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Research Paper

An Integrated Optimization-Based Approaches to Fuzzy Graph Structures

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Abstract: *Fuzzy graph theory has become an essential mathematical tool for modeling uncertainty and imprecision in complex systems. This paper presents an integrated study of fuzzy graph structures (FGSs), focusing on Weighted Mean Fuzzy Graph Structures (WMFGSs), which are constructed using an optimization parameter to combine two fuzzy graph structures. The theoretical foundations of fuzzy graphs, strong fuzzy graph structures, and regular fuzzy graphs are reviewed, and several properties of WMFGSs are established. The study also highlights applications of fuzzy graphs in communication networks, biomedical systems, traffic control, neural networks, and social network analysis. Furthermore, a fuzzy graphical framework is introduced for classifying smart cities, and the Wiener index in cubic fuzzy graphs is examined. The results demonstrate the effectiveness of optimization-based fuzzy graph models in representing real-world systems characterized by vagueness and incomplete information*

Keywords: *Fuzzy Graphs; Weighted Mean Fuzzy Graph Structures; Strong Fuzzy Graphs; Optimization; Smart Cities; Social Networks.*

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I. Introduction

Fuzzy graph theory has witnessed significant development over the past three decades due to its ability to model ambiguity in relational systems. Traditional graph theory assumes crisp relationships, whereas fuzzy graphs incorporate membership values in the interval [0,1], enabling more realistic modeling of uncertain environments. Applications include communication networks, biomedical diagnostics, traffic control, neural networks, and social network analysis.

This paper focuses on Weighted Mean Fuzzy Graph Structures (WMFGSs), which integrate two fuzzy graph structures using an optimization parameter. The study also examines strong fuzzy graph structures, regular fuzzy graphs, and their mathematical properties.

(2) Background and Related Work:

(2.1) [5] Fuzzy Graph Theory:

A fuzzy graph consists of a vertex membership function and an edge membership function. Fuzzy graphs have been applied in communication networks, medical diagnosis, traffic light control, neural networks, and social network influence modeling. Among the various utilizations of the domination theory in Fuzzy graphs, the regularly talked about is a communication network. The issue is to choose a smallest arrangement of localities at which the transmitters are set with the goal that each other site in the system is joined by an immediate correspondence connect to the site, which as a transmitter. Such huge numbers of utilizations of Fuzzy Graph in Social Network like Face book, Twitter, WhatsApp, Research Gate, Instagram expanding step by step. It is realized that graphs are models or affiliations. A graph is an advantageous method for speaking to data including connection between objects. The objects are represented by nodes and relations by arcs. Whenever there is ambiguity or vagueness in the description of items or in its connections or in both, it is common that we have to plan a fuzzy graph model.

(2.2) [5] Graph Structures in Fuzzy Environments:

A graph structure (GS) is defined as a pair $G\ ((V, E))$ of sets of vertices and edges, if set E of edges carries mutually disjoint subsets E_i $i.e., E_i \subset E, 1 \leq i \leq n$ associated with symmetric and irreflexive mapping f_i. Conveniently, a graph structure can be expressed just like a simple graph labeled with f_i $1 \leq i \leq n$.

(2.2.1) Example: let $V = \{u, v, w, x\}$ be a set of vertices for an undirected graph G and $E1 = \{uv, uw, wx\}, E2 = \{ux, vw, vx\}$ are two relations defined on V associated with the functions f1 and f2, respectively .One can easily observe that E1 and E2 are irreflexive because they do not have elements as uu, vv, ww, xx, moreover, these relations are symmetric being in undirected graph. Therefore, V along with E1 and E2 establish a graph structure as shown in following Figure (1).

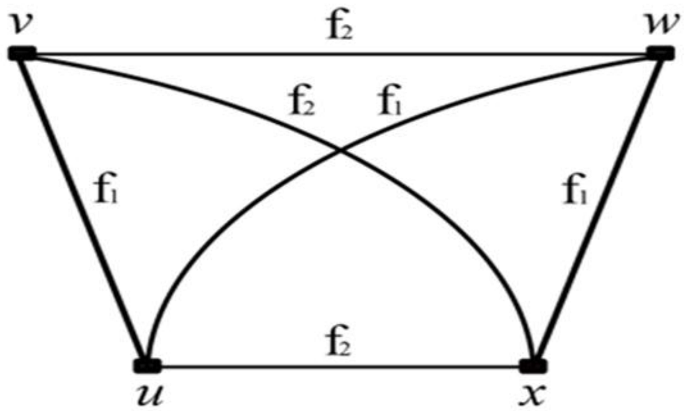

FIGURE1. A graph structure G (V E_1, E_2).

