1, 2], Vafai [3,4], Nield and Bejan [5], and Vadasz [6]. Convective instability in binary fluid systems has been extensively studied through both experimental and theoretical approaches. Notable reviews include those by Turner [7], Pltten [8], Huppert and Turner [9], Huppert and Sparks [10]. Banu and Ree [11], Platten and Legros [12] . In systems involving two diffusing components, instability typically arises when at least one component has a destabilizing effect. However, when cross-diffusion effects (such as the Soret effect) are included in the species transport equations, the behavior of the system changes significantly. In such cases, the gradient of one property can strongly influence the flux of the other, leading to more complex convection patterns. A flux of a salt caused by a spatial gradient of temperature is called the Soret effect. Similarly, a flux of heat caused by a spatial gradient of concentration is called the Dufour effect. The Dufour coefficient is of order of magnitude smaller than the Soret coefficient in liquids, and the corresponding contribution to the heat flux may be neglected.

Many studies can be found in the literature concerning the Soret and Dufour effects. A study by Rudraiah and Malashetty [13] discussed the double diffusive convection in a porous medium in the presence of Soret and Dufour effects. In another study, Rudraiah and Siddheshwar [14] investigated a weak non-linear stability analysis of double diffusive convection with cross diffusion in a fluid saturated porous medium. Recently, Gaikwad, et.al.,[15-16],studied the Soret effect on onset of convection in a binary viscoelastic fluid-saturated porous layer. many researchers have studied the onset of convection in a fluid saturated anisotropic porous layer. Gaikwad and Kamble [17], investigated double diffusive convection in a fluid saturated anisotropic porous layer with Soret effect. Altwallbeh et al [18] have investigated the linear and non-linear DDC in a saturated porous layer with Soret effect and internal heat source.

It is well known that Darcy's law is not valid for non-Newtonian fluid flows in porous media. Recently, Malashetty et.al.,[19], studied the double diffusive convection in a porous layer using a thermal non-equilibrium model., in another study, Malashetty, and Swamy, [20]. The onset of double diffusive convection in a viscoelastic fluid layer. The effect of thermal and solutal gradients in porous convection is investigated by Shivakumara, and Sureshkumar [21]. Since the Porous media provide an additional layer of complexity due to their internal structure, which affects momentum, heat, and mass transport. Traditional models often assume local thermal equilibrium (LTE) between fluid and solid phases. However in many real-world applications especially those involving high heat fluxes, low conductivity solids, or rapid transient processes the assumption of LTE is not valid. Instead of this local thermal non-equilibrium (LTNE) model must be employed where separate energy equations are considered for fluid and solid phases, allowing for temperature differences between.

Although some literature on double diffusive convection in a porous medium saturated by ordinary fluid with or without an internal heat source and Soret effect is available and has been investigated by Cheluvaraj, et.al.[22], Bennacer, and Beji [23], *Ren* [24], and Postelnicu [25]. But a very little attention has been devoted to this study in which DDC in a porous layer saturated by a binary viscoelastic fluid in presence of an internal heat source and Soret effect.

The coupled effects of LTNE and Soret-induced thermodiffusion on double-diffusive instability in porous media remain relatively unexplored. This study develops an LTNE-based stability framework incorporating the Soret effect to quantify its influence on the critical conditions and onset of convection. The results provide new insight into the coupled heat–mass transport behaviour of fluid-saturated porous layers. The main objective of this study in the present paper we intend to perform the onset criteria for stationary and oscillatory mode of convection by considering the linear stability analysis. Further we study the effect of Soret coefficient, Darcy–Prandtl number, specific heat ratio (γ) on the DDC in a fluid saturated porous layer using LTNE model. is a weight suspended from a pivot so that it can swing freely. When a pendulum is displaced sideways from its resting equilibrium position, it is subject to a restoring force due to gravity that will accelerate it back toward the equilibrium position [1] and [2] . When released, the restoring force combined with the pendulum's mass causes it to oscillate about the equilibrium position, swinging back and forth. The time for one complete cycle, a left swing and a right swing, is called the period. The period depends on the length of the pendulum and also on the amplitude of the oscillation. However, if the amplitude is small, the period is almost independent of the amplitude [3] and [4].

Double pendulum is a mechanical system that is most widely used for demonstration of the chaotic motion. It is described with two highly coupled, nonlinear, 2nd order ODE's which makes is very sensitive to the initial conditions[5] and [6]. Although its motion is deterministic in nature, sensitivity to initial conditions makes its motion unpredictable or 'chaotic' in the long turn in this paper discusses in the first part purpose of the double pendulum [7], in the second section, the system of coordinates is presented and in the third section, the equations of motion it's numerical solutions are investigated by using ODEs 45. Whereas in the final section, behavior of the system and simulation of the double pendulum are discussed by this paper and explain how to linearize the double pendulum investigate modeling the Linearization Error.

