



The Mathematical Legacy of Leonhard Euler

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Abstract: Leonhard Euler is one of the most distinguished and acclaimed mathematician in the history of world. The groundbreaking works of Euler have laid a powerful and impactful effect on the development of mathematical concepts and have elevated the applications of mathematics in other branches of science. His significant works in the different areas of mathematics have laid the foundation for modern mathematics. A brief breakdown of some of the major and pioneering contributions of Euler in the field of Mathematics is discussed in this paper.

Keywords: Leonhard Euler, Mathematical Legacy, Analysis, Number Theory, Geometry, Calculus of Variation

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I. INTRODUCTION

Leonhard Euler was a renowned Swiss mathematician and physicist of the 18th century. He was one of the most influential mathematicians and scientists in the history of the world. His contributions to the numerous fields of mathematics and physics have had an intense and far reaching impact on the development of these disciplines. His significant works in the different areas of mathematics and physics have laid the foundation for modern mathematics that has a lot of applications in the real world. Many of the mathematical notations and terminology that are in use today have been introduced by Euler. During his lifetime, he has obtained many theorems, formulae and equations that have revolutionized the fields of mathematics and sciences.

Leonhard Euler was born on 15th April 1707 in Basel, Switzerland, and died on 18th September 1783 in St. Petersburg, Russia. He had his formal education in Basel. Euler's talent for mathematics became apparent at an early age. At the age of 13 years only, he got himself enrolled at the University of Basel, where he quickly advanced in his mathematical studies under the guidance of Johann Bernoulli. During his university years, Euler exhibited his brilliant ability to solve complex mathematical problems with ease. His professors were amazed by his quick thinking and innovative approaches to problem-solving. Euler's passion for mathematics grew stronger during his university years.

Euler completed his Master's degree in philosophy in 1723, having compared and contrasted the philosophical ideas of Descartes and Newton. Euler completed his studies at the University of Basel in 1726. He had studied many mathematical works during his time in Basel. In 1727, he participated first time in the Paris Academy Prize Problem competition and he was able to win the second prize in it. Euler joined the Russian Academy of Sciences in St. Petersburg in the year 1727, when he was only 20 years old. He left St. Petersburg in the year 1741 and moved to Berlin to work at the Berlin Academy.

There are many contents and articles available online that discusses about the life and work of Euler. Many authors have also published their books that throws light on the contributions of Euler. Calinger (1996) has investigated and explored the personality and work of Euler during his first St. Petersburg years. Leonhard Euler: Life, Work and Legacy (2007), an edited book by Bradley and Sandifer has compiled various chapters written by different authors that talk over the works of Euler in diverse fields. Oliveira (2007) has discussed the Euler's contribution to classical mechanics. Hedrih (2007) has talked on life of Euler and his works in rigid body dynamics. Gautschi (2008) has explored on Euler's personality, intellect, and craftsmanship; and has discussed few of Euler's memorable contributions and its impact on modern science. The Legacy of Leonhard Euler: A Tricentennial Tribute (2010) is a book by Lokenath Debnath that presents a short history of mathematical developments and discoveries before Euler, and a brief sketch of the life, work, career, and major achievements of Euler.

In this paper, we are enlisting a brief breakdown of some of the major and pioneering contributions of Euler. These works of Euler have made a significant impact on the development of various disciplines of

Mathematics that we are studying in today's world. This paper provides an opportunity for the young scholars to have a glimpse of the great work done by Euler in the field of Mathematics, and tries to stimulate them to explore the beauty of Mathematics.

II. EULER'S WORKS AND CONTRIBUTIONS

In this section, we are providing a brief outline of some of the striking and important contributions of Euler in the field of Mathematics.

1. Euler–Cauchy equation

Euler–Cauchy equation, also known as Cauchy–Euler equation, is a linear homogeneous ordinary differential equation with variable coefficients. The general form of Euler–Cauchy equation is given as,

$$a_n x^n y^{(n)} + a_{n-1} x^{n-1} y^{(n-1)} + \dots + a_1 x y' + a_0 y = 0$$

where the symbols a_i , $i = 0, 1, \dots, n$ are constants and $a_n \neq 0$. In this equation the power of x in each of the coefficients matches the order of the derivative in that term. The Euler–Cauchy equation is important in the theory of linear differential equations because it has direct application to Fourier's method in the study of partial differential equations. The Euler–Cauchy equation provides valuable insights and tools for analyzing physical systems, engineering problems, and various applications in science and technology.