(3)[5] Weighted Mean Fuzzy Graph Structures (WMFGSs)

(3.1) Definition: If $\tilde{G}_1$ = (V_1, E_{1i},λ_1, μ_{1i}) and $\breve{G}_2$ = (V_2, E_{2j},λ_2, μ_{2j},) ,1≤ i≤ n and 1≤ j ≤n, are the two FGSs with core graph structures G_1= (V_1, E_{1i}) and G_2 = (V_2, E_{2j}), respectively. Then $\breve{G} = \breve{G}_1 * \breve{G}_2$ =(V, E_i,λ, μ_i), 1≤ i ≤n, is called WMFGS with core graph structure G= (V, E_i), 1≤ i ≤n, where

$V = V_1 \times V_2$ and $E_i = \{(u_1, v_1)(u_2, v_2) | u_1 = u_2\ and\ v_1v_2 \in E_{2j}$ For all u_1 $u_2 \epsilon$ V_1 and v_1 v_2 ϵ V_2. Fuzzy membership functions and i are defined as

$$(u\ v)\ (\ \xi(\lambda_1 u)\ v\ \lambda_2(v)) + (1-\xi)\ (\ \lambda_1(u) \wedge \lambda_2(v)$$

$$\text{and } \mu_i((u_1\ v_1)(u_2\ v_2)) =$$

$$\xi(\lambda_1(u_2) \vee \mu_{2j}(v_1v_2)) + (1 - \xi)(\lambda_1(u_1) \wedge \mu_{2i}(v_1v_2))$$

$$\textit{such that } u_1 = u_2, v_1v_2 \in E_{2j};$$

$$(\xi(\lambda_2(v_1) \vee \mu_{1i}\ (u_1u_2)) + (1 - \xi)(\lambda_2(1_1) \wedge \mu_{1i}(u_1u_2)), =$$

$$\text{such that } v_1 = v_2, u_1u_2 \in E_{1i}.$$

for $i = 1,2, \dots, n,$ and $\xi \in [0,1]$ is known as optimization parameter

(3.1) Example: Let us suppose, we have two FGS $\breve{G} = (V1, E1i, \lambda1, \mu1i)\ and\ G2 = (V2, E2j, \lambda2, \mu2j), i = 1, j = 1,2,$, that do not carry strong conditions The weighted mean product of $\widetilde{G}_1$ and $\widetilde{G}_2$, for ξ = 1, can be constructed as shown in Figure (2). From the Figure (2), we can easily observe that:

$$\mu1((u1, v1)(u1, v2)) = \lambda(u1, v1) \wedge \lambda2(u1, v2),$$

$$\mu1((u1, v1)(u2, v1)) = \lambda(u1, v1) \wedge \lambda2(u2, v1),$$

$$\mu1((u1, v2)(u2, v2)) = \lambda(u1, v2) \wedge \lambda2(u2, v2),$$

$$\mu1((u2, v1)(u2, v2)) = \lambda(u2, v1) \wedge \lambda2(u2, v2),$$

$$\mu1((u1, v3)(u2, v3)) = \lambda(u1, v3) \wedge \lambda2(u2, v3),$$

$$\mu2((u1, v1)(u1, v3)) = \lambda(u1, v1) \wedge \lambda2(u1, v3),$$

$$\mu2((u2, v1)(u2, v3)) = \lambda(u2, v1) \wedge \lambda2(u2, v3)$$

Therefore, $\breve{G} = (V, Ei, \lambda, \mu i), i = 1,2, is$ an SFGS.