This paper is not only analyzed the dynamics of the double pendulum system and discussing the physical system, but also explain how the Lagrangian and the Hamiltonian equations of motions are derived, we will analyze and compare between the numerical solution and simulation, and also change of angular velocities with time for certain system parameters at varying initial conditions.

II. GOVERNING EQUATIONS

We consider a fluid-saturated porous layer bounded by two infinitely extended, horizontal planes located at $z = 0$, and $z = d$. The porous layer is heated from below and cooled from above, thereby establishing an adverse temperature gradient across the layer. The Darcy model is adopted to describe the fluid motion within the porous medium. A Cartesian coordinate system is introduced with its origin on the lower boundary, while the z - axis is directed vertically upward.

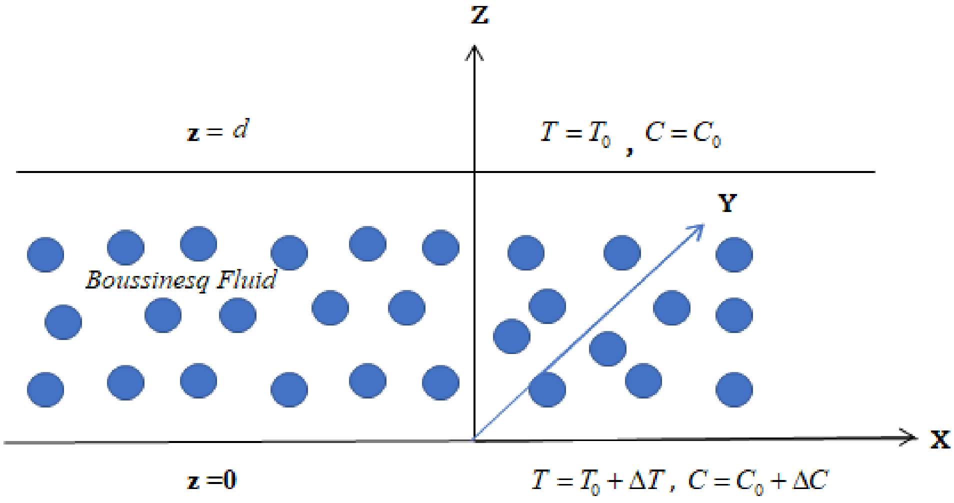

Fig.1. Physical model

The temperatures of the lower and upper boundaries are prescribed as $T_0 + \Delta T$,and T_0 , respectively. Also, the corresponding solute concentrations at the lower and upper boundaries are maintained at $C_0 + \Delta C$ and C_0 , respectively. Under these assumptions, the resulting temperature and concentration gradients drive the convective motion within the porous layer. The governing equations describing the coupled momentum, thermal, and solutal transport processes are expressed as follows:

$$\nabla . q = 0 , \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial q}{\partial t} + = -\nabla p + \rho g - \frac{\mu}{K} q , \tag{2}$$

$$\varepsilon(\rho_o \mathrm{c})_\mathrm{f} \frac{\partial T_f}{\partial t} + (\rho_o \mathrm{c})_\mathrm{f} (q.\nabla) T_f = \varepsilon k_f \nabla^2 T_f + \mathrm{h}(T_s - T_f) , \tag{3}$$

$$(1-\varepsilon)\left((\rho.\mathrm{c})_s \frac{\partial T_s}{\partial t} - k_s \nabla^2 T_s \right) = -\mathrm{h}(T_s - T_f) , \tag{4}$$

$$\varepsilon \frac{\partial q}{\partial t} + (q.\nabla) C = -\varepsilon \kappa_c \nabla^2 C + \varepsilon D \nabla^2 T_f , \tag{5}$$

$$\rho = \rho_0 [\beta_T (T_f - T_0) - \beta_S (C - C_0)] , \tag{6}$$

2.1 Basic state

The basic state of a fluid is assumed to be quiescent and is given by

$$q_b = (0,0,0), \quad p = p_b(\mathrm{z}), \quad C = C_b(\mathrm{z}), \quad T_f = T_{fb}(\mathrm{z}), \quad T_s = T_{sb}(\mathrm{z}), \quad d = 0, \tag{7}$$

The basic state of temperatures and concentration satisfy the Eqs. (1) - (6) yield,

$$\frac{d^2 T_{fb}}{dz^2} = 0 , \quad \frac{d^2 T_{sb}}{dz^2} = 0 , \quad \frac{d^2 C_b}{dz^2} = 0 , \tag{8}$$

With boundary conditions

$T_{fb} = T_{sb} = T_0 + \Delta T$, and $C_b = C_0 + \Delta C$, at z = 0, (9)

$T_{fb} = T_{sb} = T_0$, and $C_b = C_0$, at z = d, (10)