2. Euler–Lagrange equation

The Euler–Lagrange equation is a fundamental result in the calculus of variations and is used to find the path or function that minimizes (or maximizes) a certain functional. If I is defined as

$$I = \int f(x, y, y') dx$$

then I has a stationary value if the Euler–Lagrange differential equation

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$$

is satisfied.

The Euler–Lagrange equation is a powerful equation capable of solving a wide variety of optimization problems that have applications in mathematics, physics and engineering by focusing on the principle of least action. The Euler–Lagrange equations have helped in understanding our physical world, from classical mechanics, quantum mechanics and geometrical optics to modern day engineering problems.

3. Euler–Rodrigues formula

The Euler–Rodrigues formula is an efficient algorithm for rotating a vector in three – dimensional space given a specific rotation axis and angle. If v is a vector in \mathbb{R}^3 and k is a unit vector describing an axis of rotation about which v rotates by an angle θ according to the right hand rule, the Rodrigues formula for the rotated vector v' is

$$v' = v \cos \theta + (k \times v) \sin \theta + k(k \cdot v)(1 - \cos \theta).$$

This formula can also be expressed in the matrix form. This formula provides an efficient and geometrically intuitive way to represent finite rotations in the special orthogonal group $SO(3)$, preserving lengths and orientations while fixing points along the rotation axis. This formula is used in fields such as computer graphics, robotics and rigid body dynamics.

4. Euler–Poisson–Darboux equation

The Euler – Poisson – Darboux equation is a partial differential equation that is used to solve the wave equation. This equation is the simplest linear hyperbolic equation in two independent variables whose coefficients exhibit singularities, and so this equation creates interest among researchers as a paradigm to relativity theory. This equation is generally expressed as

$$\frac{\partial^2 u}{\partial t^2} + \frac{\gamma}{t} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}.$$

This equation is used in solving the Cauchy problem for hyperbolic equations, acoustics, and general relativity.

5. Euler–Bernoulli Beam equation

The Euler–Bernoulli beam equation provides a mathematical model to predict the structural behavior of beams. The equation is derived from four segments of beam theory; the combination of the equations governing the four segments formulate the Euler–Bernoulli beam equation. The four distinct segments are kinematics, constitutive, resultants, and equilibrium. The Euler–Bernoulli equation describes the relationship between the beam's deflection and the applied load. The simplicity of this equation in beam theory makes it an important tool in the sciences, especially structural and mechanical engineering. This equation assumes that plane cross-sections remain plane and perpendicular to the neutral axis after deformation while neglecting the shear deformation. The equation is given as

$$\frac{d^2}{dx^2} \left(EI \frac{d^2 w}{dx^2} \right) = q(x),$$

where the curve $w(x)$ describes the deflection of the beam in the z direction at some position x along the beam, $q(x)$ is the distributed load (force per unit length), E is the elastic modulus and I is the second moment of area of the beam's cross section. The product EI is known as the flexural rigidity and generally it is a constant, so that we have

$$EI \frac{d^4 w}{dx^4} = q(x).$$

This equation describes the deflection of a uniform, static beam and it is used widely in engineering practice.

6. Euclid–Euler theorem

The Euclid–Euler theorem establishes a one-to-one correspondence between even perfect numbers and Mersenne primes. A number is said to be perfect if it is equal to the sum of its proper divisors, that is, divisors smaller than the number itself. A Mersenne prime is a prime number of the form $2^p - 1$, where p is also a prime number. The Euclid–Euler theorem states that a positive even integer n is a perfect number if and only if it can be expressed as $n = 2^{p-1}(2^p - 1)$, where p is a prime number and $2^p - 1$ is a Mersenne prime. The theorem implies that finding new Mersenne primes is equivalent to finding new even perfect numbers, and vice versa. As of 2025, 52 even perfect numbers and thus 52 Mersenne primes are known.

