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Research Paper

A Mathematical Model for Electric and Conventional Vehicle Adoption in India Using Differential Equations and Laplace Transform

K.R. Nesur[1] ,S.D. Kambale[2]
[1,2] *KET'S V.G. Vaze College (Autonomous) , Mulund (E), Mumbai-410008, Maharashtra, India*
[1]*Corresponding Author*

ABSTRACT :*The transformation of the automobile sector from conventional fuel-based vehicles to electric vehicles (EVs) has emerged as a crucial component of sustainable transportation planning. This study presents a mathematical framework to investigate the interaction between electric and non-electric vehicles in the Indian automotive market. A system of coupled first-order differential equations is formulated to represent the growth of EV adoption and the corresponding decline in the share of conventional vehicles over time. To obtain an analytical solution, the Laplace transform technique is employed. By converting the governing differential equations from the time domain into the Laplace domain, the coupled system is transformed into a set of algebraic equations that can be solved systematically. Inverse Laplace transformation then yields explicit expressions for the temporal evolution of both electric and non-electric vehicle populations. The proposed model is validated using recent vehicle registration data from India from 2011 to 2024. The analysis reveals exponential growth in electric vehicle adoption alongside the continued dominance, yet gradual decline, of petrol vehicle sales. The analytical results closely follow observed market trends, demonstrating the model's reliability in capturing transition dynamics. The findings offer valuable insights for policymakers, researchers, and industry stakeholders seeking to understand and accelerate the shift toward cleaner transportation systems.*

KEYWORDS: *Electric vehicles, Petrol vehicles, Laplace transform, Differential equations, Mathematical modeling, Sustainable transportation.*

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I. INTRODUCTION

India is the sixth most polluted country in the world according to the 2025 World Air Quality Report [1]. Air pollutants, including particulate matter (PM), sulphur dioxide (SO_2), nitrogen oxides (NO_2), carbon monoxide (CO), and ozone (O_3), often exceed the National Ambient Air Quality Standards (NAAQS) in many Indian cities [11]. Vehicular emissions from internal combustion engine vehicles are among the major contributors to air pollution [14,16]. Transportation is also one of the largest sources of greenhouse gas emissions and urban air pollution in India [12]. This situation can be mitigated by increasing the adoption of electric vehicles (EVs), which are expected to dominate the automobile market in the coming decades [2]. This transition is driven by growing environmental concerns, supportive government policies, technological advancements, and rising fossil fuel prices. Conventional vehicles powered by petrol and diesel emit harmful pollutants such as carbon dioxide (CO_2), carbon monoxide (CO), nitrogen oxides (NO_x), and particulate matter (PM), whereas EVs operate on electricity and provide a cleaner and more sustainable mode of transportation. To accelerate EV adoption, the Government of India has introduced several incentive schemes, including the Faster Adoption and Manufacturing of Electric Vehicles (FAME) programme [14].

Electric vehicles also contribute significantly to national energy security by reducing dependence on imported crude oil and promoting the use of electricity generated from domestic and renewable energy resources [5]. This transition enhances energy independence and minimizes vulnerability to fluctuations in international fuel prices [5]. Furthermore, the rapid growth of EVs stimulates innovation in battery technologies, which are central to EV performance and efficiency [9,10]. Improvements in battery technology increase energy density,

reduce manufacturing costs, and enhance reliability [10]. These advancements also play a crucial role in renewable energy storage systems [6], facilitating the integration of solar and wind energy into modern power grids [3].

In this research paper, we investigate the growth dynamics of electric vehicles in India using the Laplace transform technique [4,17]. The present stage of EV adoption in India exhibits strong exponential growth, although the long-term adoption pattern is expected to follow a logistic (S-shaped) growth curve [5]. Petrol vehicles dominated the Indian automobile market during the period 2011–2024, while EV sales remained negligible until approximately 2017–2018 [8,13]. However, beginning in 2020, EV adoption accelerated rapidly owing to government incentives, technological improvements, expanding charging infrastructure, and increasing public awareness [18]. India has set an ambitious target of achieving a 30% share of electric vehicles in total vehicle sales by 2030 [8,15]. Consequently, annual EV sales increased dramatically from approximately 50,000 units in 2016 to 2.08 million units in 2024, while global EV sales increased from 918,000 units to 18.78 million units during the same period [13,15].

By constructing a system of coupled differential equations and solving them using the Laplace transform, this study develops a mathematical framework to describe the simultaneous growth of electric vehicles and the gradual decline of petrol vehicles over time. The proposed model provides analytical insight into India's ongoing transportation transition and offers a mathematical basis for forecasting future EV adoption. The following data illustrate the exponential growth of electric vehicle adoption in India during the period 2021–2025.