2.2 Perturbed state

The basic state is perturbed by introducing small disturbances in the following form,

$$\mathrm{q} = \mathrm{q_b} + \mathrm{q}' ,\quad \mathrm{T} = \mathrm{T_b} + \mathrm{T}' ,\quad p = p_b + p' ,\quad \mathrm{C} = \mathrm{C_b} + \mathrm{C}' ,\quad \rho = \rho_b + \rho' , \tag{11}$$

where the prime indicates the perturbation quantities. Using Eq.(11) into Eqs. (1) to (6), together with equations (7) to (8) yields the corresponding perturbation equations,

$$\nabla.\mathrm{q}' = 0 , \tag{12}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \mathrm{q}'}{\partial t} + \nabla p' = \rho g - \frac{\mu}{K}\mathrm{q}', \tag{13}$$

$$(\rho c)_f \left(\varepsilon \frac{\partial}{\partial t} + q'.\nabla \right) T_f' = \varepsilon k_f \nabla^2 T_f' + h(T_s' - T_f'), \tag{14}$$

$$(1-\varepsilon)\left((\rho.c)_s \frac{\partial}{\partial t} + k_s \nabla^2 \right)(T_s') = h\left(T_s' - T_f'\right) , \tag{15}$$

$$\left(\varepsilon \frac{\partial}{\partial t} + q'.\nabla - \varepsilon k_c \nabla^2 \right) C' = \varepsilon D \nabla^2 T_f{}' , \tag{16}$$

$$\rho = -\rho_0\left(\beta T_f' - \beta_c C'\right) , \tag{17}$$

Using the following transformations, then the resulting Eqs.(12) to (17) are non-dimensionalised ,

$$\left(x, y, z\right) = d\left(x*, y*, z*\right) ,\quad t = \frac{(\rho.c)_f d^2}{k_f} ,\quad \left(u', v', w'\right) = \frac{\varepsilon k_c}{(\rho.c)d}\left(u*, v*, w*\right),$$
$$T_f{}' = \Delta T_f^* ,\quad C' = \Delta C^* . \tag{18}$$

The pressure term is eliminated by applying the curl operator twice on the momentum equation. Then the resulting dimensionless equations are obtained in the form (on dropping the asterisks),

$$\left(1 + \frac{1}{\mathrm{Pr}_D}\right)\nabla^2\Psi + Ra_T \frac{\partial T_f}{\partial x} - Ra_C \frac{\partial C}{\partial x} = 0 , \tag{19}$$

$$\left(\frac{\partial}{\partial t} + q.\nabla - \nabla^2 \right) T_f = w + H\left(T_s - T_f\right) , \tag{20}$$

$$\left(\alpha \frac{\partial}{\partial t} - \nabla^2 \right) T_s = -H\gamma\left(T_s - T_f\right) , \tag{21}$$

$$\left(\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2 \right) C - Sr\frac{Ra_T}{Ra_c}\nabla^2 T_f = w - \left(q.\nabla\right) C . \tag{22}$$

Where $Ra_T = \dfrac{\beta_T g \Delta T d K}{\varepsilon \nu \kappa_f}$, $Ra_C = \dfrac{\beta_c g \Delta C d K}{\varepsilon \nu \kappa_f}$, $\Pr_D = \dfrac{\varepsilon \nu d^2}{k_f K}$, $\Pr = \dfrac{\nu}{k_f}$, $H = \dfrac{h d^2}{k_f \varepsilon}$,

$\gamma = \dfrac{k_f \varepsilon}{k_s(1-\varepsilon)}$, $\gamma H = \dfrac{h d^2}{k_s(1-\varepsilon)}$, $\alpha = \dfrac{k_f}{k_s}$.

Where the above system of equations will be solved by neglecting the Jacobean terms and considering stress free and isothermal boundary conditions as mentioned below,

$$\Psi = \frac{\partial^2 \Psi}{\partial z^2} = \Theta = \Phi = \xi = 0, \text{ on z = 0, and z =1.} \tag{23}$$

III. LINEAR STABILITY ANALYSIS

The stability of the basic state is examined by applying linear stability theory to Eqs. (16) – (18), subject to the boundary conditions in Eq. (23). Small-amplitude, time-dependent periodic disturbances are introduced in the horizontal directions in the normal-mode form,

$$(\Psi, T_f, T_c, C) = (W, \Theta, \Phi, \zeta) \, exp\,(i(lx + my) + \sigma\, t), \tag{24}$$

Where *l,* and *m* are horizontal wave numbers, and the growth rate be $\sigma = \sigma_r + i\sigma_i$.