7. Euler–Maclaurin formula

The Euler–Maclaurin formula is a formula for the difference between an integral and a closely related sum. It can be used to approximate integrals by finite sums, or conversely to evaluate finite sums and infinite series using integrals and the machinery of calculus. This formula is used to obtain the value of sums that are difficult to compute directly, such as the harmonic series or sums of powers. The formula is also used for detailed error analysis in numerical quadrature. It explains the superior performance of the trapezoidal rule on smooth periodic functions and is used in certain extrapolation methods.

8. Euler's formula in Complex Analysis

The Euler's formula in complex analysis is given as $e^{ix} = \cos x + i \sin x$, where x is any real number and i is the imaginary number ($i^2 = -1$). This formula gives the relationship between trigonometric functions and complex exponential functions. This formula has several applications and it leads to simplifications in a lot of calculations.

9. Euler's identity

The Euler's identity is given as $e^{i\pi} + 1 = 0$. This identity combines five fundamental constants: e , i , π , 1 and 0 . Here e is the base of natural logarithms ($e = 2.71828 \dots$), i is the imaginary number ($i^2 = -1$), π is the ratio of the circle's circumference to its diameter ($\pi = 3.14159 \dots$), 1 is the multiplicative identity and 0 is the additive identity. The Euler's identity also involves the three of the basic arithmetic operations: addition, multiplication and exponentiation. The Euler's identity is described as the most beautiful equation in Mathematics. This identity is a special case of the Euler's formula $e^{ix} = \cos x + i \sin x$, obtained by taking $x = \pi$.

10. Euler number (physics)

The Euler number (Eu) is a dimensionless number used in fluid flow calculations. It expresses the relationship between a local pressure drop caused by a restriction and the kinetic energy per volume of the flow, and is used to characterize energy losses in the flow, where a perfect frictionless flow corresponds to an Euler number of 0.

The Euler number is defined as

$$Eu = \frac{p_u - p_d}{\rho v^2},$$

where ρ is the density of the fluid, p_u is the upstream pressure, p_d is the downstream pressure and v is a characteristic velocity of the flow.

11. Euler's rotation theorem

Euler's rotation theorem states that, in three-dimensional space, any displacement of a rigid body such that a point on the rigid body remains fixed, is equivalent to a single rotation about some axis that runs through the fixed point. It also means that the composition of two rotations is also a rotation. The axis of rotation is known as an Euler axis and it is represented by a unit vector. Its product by the rotation angle is known as an axis-angle vector. The extension of the theorem to kinematics yields the concept of instant axis of rotation, a line of fixed points. Euler's rotation theorem simplifies complex motion into axis-angle representation that is applied in robotics, computer graphics, animation, aerospace navigation, plate tectonics and rigid-body mechanics.

12. Euler's rotation equations

Euler's rotation equations are a set of three quasilinear first-order ordinary differential equations that describe the rotational motion of a rigid body using a rotating reference frame fixed to the body. This equations relates the change in angular momentum of a rigid body to the applied torque. The equations are given as

$$\begin{aligned} I_1 \dot{\omega}_1 + (I_3 - I_2) \omega_3 \omega_2 &= M_1 \\ I_2 \dot{\omega}_2 + (I_1 - I_3) \omega_1 \omega_3 &= M_2 \\ I_3 \dot{\omega}_3 + (I_2 - I_1) \omega_2 \omega_1 &= M_3 \end{aligned}$$

where (M_1, M_2, M_3) are the components of the applied torques, (I_1, I_2, I_3) are the principal moments of inertia and $(\omega_1, \omega_2, \omega_3)$ are the components of the angular velocity. Euler's rotation equations are used to predict the stability of rotating bodies and they simplify the analysis of complex 3D rotational dynamics, making them fundamental in engineering and physics. These equations are applied in designing control systems for satellites and spacecraft; analyze flywheels, gears, and turbines in rotating machineries; and to model the dynamics of rotating components in robotics. These equations are significant in analysis of Newton–Euler Coupling and they play a key role in solving specialized problems in mechanics.