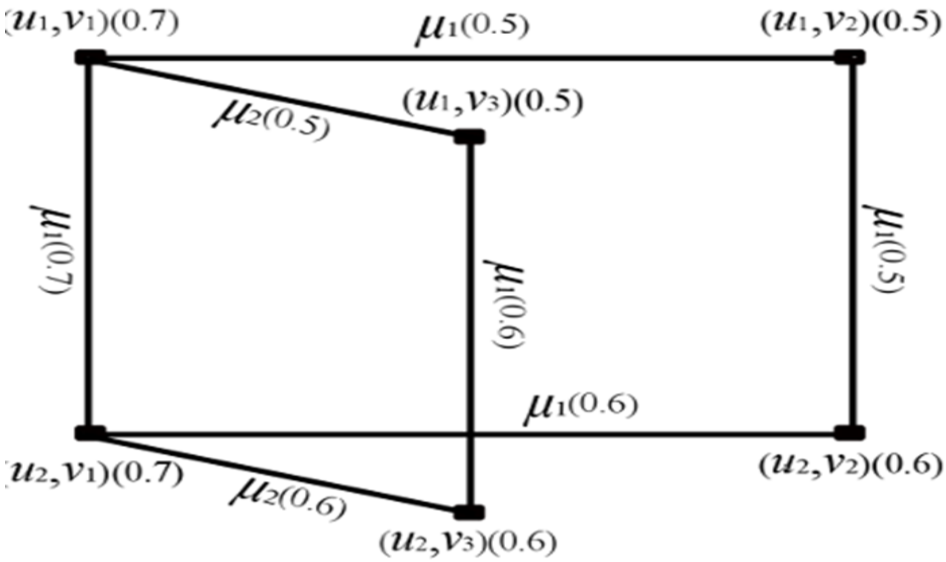

Figure2. The WMFGS (Weighted Mean Fuzzy Graph Strctures) of $\breve{G}_1$and $\breve{G}_2$ i.e. $\breve{G}_1 * \breve{G}_2$.

while μ_i-degree of a vertex in $\breve{G}$ is defined as

$\mu_i - d_{\breve{G}}$ (ui vj)

$$= \sum_{u_i u_k \in E_{1i}, v_j = v_l} \xi\left(\lambda_2(v_j) \vee \mu_{1i}(u_i u_k)\right) + (1-\xi) \times \left(\lambda_2(v_j) \wedge \mu_{1i}(u_i u_k)\right) + \sum_{\substack{v_j v_l \in E_{2j} \\ u_i = u_k}} \xi\left(\lambda_1(u_i) \vee \mu_{2i}(v_j v_l)\right) + (1-\xi)\left(\lambda_1(u_i) \wedge \mu_{2i}(v_j v_l)\right)$$

(3.2) Theorem: if $G_1 = (V_1, E_{1i}, \lambda_1, \mu_{1i}), 1 \leq i \leq n$, and $\tilde{G}_2 = \left(V_2, E_{2j}, \lambda_2, \mu_{2j}\right), 1 \leq j \leq n$ are two FGSs such that $\lambda_1 \leq \mu_{2j}$ then a vertex degree in WMFGS $\breve{G} = \breve{G}_1 * \breve{G}_2$ is evaluated as

$$d_{\tilde{G}}(u_i, v_j) = \xi\left(d_{G_1}(u_i)\lambda_2(v_j) + d_{\tilde{G}_2}(v_j)\right) + (1-\xi)\left(d_{G_2}(v_j) \times \lambda_1(u_i) + d_{\tilde{G}_1}(u_i)\right).$$

Proof: *Suppose that* $\tilde{G}_1 = (V_1, E_{1i}, \lambda_1, \mu_{1i}), 1 \le i \le n,$ *and* $\tilde{G}_2 = (V_2, E_{2j}, \lambda_2, \mu_{2j}), 1 \le j \le n$, are two FGSs such that $\lambda_1 \le \mu_{2j}$then $\mu_{1i} \le \lambda_2$. Furthermore, by definition of vertex degree in WMFGS $\breve{G}$, we have

$$
\begin{aligned}
&d_{\tilde{G}}(u_i, v_j) \\
&= \sum_{u_i u_k \in E_{1i}, v_j = v_l} \xi\left(\lambda_2(v_j) \vee \mu_{1i}(u_i u_k)\right) + (1-\xi)(\lambda_2 \\
&\quad \times (v_j) \wedge \mu_{1i}(u_i u_k)\Big) + \sum_{v_j v_l \in E_{2j}, u_i = u_k} \xi\left(\lambda_1(u_i) \vee \mu_{2j}(v_j v_l)\right) \\
&\quad +(1-\xi)\left(\lambda_1(u_i) \wedge \mu_{2j}(v_j v_l)\right) \\
&= \sum_{u_i u_k \in E_{1i}, v_j = v_l} \xi\left(\lambda_2(v_j)\right) + (1-\xi)\left(\mu_{1i}(u_i u_k)\right) + \sum_{v_j v_l \in E_{2j}, u_i = u_k} \\
&\quad \times \xi\left(\mu_{2j}(v_j v_l)\right) + (1-\xi)\left(\lambda_1(u_i)\right) \\
&= \xi \sum_{u_i u_k \in E_{1i}, v_j = v_l} \left(\lambda_2(v_j) + \mu_{2j}(v_j v_l)\right) + (1-\xi) \sum_{v_j v_l \in E_{2j}, u_i = u_k} \\
&\quad \times \left(\mu_{1i}(u_i u_k) + \lambda_1(u_i)\right) \\
&= \xi\left(d_{G_1}(u_i)\lambda_2(v_j) + d_{\tilde{G}_2}(v_j)\right) + (1-\xi)\left(d_{G_2}(v_j)\lambda_1(u_i)\right. \\
&\quad \left. + d_G(u_i)\right).
\end{aligned}
$$

(3.3) Definition: An FGS $\tilde{G} = (V, E_i, \lambda, \mu_i), 1 \le i \le n,$ is said to be regular fuzzy graph structure (RFGS), if each vertex of G carries same degree, and is mμ_k-regular if all vertices of $\tilde{G}$ have same degrees mϵ[0,1]under membership function μ_k, k $\in \{1,2, \dots, n\}$.