Now Substitution of these disturbances Eq. (24) into the linearized Eqs.(16) to (18) transforms the governing system into an eigenvalue problem, which is used to determine the conditions for the onset of convection.

$$\left(1 + \frac{\sigma}{\Pr_D}\right)\delta^2 \Psi_0 - aRa_T \Theta_0 + aRa_C \xi_0 = 0, \tag{25}$$

$$a\Psi_0 - (\sigma + H + \delta^2)\Theta_0 + H\Phi_0 = 0, \tag{26}$$

$$H\gamma\Theta_0 - (\alpha\sigma + H\gamma + \delta^2)\Phi_0 = 0, \tag{27}$$

$$-a\Psi_0 + Sr\delta^2 \frac{Ra_T}{Ra_c}\Theta_0 + (\sigma + \frac{\delta^2}{Le_c})H\xi_0 = 0, \tag{28}$$

Since, D =d/dz, and $a^2 = l^2 + m^2$, and *W,* Θ, Φ, ξ are the amplitudes of the stream function, Ψ. We assume the solutions W, Θ, Φ, and ξ as –

$$(W, \Theta, \Phi, \zeta) = (W_0, \Theta_0, \Phi_0, \zeta_0) Sin(n\pi z), \quad \text{(n = 1, 2 3, ...),} \tag{29}$$

Where Since the fundamental mode n = 1 represents the most unstable mode, we set n = 1 in the expressions given above and substitute them into Eqs. (25) – (28). This yields the homogeneous matrix system,

AX = 0.

That is.,

$$\begin{pmatrix} \left(1+\frac{\sigma}{\Pr_D}\right)\delta^2 & -aRa_T & 0 & aRa_C \\ a & -\left(\sigma+\delta^2+H\right) & H & 0 \\ 0 & \gamma H & -\left(\alpha\sigma+\delta^2+\gamma H\right) & 0 \\ -a & \frac{Ra_T\delta^2 Sr}{Ra_C} & 0 & \frac{\delta^2}{Le}+\sigma \end{pmatrix} + \begin{pmatrix} W_0 \\ \Theta_0 \\ \Phi_0 \\ \Psi_0 \end{pmatrix} = 0. \qquad (30)$$

For a non-trivial solution, the determinant of matrix, A must vanish. The resulting condition provides the thermal Rayleigh number in the form,

$$Ra_T = \frac{\left(a^2Ra_C+\delta^2\left(1+\frac{\sigma}{\Pr_D}\right)\left(\sigma+\frac{\delta^2}{Le}\right)\right)\left\{\left(\sigma+H+\delta^2\right)\left(\alpha\sigma+\delta^2+H\gamma\right)-\gamma H^2\right\}}{a^2\left(\sigma+\frac{\delta^2}{Le}+\delta^2 Sr\right)\left(\gamma H+\alpha\sigma+\delta^2\right)}, \qquad (31)$$

Where $\delta^2 = a^2+\pi^2$. In general, the growth rate , σ be a complex quantity so that $\sigma = \sigma_r + i\sigma_i$. The system with $\sigma_r < 0$ is always stable while it will becomes unstable for $\sigma_r > 0$. And for neutral stability $\sigma_r = 0$.

3.1. Marginal State

For the threshold of stationary instability, the growth rate is set to $\sigma = 0$. Under this marginal stability condition, the thermal Rayleigh number corresponding to the stationary mode is obtained as

$$Ra_T^{St} = \frac{\left(a^2Ra_C Le+\delta^2\left(\delta^2\right)\right)\left\{\left(H+\delta^2\right)\left(\delta^2+H\gamma\right)-\gamma H^2\right\}}{a^2\delta^2\left(1+SrLe\right)\left(\gamma H+\delta^2\right)}, \qquad (32)$$

It should be noted that the critical Rayleigh number Ra_{Tc}, is a function of the critical wave number $a = a_c^{St}$ and it depends on H, solute Rayleigh number Ra_C , Lewis number, soret parameter of the system.

For the solute Rayleigh number, $Rac = 0$, by Eq. (32), we get

$$Ra_T^{St} = \frac{\delta^2\left(H(1+\gamma)\delta^2+\delta^4\right)}{a^2\left(1+SrLe\right)\left(\gamma H+\delta^2\right)}, \qquad (33)$$

In the absence of soret parameter i.e., $Sr = 0$, Eq.(33) gives

$$Ra_T^{St} = \frac{\delta^2\left(H(1+\gamma)\delta^2+\delta^4\right)}{a^2\left(\gamma H+\delta^2\right)}, \qquad (34)$$

This result agrees with the classical findings of Banu and Rees[29] for single component convection in a porous layer under the LTNE framework.

The system without heat source, i.e., when H = 0, then Eq.(32) reduces to

$$Ra_{T}^{St} = \frac{(a^2+\pi^2)^2}{a^2}, \tag{35}$$

This is the classical result obtained by Horton and Rogers [26], and Lapwood [27], which has the critical value $Ra_{T,c}^{St} = 4\pi^2$ for $a_c^{St} = \pi$.