13. Euler product formula

Euler's product formula is one of the most important results in the history of number theory and it has made way for fusing arithmetic and analysis into analytic number theory. This formula has been referred as Golden Key to indicate its significance in the development of analytic number theory and number theory in general. It establishes relationship between the prime numbers and the Riemann zeta function. The Euler product formula is

$$\zeta(s) = \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}}$$

Here the left hand side is the famous Riemann zeta function $\zeta(s)$:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \dots$$

and the product on the right hand side extends over all prime numbers p :

$$\prod_{p \text{ prime}} \frac{1}{1 - p^{-s}} = \frac{1}{1 - 2^{-s}} \cdot \frac{1}{1 - 3^{-s}} \cdot \frac{1}{1 - 5^{-s}} \dots \frac{1}{1 - p^{-s}} \dots$$

The Euler product formula is crucial for the study of the distribution of primes and it leads to important results in analytic number theory. It is a key tool in evaluation of series, probability theory methods and it serves as basis for advanced mathematical theories.

14. Euler–Mascheroni constant

Euler–Mascheroni constant is a mathematical constant that is defined as the limiting difference between the harmonic series and the natural logarithm. This constant is denoted by γ with a value of approximately 0.5772156649015... and it is given as

$$\gamma = \lim_{n \rightarrow \infty} \left(\sum_{k=1}^n \frac{1}{k} - \log n \right)$$

This constant appears in many problems in analytic number theory and calculus. It is used in many problems that require an estimation of a harmonic numbers. This constant is considered as the third most important non-trivial mathematical constant after π and e and it plays a very crucial role in bridging discrete sums and continuous logarithms.

15. Euler–Lotka equation

The Euler–Lotka equation is a mathematical expression used to study population dynamics and growth, particularly in the context of demography and ecology. This equation is used in the study of stable age populations. This equation serves as valuable tool for analyzing and modeling population growth rates, age distribution of individuals, and reproductive patterns within a population. Based on the age demographic of females in the population and female births, this equation allows for an estimation of how a population is growing. The Euler–Lotka equation is given as

$$\sum_{x=1}^{\infty} e^{-rx} l(x) m(x) = 1$$

where α and β are the boundary ages for reproduction defining the discrete growth rate, $\lambda = e^r$; $l(x)$ is the fraction of individuals surviving to age x and $m(x)$ is the number of offspring born to an individual of age x during the time step.

16. Euler–Tricomi equation

The Euler–Tricomi equation is a linear partial differential equation of mixed elliptic – hyperbolic type for $u(x,y)$ with the form: $u_{xx} + x u_{yy} = 0$. The equation is hyperbolic in the half plane $x < 0$, elliptic in the half plane $x > 0$, and degenerates on the line $x = 0$. Its characteristic equation is $dy^2 + x dx^2 = 0$, whose solutions are $y \pm \frac{2}{3}(-x)^{3/2} = C$ for any constant C , which are real for $x < 0$. The characteristics comprise two families of semicubical parabolas lying in the half plane $x < 0$, with cusps on the line $x = 0$. This is of hyperbolic

degeneracy, for which the two characteristic families coincide, perpendicularly to the line $x = 0$. Many important problems in fluid mechanics and differential geometry can be reduced to corresponding problems for the Euler–Tricomi equation, particularly transonic flow problems and isometric embedding problems.

17. Euler's number $e \approx 2.71828\dots$

Euler's number, denoted as e , is the base for natural logarithms and is one of the non-terminating constants. Euler's number is one of the most important constants in mathematics and is approximately equal to 2.71828. Euler's number helps mathematicians, physicists, statisticians, and other scientists explain exponential growth in various phenomena, from population to radioactive decay. In finance, Euler's number is used to calculate how wealth can grow due to compound interest.

The main principle for defining Euler's number revolves around the concept of continuous growth and change. The Euler's number e is given as

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

The definition of e through limits highlights its role in modeling continuous and exponential growth, compounding interest, and various natural phenomena. It can also be expressed as the sum of infinite numbers:

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \dots$$

This series is fundamental in calculus and other areas of mathematics because it provides a precise method to define e using a sum of infinitely many fractions.

18. Euler's partition theorem

In number theory, a partition of a positive integer n is a way of writing n as a sum of positive integers. The order of the summands is ignored in a partition. Two sums that differ only in their order are considered the same partition. Distinct partitions are those in which each summand is different, and Odd partitions are those in which each summand is odd. Euler's Partition Theorem states that for each positive number, the number of partitions with only distinct parts is equal to the number of partitions with only odd parts.