(4) [11] Social Networks: Analysis, Algorithms and Their Implementation:

(4.1) Networks and its analysis:

A network is a set of nodes (such as people, organizations, websites, or government entities) and the relationships between them (oriented or not). For example, for the postal network is the principle to order the nodes (senders and recipients). A social network organized by the software is not usually an oriented network of friends (users). Network analysis makes good results when all network nodes belong to the same object class. To explore more than one type of object (such as bloggers and commentators), "two-level analysis," which involves your own set of distinctions have to be used.

There are the following types of networks:

- Solid networks (e-mail, mailing lists, and social networks of the Internet space): links are clearly defined, it is easy to form a group structure and identify specific users. They are built using lists, which allows you to reach any user
- Partial networks: they are a compromise between the desire to cover a whole network and the fact that some whole networks are just too massive to cover them completely. The researcher can start with a single web page or several pages (so-called "sowing"), then he will look for pages related to that sowing, and then pages related to these pages. The sampling process ends when a sufficient number of pages have been collected; when all possible pages are collected; or when the sample meets certain criteria (for example, when all pages with more than 400 words are collected);
- Ego networks: they are represented by a spontaneous sample of users, the researcher can collect data or a star-shaped network (ego-node and its connections with other nodes), or a complete ego network (which also includes connections of other nodes with each other). Data collection on ego networks can be based on the already available results of various research techniques and interviews.
- Social networks: people are together in pre-established interpersonal relationships such as kinship, friendship, classmates, colleagues, business partners, etc. A connection is built one at a time. The main reason for people to join a social network is to maintain old relationships and create new ones to expand

their network. There are social networks for communication (Facebook), for sharing media content (Instagram), for collective bargaining (Quora), for authoring (Twitter), social bookmarking services (Pinterest), for interests (Goodreads).

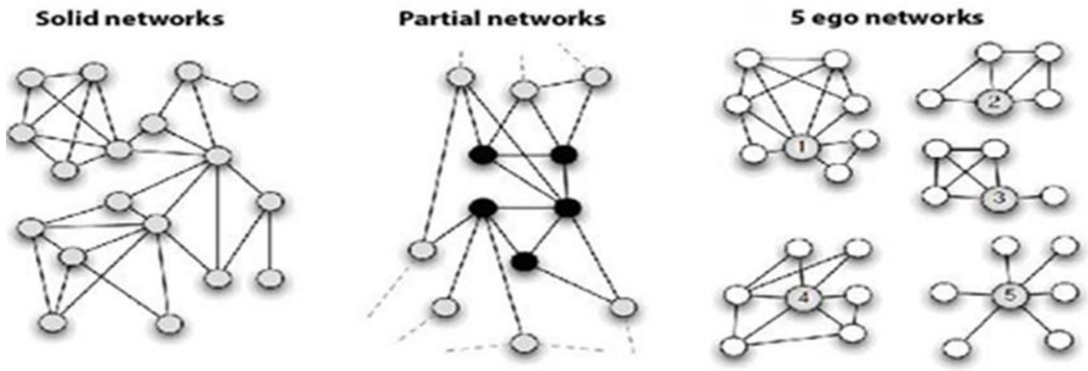

Figure.3. The examples of solid, partial and ego networks are shown

(4.2) Determination of weak and strong ties:

The powerful factor for the tie construction is a relationship between friends and friends of friends. The connections between them are the most important thing to create a graphical representation of a graph of friends. To detect the strength of communication (weak or strong), the program will make a comparison between all friends on the following parameters:

- School - the system will compare data of friends and find friends:
  a. who studies in the same schools,
  b. go to school, which is located in the same region in a big city. c. or all schools from a small town (where 2-5 schools) and shops,
- Work - the program compares data of friends and finds friends:
  a. in profiles whose job data match,
  b. or companies, where they work, are located in the same city and work in the same field,
- Communities - the program will compare groups of friends and search for shared groups that both have. It is also a criterion for determining the strength of the connection between a user.
- Music (selective) - this criterion for those people who want to find among their friend's people who have common favors in music.
- Photo - the program will analyze the number of tags in the photo of certain person, in the photos which are in the profiles of friends. The weight of these connections is calculated.
- Information that meets the above criteria is broken down into a database. And then the system has a complete database with information about all friends from the list of friends in a social network. Then the collected data about each friend will be compared with others. There is a gradation of bond strength: weak ties, medium and strong ties.