3.2. Oscillatory State

For the onset of oscillatory convection, the growth rate is taken as purely imaginary, i.e., σ = iσ, where σ denotes the oscillation frequency. Substituting this relation into Eq. (31) and rationalizing the denominator to eliminate the complex quantity gives the following expression:

$$Ra_T = \Delta_1 + i\sigma_i\Delta_2, \tag{36}$$

Where the expression for Δ_1, Δ_2 are not presented for brevity.

Since Ra_T is a physical quantity, it must be real. So form Eq. (36), it follows that either $\sigma = 0$ (steady onset), or $\Delta_2 = 0$ ($\sigma \neq 0$,Oscillatory onset).

Now for oscillatory onset, setting $\Delta_2 = 0$ ($\sigma \neq 0$) and it gives a dispersion relation of the form (on dropping the subscript *i*)

$$p\left(\sigma^2\right)^2 + q\left(\sigma^2\right) + r = 0, \tag{37}$$

Now the Eq.(36) with $\Delta_2 = 0$, reduces to,

$$Ra_{T}^{Osc} = b_0(b_1 + b_2\sigma^2), \tag{38}$$

Where p,q,r and b_0, b_1, b_2 are not presented here for brevity.

Then from Eq.(38), we find the oscillatory neutral solutions.

IV. RESULTS AND DISCUSSIONS:

An analytical study of linear double diffusive convection in a horizontal fluid saturated porous layer is carried out by considering the thermal non-equilibrium model. The onset criteria for both marginal and oscillatory convection is derived in the linear theory. The expression for stationary and oscillatory Rayleigh numbers are derived analytically and the graphs are plotted for different values of different parameters and the results are discussed here.

The neutral stability curves in the Ra_T-a plane for various parameter values are as shown in **Figure** (1) - (6). We fixed the values for the parameters except the varying parameter.

In figure 1(a, b), the neutral stability curves for different values of the Soret parameters, ***Sr***, are shown. From this figure, we observe that with an increase in the values of the negative Soret parameter, then the stationary Rayleigh number increases, indicating that the negative Soret number stabilizes the system. On the other hand, for positive Soret parameter, the minimum of the stationary Rayleigh number decreases with an increase of the Soret parameter, indicating that the positive Soret parameter destabilizes the system in the stationary mode. In the case of oscillatory mode, we find that the negative Soret coefficient has destabilizing effect, whereas the positive Soret coefficient has stabilizing effect.

Figure 2(a, b), Increasing the solutal Rayleigh number Ra_C, raises the stationary critical Rayleigh number and consequently delays the onset of convection. For the oscillatory mode, the neutral curves show a shift in their minima with increasing Ra_C, indicating a significant influence of solutal buoyancy on the stability characteristics.

Figure 3(a, b), depicts the neutral stability curves for different values of Lewis number ***Le***, are drawn. it is observed that with the increase in the value of Lewis number, the critical values of Rayleigh number and the corresponding wave number for the overstable mode increases, while those for stationary mode decreases. Therefore, the effect of Lewis number is to advance the onset of oscillatory convection, whereas its effect inhibits the stationary onset.

In Figure. 4 (a, b), It indicates the effect of porosity modified conductivity ratio , γ on the marginal stability curves. We observe that with the increase in the value of γ, the minimum of oscillatory Rayleigh number decreases. The effect of gamma therefore is to advance the onset of oscillatory convection. It is interesting to note that the bifurcation from the oscillatory to stationary mode occurs at the same value of the wavenumber for all values of γ**.**

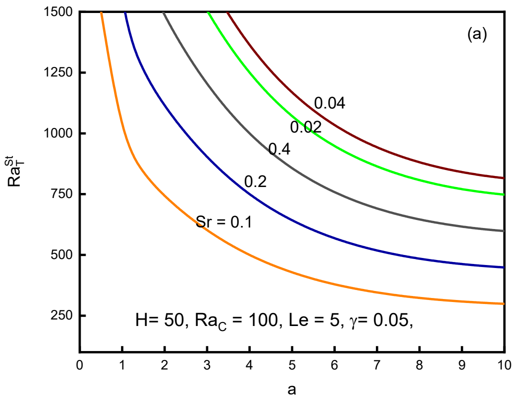

Fig. 1. Stationary netural curves for various vales of *Sr*.

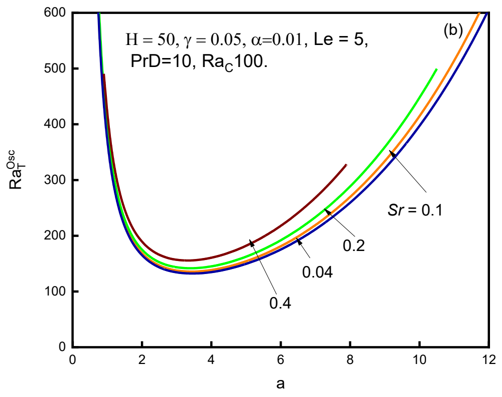

Fig.1.Oscillatory neutralcurves for various values of *Sr*.