Euler's partition theorem has several applications in Number Theory, Combinatorics, and Cryptography. In Number Theory, it is used to prove relationships between different types of partitions and their generating functions. In Combinatorics, it provides a basis for enumerating partitions of integers and analyzing related structures, such as those found in symmetric groups. In Cryptography, it is used for the development of secure encryption algorithms, which utilize the principles of partitions and compositions for key generation and functions.

19. Euler's homogeneous function theorem

In mathematics, a homogeneous function is defined as a function of many variables. The function is such that if all the variables of a function are multiplied by a scalar, then the value of the function is multiplied by some power of the same scalar. Thus, a function $f(x, y)$ will be a homogeneous function in x and y of degree n if $f(tx, ty) = t^n f(x, y)$. The identification of homogeneous and non-homogeneous functions was made easier by using the Euler's homogeneous function theorem. This theorem is used to establish a relationship between the partial derivatives and the function product with its degree. This theorem states that if ' u ' is a homogeneous function of three variables x, y and z of degree n , then

$$x \frac{du}{dx} + y \frac{du}{dy} + z \frac{du}{dz} = nu.$$

20. Euler's quadrilateral theorem

Euler's quadrilateral theorem is one of the most important and classical theorem of elementary geometry. It is a direct generalization of the two great problems in mathematics: the parallelogram law and the Pythagorean theorem. The theorem relates the four side lengths, the two diagonals, and the distance between the midpoints of the diagonals of any convex quadrilateral. It states that the sum of the squares of the four sides is equal to the sum of the squares of the diagonals plus four times the square of the distance between the midpoints of the diagonals. Mathematically, for a quadrilateral $ABCD$ with side lengths AB, BC, CD, DA , diagonals AC and BD , and MN representing the segment connecting the midpoints of the diagonals, the theorem is expressed as:

$$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 + 4MN^2$$

21. Euler's idoneal numbers

In mathematics, Euler's idoneal numbers (also called suitable numbers or convenient numbers) are the positive integers D such that any integer expressible in only one way as $x^2 \pm Dy^2$ (where x^2 is relatively prime to Dy^2) is a prime power or twice a prime power. Thus, a composite number cannot be represented as a sum or difference of two squares in a single way using an idoneal number D .

A positive integer n is idoneal if and only if it cannot be written as $ab + bc + ac$ for distinct positive integers a, b , and c with $0 < a < b < c$.

Euler identified the first 65 idoneal numbers, with the largest being 1848. He conjectured that this set of 65 numbers was complete. Idoneal numbers have applications in areas of Cryptography, Coding Theory and Graph Theory. The study of idoneal numbers can lead to algorithms for optimizing certain computational processes.

22. Euler's Theorem

Euler's Theorem is a key concept in number theory that provides a powerful tool for solving problems related to prime numbers and modular arithmetic. It has far-reaching applications in various fields such as cryptography, number theory, and combinatorics. The theorem states that for any positive integer a and any positive integer m that is relatively prime to a , the following congruence relation holds: $a^{\varphi(m)} \equiv 1 \pmod{m}$. Here, $\varphi(m)$ is Euler's totient function, which gives the number of positive integers that are relatively prime to m .

Euler's theorem is foundational in modern cryptography, particularly in the RSA (Rivest-Shamir-Adleman) encryption algorithm. RSA utilizes Euler's theorem in the process of encryption and decryption. The theorem is used in primality testing algorithms, such as the Fermat primality test. Euler's theorem is a powerful tool in solving number theory problems involving divisibility, remainders, and the properties of numbers in different number systems. The theorem is helpful in establishing many mathematical assertions, particularly those involving divisibility and modular arithmetic. It aids in making proofs clear and trustworthy.

23. Euler's theorem in geometry

In geometry, Euler's theorem (also known as the Euler triangle formula) provides a relationship between the distance separating the incenter and circumcenter of a triangle, and the radii of its inscribed and circumscribed circles. The theorem states that the distance d between the circumcenter (O) and the incenter (I) of any triangle is given by the formula $d^2 = R(R - 2r)$ where, d is the distance between the circumcenter and incenter; R is the circumradius (radius of the circumscribed circle) and, r is the inradius (radius of the inscribed circle). Since the distance d^2 must be non-negative ($d^2 \geq 0$), the Euler's theorem directly leads to Euler's inequality $R \geq 2r$. Euler's theorem in geometry is used in solving complex geometric problems, establishing relationships between different triangle centers, and proving geometric inequalities.