Weak ties according to the system of criteria are those ties that will correspond to 0 or 1 criterion. Medium ties according to the system of criteria are those ties that will correspond to 2 criteria. Strong ties according to a system of criteria are those that will correspond to 3 or more criteria.

This is not a definitive system of criteria. There are many other variations that can be used to strengthen and define relationships.

(5) Database design

There are the following basic elements: Person, Playlist, Song, List Communities, Community, Photo, Album, Tag (see Figure 4). The Person table is used to store all users who can be parsed using the API. All users' data will be compared with others. The Playlist is the connecting table between the Person table and the Songs table. For example, each person has music that is in a playlist. The Song table describes the song in the playlist for future comparison of the two playlists. Usually, everyone has albums that have photos, but often there can be many such albums. The album table gives a foreign key to person and has only one user field. Each photo in this

Photo table is described by three fields: idphoto, name, date. The tag table has information about each tag of our user in the photos of other users. The List Communities table is the same as the Album or Playlist. Everyone can visit some communities. There are many communities, but the list of communities that a particular person attends is unique, even if there are two people with the same communities on their list. The information in this table is also used to find the strength of the links.

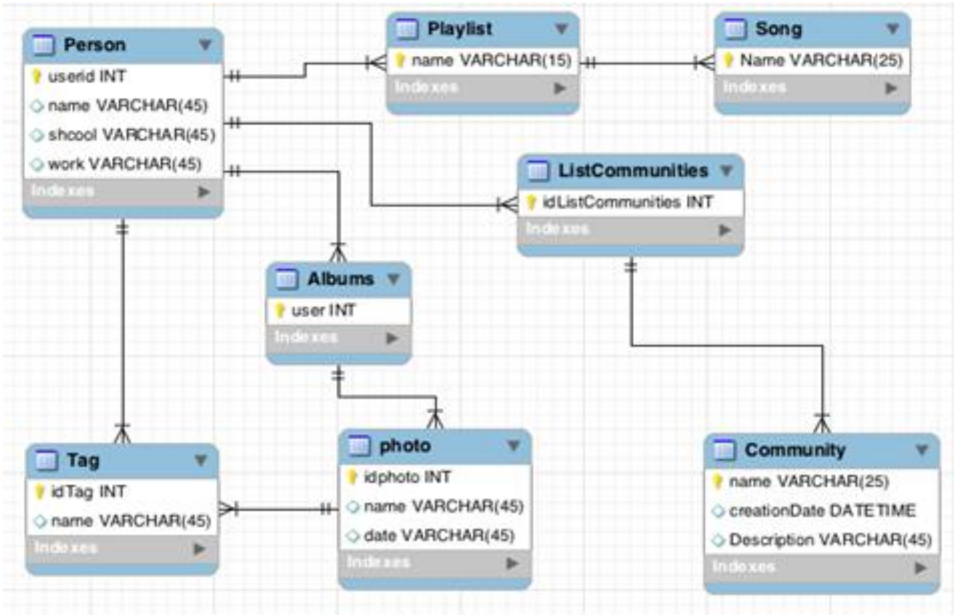

Figure 4: The database schema

The Community table is a table that describes the community and has three fields: name, creation Date, Description. Basic tools of development RabbitMQ is a platform that implements a messaging system between software system components based on the AMQP (Advanced Message Queuing Protocol) standard. Uses the standard AMQP (Advanced Message Queuing Protocol), supports horizontal scaling to build a cluster architecture, supports data storage on disk, support for HTTP, XMPP and STOMP, is the implementation of clients to access RabbitMQ for some programming languages: Java, .NET, Perl , Python, Ruby, PHP, etc., there are various plug-ins (such as a plug-in for monitoring and management via HTTP or web interface or a plug-in "Shovel" for transferring messages between brokers). NetworkX Python library for studying graphs and networks. NetworkX is free software that is distributed under a BSD license. Gives you the ability to convert graphics from multiple formats, build random graphs or build them gradually, find subgraphs, clicks, K-cores, explore contiguity, degree, diameter, radius, center, intermediate, etc., draw networks in 2D and 3D. Matplotlib is a Python programming library for data visualization with 2D graphics (3D graphics are also supported). The resulting images can be used as illustrations in publications. Matplotlib is a flexible, easily configurable package that, along with NumPy, SciPy, and IPython, provides features similar to MATLAB. The package currently works with several graphics' libraries, including wxWindows and PyGTK.