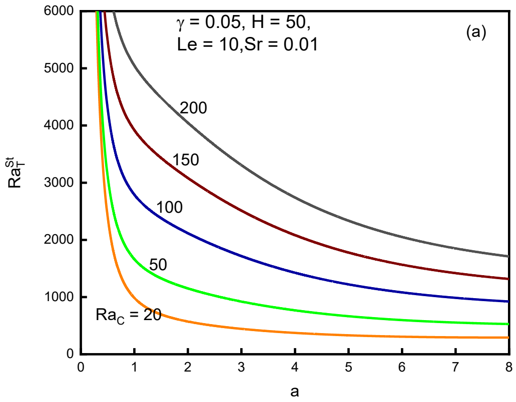

Fig. 2. Stationary neutral curves for various values of Ra_C

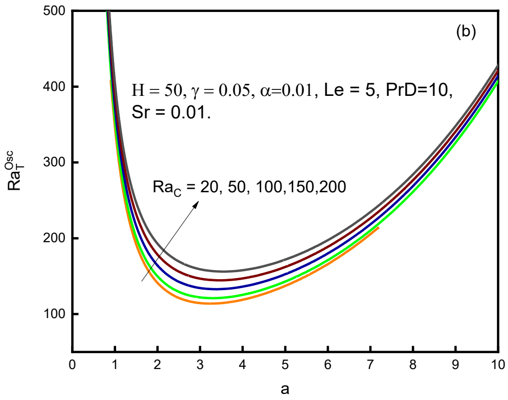

Fig.2. Oscillatory neutral curves for various vales of Ra_C.

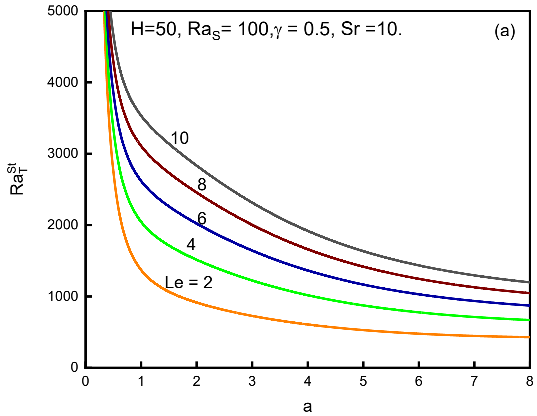

Fig.3. Stationary neutral curves for various values of *Le* .

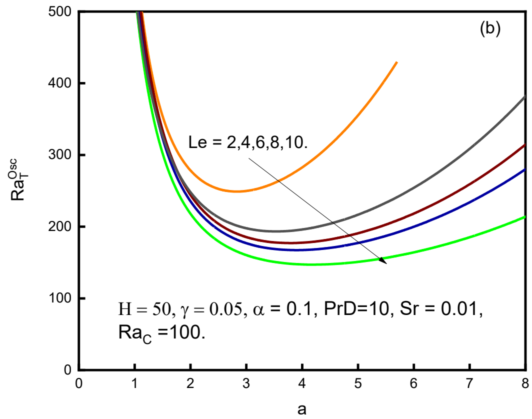

Fig.3. Oscilolatory neutral curves for various values of *Le*.

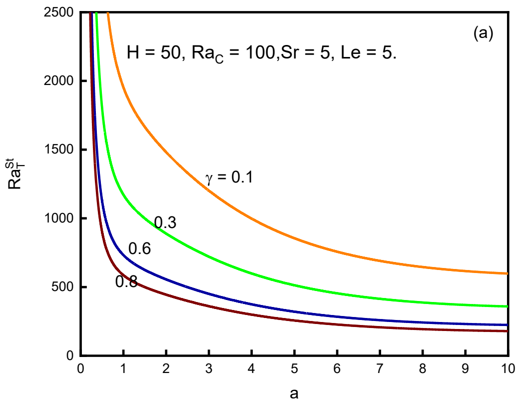

Fig.4. Stationary neutral curves for various values of γ .