24. Euler's theorem (differential geometry)

In differential geometry, Euler's theorem is a result on the curvature of curves on a surface. The theorem establishes the existence of principal curvatures and associated principal directions which give the directions in which the surface curves the most and the least. The theorem states that at any point on a smooth surface, the normal curvature (k_n) in any given direction is given by the formula:

$$k_n = k_1 \cos^2 \theta + k_2 \sin^2 \theta,$$

where k_1 and k_2 are the principal curvatures and θ is the angle between the given direction and the principal direction associated with k_1 . This means that any other normal curvature can be expressed as a combination of the maximum and minimum normal curvatures at that point. Thus, the curvature behavior of a surface at a point is completely determined by just two values, the principal curvatures, and their associated directions.

25. Euler's four-square identity

Euler's four square identity states that the product of any two numbers a and b can be expressed as a sum of four squares if a and b both can individually be expressed as the sum of four squares. This means that if

$$a = a_1^2 + a_2^2 + a_3^2 + a_4^2 \text{ and } b = b_1^2 + b_2^2 + b_3^2 + b_4^2 \text{ then}$$

$$a b = (a_1^2 + a_2^2 + a_3^2 + a_4^2)(b_1^2 + b_2^2 + b_3^2 + b_4^2) = c_1^2 + c_2^2 + c_3^2 + c_4^2$$

where $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, c_1, c_2, c_3, c_4$ are any integer. In other words, the product of the sums of four squares is itself the sum of four squares.

Euler's identity is a crucial part of the proof for Lagrange's four-square theorem, which states that every natural number can be written as the sum of four non-negative integer squares. This identity also has a direct connection to quaternion algebra, where it is equivalent to the property that the norm of a product is the product of the norms: $|XY|^2 = |X|^2 + |Y|^2$. While the identity is a powerful theoretical tool, it also finds practical use in programming, such as finding a sum of four squares for a given product using computational methods.

26. Euler's totient function

Euler's totient function, denoted by $\varphi(n)$, counts the positive integers up to a given integer n that are relatively prime to n . In other words, $\varphi(n)$ is the number of integers m such that $1 \leq m < n$ and $\gcd(m, n) = 1$. The Euler's totient function appears in many applications of elementary number theory, including Euler's theorem, primitive roots of unity, cyclotomic polynomials, and constructible numbers in geometry.

27. Euler's Polyhedral Formula

Euler's Polyhedral Formula (sometimes called as Euler's formula), relates the number of vertices, edges, and faces of a polyhedron. It is expressed as $V - E + F = 2$, where V is the number of vertices, E is the number of edges and F is the number of faces. Euler's Formula tells us something very deep about shape and space and it is a fundamental result in topology and combinatorial geometry.

28. Lucky numbers of Euler

The lucky number of Euler is a number p such that the prime-generating polynomial $n^2 - n + p$ is prime for $n = 1, 2, \dots, p - 1$. Only 6 lucky numbers of Euler exist, namely 2, 3, 5, 11, 17 and 41. The lucky numbers of Euler are related to the imaginary quadratic field in which the ring of integers is factorable. Specifically, the lucky numbers of Euler (excluding the trivial case $p = 3$) are those numbers p such that the imaginary quadratic field $\mathbb{Q}(\sqrt{1 - 4p})$ has class number 1. For $p = 41$, the formula $n^2 - n + p$ generates prime numbers for $n = 1, 2, \dots, 40$.

29. Euler's formula for the critical load of a column

Euler's buckling theory is a fundamental concept in structural mechanics used to predict the buckling load of slender columns. Buckling is a form of structural instability that occurs when a column under axial compression suddenly deforms laterally. According to Euler's buckling theory, the critical load (P_{cr}) for a column with hinged ends is given by a specific formula. This Euler's critical load formula, also known as Euler's buckling formula, is a fundamental principle in the analysis and design of columns and other compression members in structural engineering. This formula relates the column's material properties, cross-sectional properties, and its length to the load it can withstand before buckling. This formula provides a way to calculate the critical load at which a long, slender column will buckle under axial compression. The formula for the critical load for a column with hinged ends is given as:

$$P_{cr} = \frac{\pi^2 E I}{l^2}$$

where P_{cr} is the critical buckling load, E is the Young's modulus of the column material (a measure of its stiffness), I is the minimum moment of inertia of the column's cross-section (a measure of its resistance to bending) and l is the actual length of the column. The concept of Euler's critical load formula is crucial in structural engineering as it helps designers to determine the minimum required size and reinforcement of columns to ensure their stability and safety under compressive loads.