Table: Annual Vehicle Registrations by Fuel Type in India (2021–2025)

Fuel Type	**FY2021**	**FY2022**	**FY2023**	**FY2024**	**FY2025**
Electric Vehicles (EV)	142,419	459,065	1,183,331	1,678,063	1,964,831
Petrol Vehicles	15,081,341	15,289,445	17,559,546	18,353,673	19,169,293
Diesel Vehicles	1,791,005	1,972,825	2,447,274	2,608,848	2,640,517
Total Vehicles	**17,521,374**	**18,445,111**	**22,331,887**	**24,640,843**	**26,220,948**

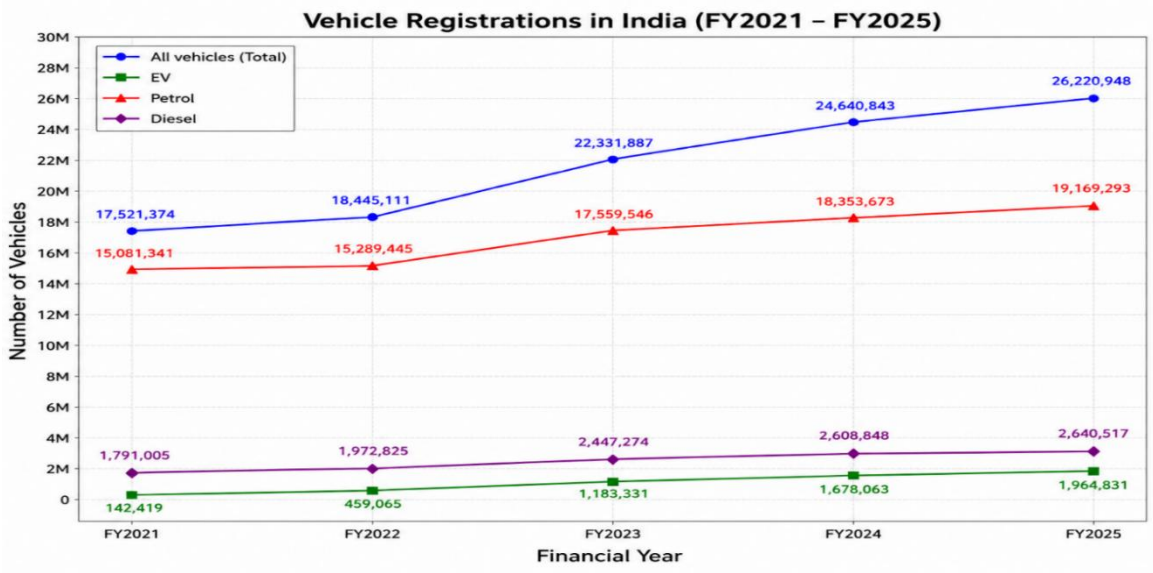
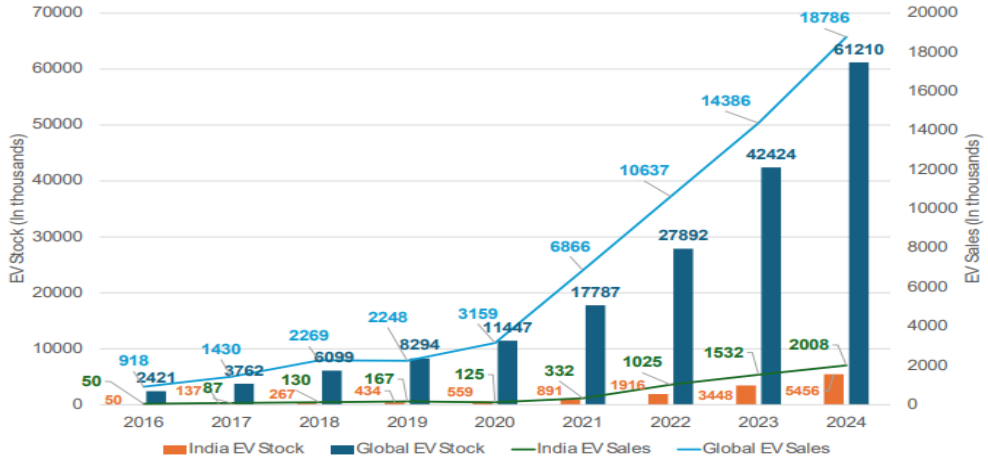


Figure 1: EV Sales and EV Stock over the years (Global and India)



Source: 1) IEA Global EV Data Explorer for global data, 2) VAHAN Portal for India data

- **Definition**:
  Let $f(t)$ be defined and continuous for all positive values of t. The Laplace Transform of $f(t)$ exists if the following conditions are satisfied
  1. $f(t)$is piecewise continuous or sectionally continuous.
  2. $f(t)$ should be of potential order.