(6) [5] The Smart Cities and the Fuzzy Graphs:

(6.1) Comparisons of Cities using Fuzzy graphs:

Some cities are known for their constructed highways, while others have a variety of universities. For example, Islamabad and Multan are two signs can't and famous cities in Pakistan. Islamabad is the Pakistan's most advanced and modern city, but Multan appears to be underdeveloped. Everything that Multan would have in future, Islamabad already has. There are some characteristics that every city is famous for. For example, Faisalabad is famous for the Textile industry, Lahore is famous for its grand hospitals and educational institutes, Karachi is the premier industrial and financial center of Pakistan. We can use a fuzzy-graphic framework to show the most advanced and evolved city after comparing any two in a given time frame. We can also say the degree of the slow and under-developed phase at that time, with the assistance of the membership function. The fuzzy-graphic structure of the most developed and evolving cities can be very useful for a country to be

TABLE1. Development levels of five cities

City	Development level
Islamabad	0.9
Karachi	0.8
Lahore	0.8
Faislabad	0.7
Multan	0.6

concentrated on. Let us have a set C of five cities of Pakistan: $C = \{Islamabad, Karachi, Lahore, Faisalabad, Multan\}$. Let λ be a fuzzy mapping on C i.e., C↔ [01] that evaluates the level of a developed city in Pakistan under four considered parameters, as shown in Table 1. Tables 2-6 represent the development levels for each pair of cities using the law.

$\mu(v_1 v_2) = \sigma(v_1) \wedge \sigma(v_2), \forall v_1, v_2 \in C.$

Now against set C we can de ne several labeling functions, for instance, let us have as: $f_1 =$ Hospitals, $f_2 =$ Educational Facilities, $f_3 =$ Industry development, f4 = Roads network, so that G = (C ,E_1, E_2 E_3 E_4), is a graph structure, where E_1 E_2 E_3 and E_4 are disjoint subsets of edges

TABLE 2 Comparison of Islamabad with other cities.

Criteria	(Islamabad,Karachi)	(Islamabad,Lahore)	(Islamabad,Faisalabad)	(Islamabad,Multan)
Hospitals	0.8	0.8	0.7	0.6
Educational facilities	0.8	0.8	0.7	0.6
Industry development	0.7	0.7	0.6	0.5
Roads' network	0.8	0.8	0.7	0.6

Table 3 Comparison of Lahore with other cities

Criteria	(Lahore,Islamabad)	(Lahore,Karachi)	(Lahore,Faisalabad)	(Lahore,Multan)
Hospitals	0.8	0.8	0.6	0.5
Educational facilities	0.7	0.7	0.7	0.6
Industry development	0.7	0.7	0.6	0.6
Roads' network	0.6	0.6	0.7	0.5

Table 4**.** Comparison of Faisalabad with other cities

Criteria	(Faisalabad,Islamabad)	(Faisalabad,Karachi)	(Faisalabad,Lahore)	(Faisalabad,Multan)
Hospitals	0.6	0.6	0.6	0.6
Educational facilities	0.6	0.5	0.6	0.5
Industry development	0.7	0.7	0.7	0.6
Roads' network	0.7	0.6	0.6	0.6

Table 5. Comparison of Multan with other cities

Criteria	(Multan,Islamabad)	(Multan,Karachi)	(Multan,Lahore)	(Multan,faisalabad)
Hospitals	0.4	0.6	0.5	0.5
Educational facilities	0.3	0.5	0.4	0.4
Industry development	0.4	0.5	0.4	0.3
Roads' network	0.5	0.4	0.5	0.4

Table 6. Comparison of Karachi with other cities

Criteria	(Karachi,Islamabad)	(Karachi,Lahore)	(Karachi,Faisalabad)	(Karachi,Multan)
Hospitals	0.7	0.6	0.6	0.5
Educational facilities	0.6	0.7	0.5	0.4
Industry development	0.8	0.8	0.7	0.6
Roads' network	0.5	0.6	0.6	0.5