30. Euler equations (fluid dynamics)

In fluid dynamics, the Euler equations are a set of partial differential equations governing adiabatic and inviscid flow. They correspond to the Navier-Stokes equations with zero viscosity and zero thermal conductivity. These equations can be applied to both incompressible and compressible flows. The Euler's equations can be written in Eulerian form as:

$$\begin{aligned} \frac{D\rho}{Dt} &= -\rho \nabla \cdot \mathbf{u} \\ \frac{D\mathbf{u}}{Dt} &= -\frac{\nabla p}{\rho} + \mathbf{g} \\ \frac{De}{Dt} &= -\frac{p}{\rho} \nabla \cdot \mathbf{u} \end{aligned}$$

where ρ is the fluid mass density, \mathbf{u} is the flow velocity vector field, p is the mechanic pressure, \mathbf{g} is the body acceleration (force per unit mass) acting on the fluid and e is an internal energy per unit mass of fluid. The Euler's equations in fluid dynamics play a significant role in simplifying complex flow problems, designing high-speed aerodynamics, and modeling ideal flows. They are crucial for understanding the dynamics of fluid behavior and are foundational equations for studying computational fluid dynamics.

31. Euler number (algebraic topology)

In algebraic topology the Euler number or Euler characteristic is a topological invariant, a number that describes a topological space's shape or structure regardless of the way it is bent. It is commonly denoted by χ (Greek lower-case letter chi).

III. DISCUSSION

Euler was famous as the leading mathematician of the eighteenth century. There was almost no mathematical discipline in the eighteenth century to which Euler did not contribute. Euler authored more than 800 research publications in his lifetime, covering a wide range of topics. Euler has contributed significantly in the following areas of Mathematics and Physics: Analysis, Number Theory, Differential Equations, Mechanics, Calculus of Variations, Graph Theory, Differential Calculus, Integral Calculus, Algebra, Astronomy, Differential Geometry, Fluid Dynamics, Quantum Mechanics, Physical Systems, etc. Euler's extensive and influential work in various mathematical fields continues to inspire and guide mathematicians to this day.

Euler introduced and popularized several notational conventions. He introduced the concept of a function and was the first to write $f(x)$ to denote the function f applied to the argument x . He also introduced the modern notation for the trigonometric functions. The use of the Greek letter π to denote the ratio of a circle's circumference to its diameter was also popularized by Euler, although it did not originate with him. He also introduced the use of the exponential function and logarithms in analytic proofs. He expressed various

logarithmic functions using power series and defined logarithms for negative and complex numbers. He developed tools that made it easier to apply calculus to problems of physical world.

The development of calculus of variations by Euler has laid the foundation for optimizing mathematical functions. In number theory, Euler made significant contributions in areas like prime numbers, quadratic residues, perfect numbers, theory of partitions, etc. Euler formulated the laws of motion that expanded Newton's laws and provided a more comprehensive understanding of mechanics. Euler's works on graph theory has given a new horizon to the study of networks and their applications. Euler's contributions in fluid dynamics has strengthened the foundations of this field and it provides a powerful mathematical framework for studying fluid characteristics.

Euler's list of works and achievements in the area of Mathematics is extensive and it extends far beyond our discussions done in this paper. Euler's brilliance, creativity and his great amount of work done in the field of Mathematics have given him the status as one of the greatest mathematicians in history. His life and work continues to inspire the researchers to explore the beauty of mathematics. It was with great reason that Laplace told his students to "Read Euler...He is the master of us all."

IV. CONCLUSION

We have discussed the mathematical legacy of Leonhard Euler in this paper. We have taken a general survey of and discussed some of the major and pioneering contributions of Euler in the field of Mathematics. These groundbreaking works of Euler have laid a powerful and impactful effect on the development of mathematical concepts and has elevated the applications of mathematics in other branches of science.

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