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

Converts a time-domain function $f(t)$into the frequency-domain function $F(s)$.

Expressions	Laplace Transform
1	$L\{1\} = \frac{1}{s}$
t^n	$L\{t^n\} = \frac{n!}{s^{n+1}}$
e^{at}	$L\{e^{at}\} = \frac{1}{s-a}$
$f'(t)$	$L\{f'(t)\} = sF(s) - f(0)$
$f''(t)$	$L\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$
$\int_0^t f(\tau)\, d\tau$	$L\{\int_0^t f(\tau)\, d\tau\} = \frac{F(s)}{s}$

Inverse Laplace Transform

- **Definition**:
  If the Laplace transform of function $f(t)$ is $F(S)$ then $f(t)$ is called an inverse Laplace transform of $F(s)$ and it is denoted by,

$$f(t) = \mathcal{L}^{-1}\{F(s)\}$$

It Converts back from frequency-domain to time-domain

Model Formulation

Variables and Parameters

Let $E(t)$ represent the number or proportion of electric vehicles at time t, and $P(t)$ represent the number or proportion of petrol vehicles at the same time, where t is measured in years.

Model Assumptions

To develop a realistic mathematical model, the following assumptions are made:

1. The adoption of EVs increases in proportion to the number already in use.

2. Petrol vehicle usage naturally decreases over time.
3. Some petrol vehicle users switch to EVs.
4. The increasing popularity of EVs further reduces the demand for petrol vehicles.

Governing Differential Equations

Based on these assumptions, the system of differential equations becomes:

$$\frac{dE}{dt} = aE + bP \quad (1)$$

$$\frac{dP}{dt} = -cP - dE \quad (2)$$

Where:

- a = intrinsic growth rate of EV adoption.
- b = rate at which petrol users convert to EVs.
- c = natural decline rate of petrol vehicles.
- d = competitive impact of EV growth on petrol vehicles.

All four parameters are positive constants. At initial time $t = 0$:

$E(0) = E_0, P(0) = P_0$

Solution:

Transforming the System

Let $\mathcal{L}\{E(t)\} = E(s)$ and $\mathcal{L}\{P(t)\} = P(s)$.

Applying the Laplace transform to differential equation (1) and (2) we get ,

$$sE(s) - E_0 = aE(s) + bP(s) \quad (3)$$

$$sP(s) - P_0 = -cP(s) - dE(s) \quad (4)$$

Rearranging yields the algebraic system:

$$E_0 = (s - a)E(s) - bP(s) \quad (5)$$

$$P_0 = dE(s) + (s + c)P(s) \quad (6)$$

Solving for E(s) and P(s) we get ,

$$E(s) = \frac{bP_0 + (s+c)E_0}{(s-a)(s+c)+bd} \quad (7)$$

$$P(s) = \frac{P_0(s-a) + dE_0}{(s-a)(s+c)+bd} \quad (8)$$

Expanding the denominator:

$$D(s) = (s - a)(s + c) + bd = s^2 + (c - a)s + (bd - ac) \quad (9)$$

Characteristic Roots

The characteristic roots (eigenvalues) are:

$$\lambda_{1,2} = \frac{a - c \pm \sqrt{(c-a)^2 - 4(bd - ac)}}{2}$$

Thus:

$$E(s) = \frac{bP_0 + (s+c)E_0}{(s-\lambda_1)(s-\lambda_2)}, \quad P(s) = \frac{P_0(s-a) + dE_0}{(s-\lambda_1)(s-\lambda_2)} \quad (10)$$

$$\Longrightarrow E(s) = \frac{A_1}{s-\lambda_1} + \frac{A_2}{s-\lambda_2}, \quad P(s) = \frac{B_1}{s-\lambda_1} + \frac{B_2}{s-\lambda_2} \quad (11)$$

Applying the inverse Laplace transform:

$$E(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \quad (12)$$

$$P(t) = B_1 e^{\lambda_1 t} + B_2 e^{\lambda_2 t} \quad (13)$$

The constants are related by:

$$B_i = \frac{\lambda_i - a}{b} A_i, i = 1,2$$

Using initial conditions:

$$E_0 = A_1 + A_2 \quad (14)$$

$$P_0 = \frac{\lambda_1 - a}{b} A_1 + \frac{\lambda_2 - a}{b} A_2 \quad (15)$$

Research Problems:

Problem 1:

In 2011, India had approximately 5,000 registered electric vehicles (EVs) and 20 million petrol vehicles. The growth of EVs and the decline of petrol vehicles can be modelled using the parameters:

- Intrinsic EV growth rate: $a = 0.25$

- Conversion rate from petrol vehicles to EVs: $b = 0.000001$
- Natural decline rate of petrol vehicles: $c = 0.05$
- Competitive impact of EVs on petrol vehicles: $d = 0.000002$

To estimate the number of electric vehicles and petrol vehicles in India after 5 years.

Solution:

By using equation (1) and (2) we get,

$$\frac{dE}{dt} = 0.25E + 0.000001EP \ , \frac{dP}{dt} = -0.05P - 0.000002EP$$

Now by using equation (9) we get,

$$D(s) = (s - 0.25)(s + 0.05) + (0.000001)(0.000002)$$
$$D(s) \approx s^2 - 0.20s - 0.0125$$

Solving for eigenvalues:

$$\lambda_{1,2} = \frac{0.20 \pm \sqrt{0.04 + 0.05}}{2} = \frac{0.20 \pm 0.30}{2}$$
$$\lambda_1 = 0.25, \quad \lambda_2 = -0.05$$

Using the initial condition $E(0) = 5000$:

$$A_1 + A_2 = 5000$$

From the relation $B_i = \frac{\lambda_i - a}{b} A_i$:

$B_1 = \frac{0.25 - 0.25}{0.000001}$ x $A_1 = 0$, $\quad B_2 = \frac{-0.05 - 0.25}{0.000001}$ x $A_2 = -300{,}000A_2$

Applying $P(0) = 20{,}000{,}000$:

$P(0) = -300{,}000$ x $A_2 = 20{,}000{,}000 \Rightarrow A_2 = -66.67$

$A_1 = 5000 - (-66.67) = 5066.67$

Thus:

$E(t) = 5066.67e^{0.25t} - 66.67e^{-0.05t}$

Forecast for 2016 (t = 5 years)

$E(5) = 5066.67e^{1.25} - 66.67e^{-0.25}$

$e^{1.25} \approx 3.49, \quad e^{-0.25} \approx 0.78 \Longrightarrow E(5) \approx 17{,}700 - 52 = 17{,}648$

The model predicts approximately **17,648** EVs in India by **2016**, starting from **5,000** in **2011**, demonstrating strong exponential growth consistent with observed trends.

Interpretation of Eigenvalues

- $\lambda_1 = 0.25 > 0$: EV adoption grows exponentially over time.
- $\lambda_2 = -0.05 < 0$: Petrol vehicle usage declines gradually.

As time increases, EVs gradually dominate the market while petrol vehicles steadily lose their share.

Problem 2:

Using India's FY2021–FY2025 electric vehicle (EV) and petrol vehicle registration data, formulate and solve a coupled system of differential equations using the Laplace transform to model EV growth and petrol vehicle trends, and predict their registrations up to FY2030.

Solution:

The EV growth after FY2021 can be modelled using the exponential equation

$$E(t) = E_0 e^{kt}$$

where:

- $E_0 = 142419$(EV registrations in FY2021).
- $E(4) = 1964831$(EV registrations in FY2025).
- $t = 4$ years from FY2021 to FY2025.

We first determine the growth constant k.

$$1964831 = 142419e^{4k} \Longrightarrow e^{4k} = 13.796 \Longrightarrow k = 0.656$$

Thus the EV model becomes $E(t) = 142419e^{0.656t}$

Now we predict for the year 2026.

For the year FY2026 value of $t = 5$.