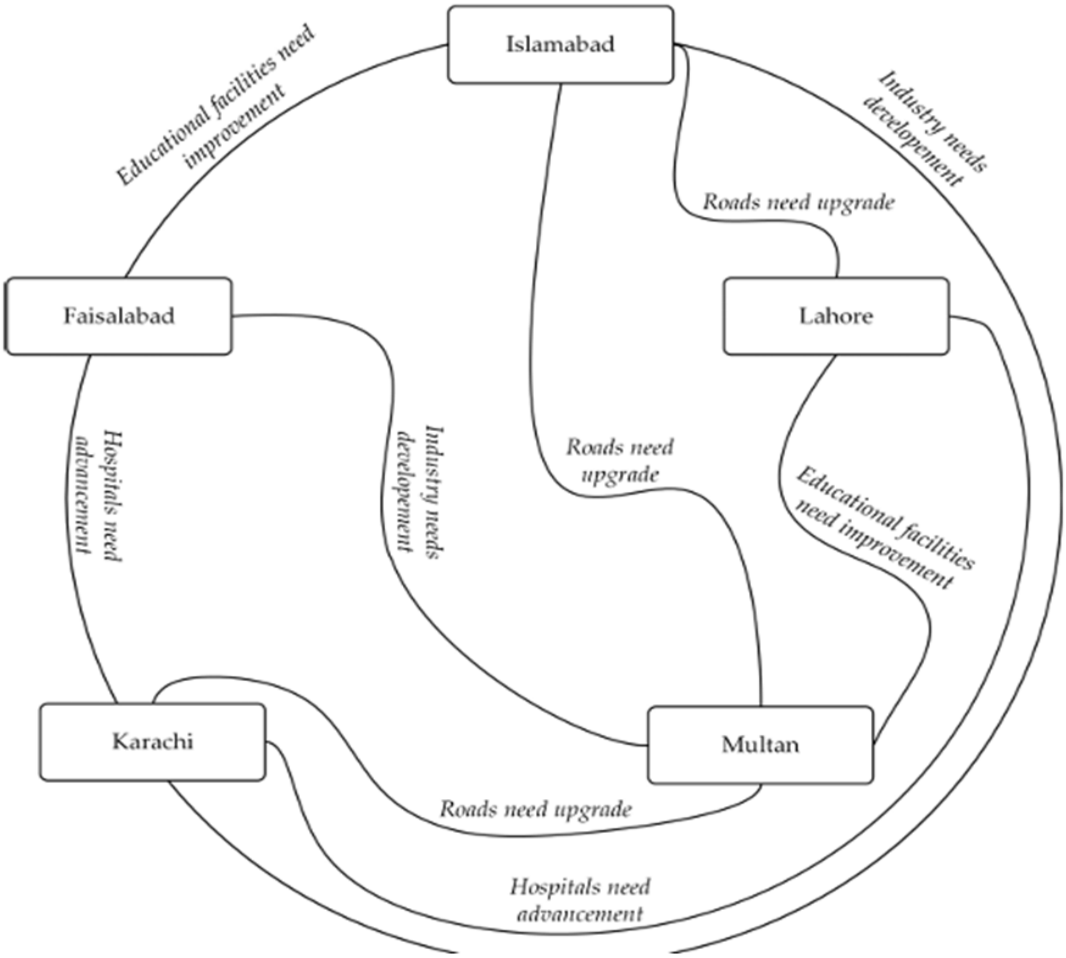

FIGURE 5. An FGS representing the parameter that needs improvement for each pair of cities. regarding pairs of cities. Let us consider

E_1 = {(Islamabad , Karachi), (Islamabad , Multan), (Karachi , × Faisalabad), (Lahore , Islamabad)},
E_2 = {(Faisalabad , Islamabad), (Lahore , Multan)},
E_3 = {(Karachi,Lahore), (Faisalabad , Multan)},
E_4 = {(Multan,Lahore), (Lahore,Islamabad)},