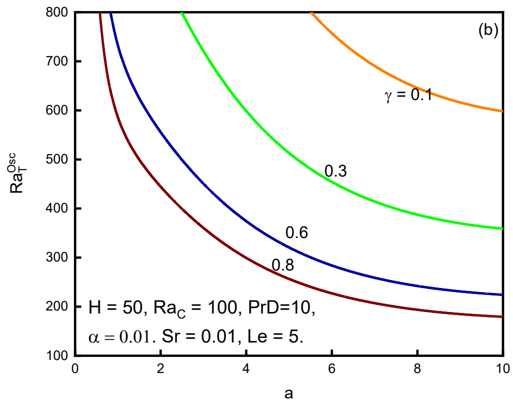

Fig.4. Oscillatary neutral curves for various values of γ

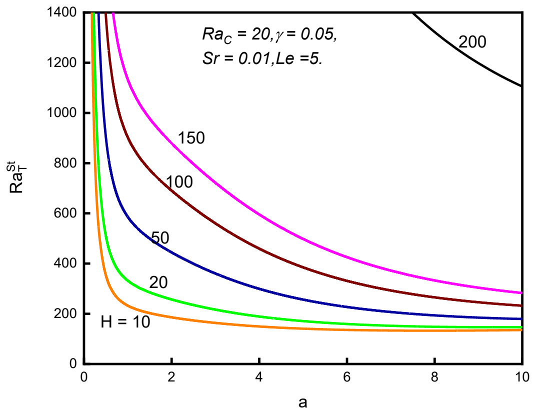

Fig.5. Stationary neutralcurves for various values of *H*

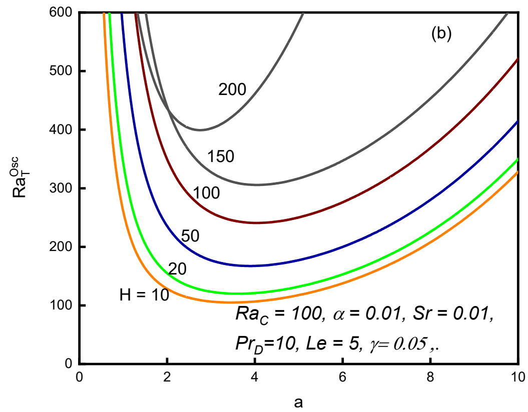

Fig.5. Oscillatory neutral curves for various values of H.

The effect of interface heat transfer coefficient ***H***, on the neutral stability curve is shown in Figure 5(a, b). For the fixed values of the solute Rayleigh number, porosity modified conductivity ratio, Lewis number, soret parameter. It follows from this figure that for the set of values chosen for the parameters, the onset of convection is through oscillatory state. Further we observe that the minimum of Rayleigh number for oscillatory mode increases with ***H***, indicating that the effect of the interface heat transfer coefficient ***H*** is to stabilize the system.

The effects of the interface heat-transfer coefficient H on the critical thermal Rayleigh number, $Ra_{T,c}^{Osc}$ are presented in Figures (6)– (9) for different values of the governing parameters. In all cases, H is plotted on a logarithmic scale. The results demonstrate that the transition from the local thermal non-equilibrium (LTNE) regime to the limiting thermal-equilibrium behavior is strongly influenced by the interphase heat transfer.

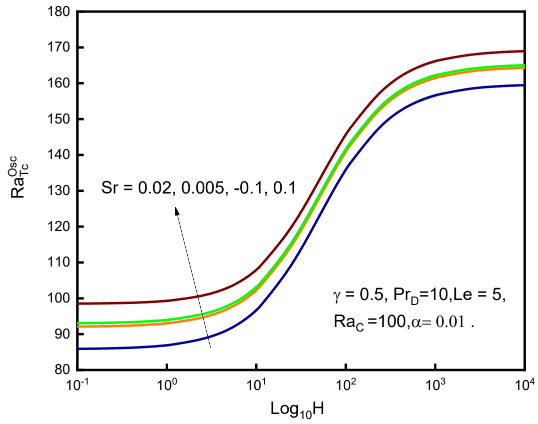

Fig. 6. Variation of Ra_{Tc}^{Osc} with *H* for different values of *Sr*.

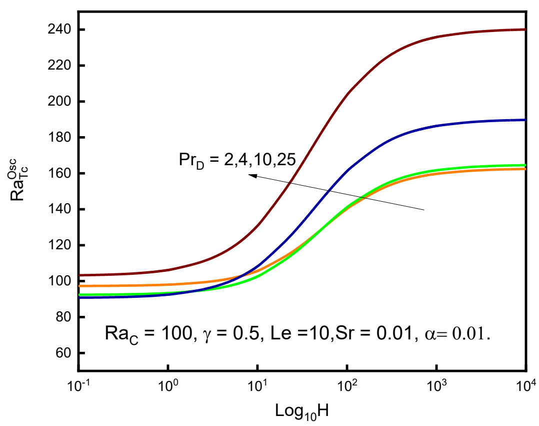

Fig.7. Variation of Ra_{Tc}^{Osc} with *H* for different values of Pr_D

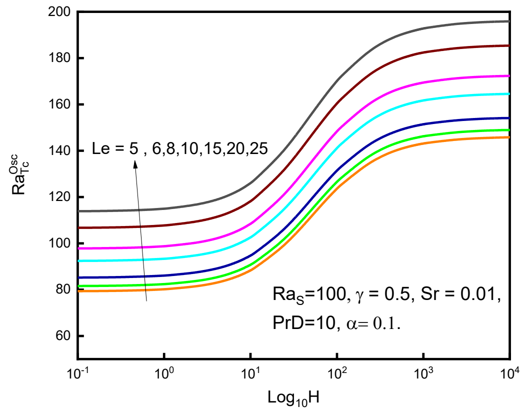

Fig.8. Variation of Ra_{Tc}^{Osc} with $Log_{10}H$ for different values of Le.