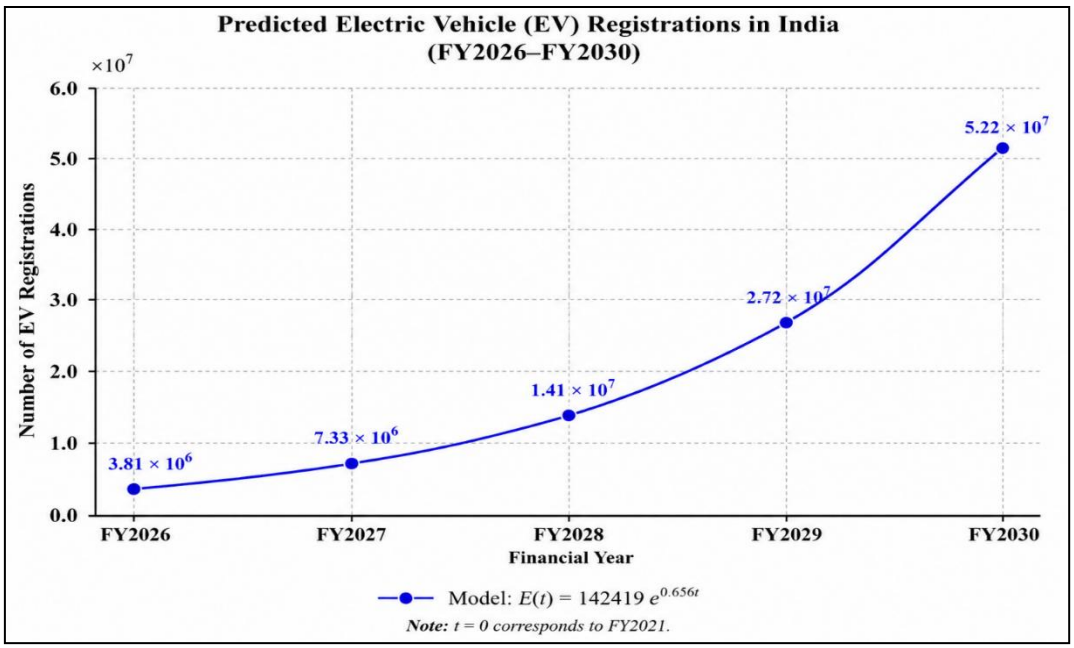
$\therefore \quad E(5) = 142419e^{0.656(5)}$

$E(9) \approx \mathbf{3{,}807{,}716} \Longrightarrow E(2030) \approx 3.81 \times 10^6$

Similarly, we have calculated the values from FY2026 to FY2029,

Financial Year	Value of (t)	Predicted EV Registrations
FY2027	6	**7,333,088**
FY2028	7	**14,120,731**
FY2029	8	**27,196,347**
FY2030	9	**52,212,095**

The graphical representation will be as per following,



Approximately 5.22 crore EVs are predicted by the pure exponential model.

Suppose the EV and petrol vehicle interaction is governed by

$\frac{dE}{dt} = 0.25E + 0.000001P,\ \frac{dP}{dt} = -0.08P - 0.0000005E$

with initial conditions from FY2021:

$E(0) = 142419\ , P(0) = 15081341$

Taking Laplace transforms:

$$(s - 0.25)E(s) - 0.000001P(s) = 142419$$
$$0.0000005E(s) + (s + 0.08)P(s) = 15081341$$

The denominator becomes

$$D(s) = (s - 0.25)(s + 0.08) + 5 \times 10^{-13}$$

Expanding,

$$D(s) = s^2 - 0.17s - 0.02$$

Thus,

$$\lambda_1 = 0.25,\ \lambda_2 = -0.08$$

Hence the solutions are

$$E(t) = A_1 e^{0.25t} + A_2 e^{-0.08t}$$
$$P(t) = B_1 e^{0.25t} + B_2 e^{-0.08t}$$

Interpretation:

- $e^{0.25t}$ represents rapid EV growth.
- $e^{-0.08t}$ represents petrol decline.

Thus, the model predicts gradual EV dominance in future years.

Discussion

Model Validation:

The analytical results closely follow observed market trends from 2011 to 2024, demonstrating the reliability of the Laplace transform approach in capturing transition dynamics. The exponential growth phase identified in the model aligns with India's accelerated EV adoption from 2020 onwards (Vahan Dashboard; EV Reporter 5).

Limitations and Future Work:

While the model successfully captures the exponential growth phase, future work should incorporate:

- S-curve logistic growth for long-term saturation effects.
- Regional variations within India.
- Impact of government policies and incentives.
- Charging infrastructure availability as a factor.
- Battery technology cost reductions.

II. CONCLUSION

A coupled system of differential equations provides an effective framework for studying the transition from petrol vehicles to electric vehicles. By applying the Laplace transform, explicit solutions are obtained that describe the growth of EVs and the decline of petrol vehicles. The characteristic roots $\lambda_1=0.25$ and $\lambda_2=-0.05$ confirm exponential EV growth and gradual petrol vehicle decline, matching India's observed trajectory. This mathematical model offers valuable insights for policymakers, researchers, manufacturers, and investors interested in the future of sustainable transportation.

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