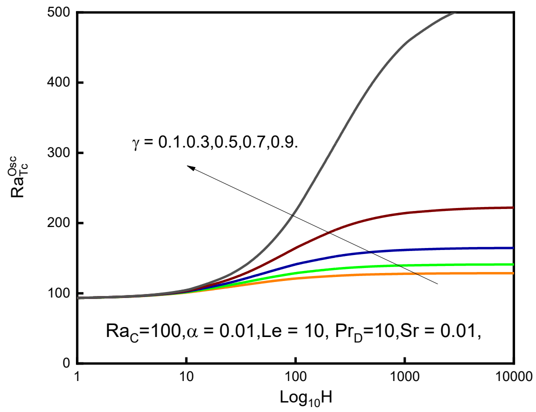

Fig 3. Variation of Ra_{Tc}^{Osc} with $Log_{10}Ha$ for different values of γ.

Figure (6), indicates the variation of $Ra_{T,c}^{Osc}$ with *H* for different values of the Soret parameter *Sr*, with fixed other parameters. For small values of *H*, the critical Rayleigh number remains relatively low and changes only slightly with the interface heat-transfer coefficient. As *H* increases, $Ra_{T,c}^{Osc}$ rises rapidly over an intermediate range of *H*. For sufficiently large *H*, the curves approach nearly constant values, indicating that the two phases tend toward thermal equilibrium.

The magnitude and direction of the Soret parameter noticeably affect the critical Rayleigh number. Positive and negative values of *Sr*, produce different stability thresholds, showing that thermodiffusion modifies the interaction between the thermal and solutal fields. In general, the increase in $Ra_{T,c}^{Osc}$ with *H* indicates that stronger thermal coupling between the fluid and solid phases delays the onset of oscillatory convection. Thus, the Soret effect and LTNE mechanism interact significantly in determining the stability characteristics of the porous layer.

Figure 7 shows the influence of the Darcy–Prandtl number $\Pr_D$ on $Ra_{T,C}^{Osc}$, with *H* for different values of the, *Sr* while other parameters are kept fixed. For every value of $\Pr_D$, the critical Rayleigh number increases as *H* increases. However, the rate and magnitude of this increase depend strongly on $\Pr_D$. At low *H*, the curves are relatively close to one another, whereas their separation becomes more pronounced as *H* increases. The larger values of $\Pr_D$, correspond to higher critical Rayleigh numbers in the strongly coupled regime. This indicates that an increase in the Darcy–Prandtl number generally stabilizes the system by increasing the thermal driving required for the initiation of oscillatory convection. The limiting behaviour at large *H* again reflects the approach toward the LTE state.

The influence of the Lewis number, *Le* is depicted in **Figure 8** for various values of *Le* with the remaining parameters are fixed. A clear increase in $Ra_{T,C}^{Osc}$ is observed as *Le* increases. For small *H*, the curves exhibit relatively weak variation, whereas a pronounced increase occurs when the interphase heat transfer becomes sufficiently strong. The higher critical Rayleigh numbers associated with larger *Le* indicate enhanced stability of the system. Since the Lewis number represents the relative importance of thermal and solutal diffusion, its increase modifies the balance between heat and mass transport and consequently affects the oscillatory instability. The separation between the curves at large *H* also demonstrates that the influence of mass-diffusion characteristics becomes more important when the fluid and solid phases are strongly thermally coupled.

Figure (9), presents the variation of $Ra_{T,C}^{Osc}$ with *H* for different values of the porosity parameter γ ,and the remaining parameters are fixed. The critical Rayleigh number exhibits a strong dependence on γ , particularly at large values of *H*. For small *H*, the curves are comparatively close, indicating that the effect of porosity is relatively weak in the highly LTNE regime. With increasing *H*, however, the curves separate considerably. The critical Rayleigh number increases markedly for certain values of γ , with the largest values occurring for the higher-porosity cases shown in the figure. This behaviour demonstrates that the permeability characteristics of the porous matrix substantially influence the onset of oscillatory convection once efficient heat exchange between the phases is established.

V. CONCLUSION

The stability of double-diffusive convection in a fluid-saturated porous layer has been investigated by incorporating local thermal non-equilibrium (LTNE) and the Soret effect. The analysis shows that the Soret parameter, solutal Rayleigh number, and Lewis number significantly influence the critical conditions for the onset of convection. Both stationary and oscillatory modes are affected by these parameters, with the neutral curves exhibiting distinct critical wave numbers and Rayleigh numbers. Overall, the results demonstrate that thermodiffusion and interphase heat transfer play important roles in controlling the stability and onset of convection in porous media.

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