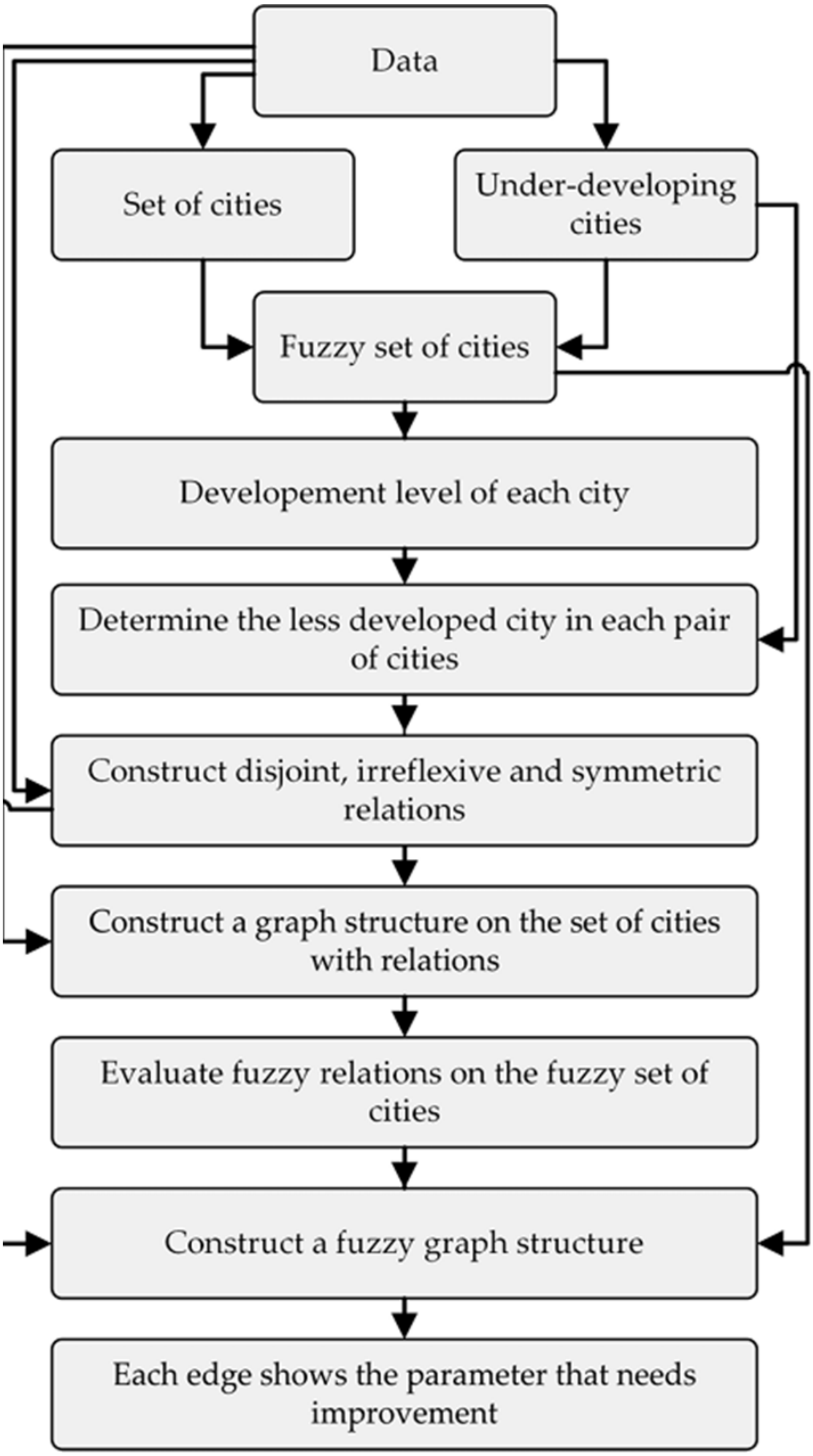

Figure 6. Flowchart of the procedure to determine the criteria which require enhancement and the corresponding fuzzy sets under membership function$\mu_{1,}\mu_{2,}\mu_3$ and μ_4, respectively, are:

$$\begin{aligned}
\mu_1 = \; & \{((\text{ Islamabad,Karachi }),0.8), \\
& \times ((\text{ Islamabad,Multan }),0.6), \\
& \times ((\text{ Karachi,Faisalabad }),0.7), \\
& \times ((\text{ Lahore,Islamabad }),0.8)\}, \\
\mu_2 = \; & \{((\text{ Faisalabad,Islamabad }),0.7), \\
& \times ((\text{ Lahore,Multan }),0.5)\}, \\
\mu_3 = \; & \{((\text{ Karachi,Lahore }),0.8), \\
& \times ((\text{ Faisalabad,Multan }),0.6)\}, \\
\mu_4 = \; & \{((\text{ Multan,Lahore }),0.5), \\
& \times ((\text{ Lahore,Faisalabad }),0.7)\}.
\end{aligned}$$

It is clear that $\breve{\ddot{G}} = (V, E_i, \lambda, \mu_i), 1 \leq i \leq 4$, is an FGS as shown in Figure (5).

In Figure (3), each edge of FGS is used to represent the parameter that needs to be improved in corresponding city. For instance, according to this FGS, Faisalabad and Multan have to improve their educational institutions as compare to Islamabad and Lahore, respectively. In the same way, Faisalabad needs to enhance hospitals in comparison with Karachi, while Karachi has to improve hospitals as compare to Lahore. An FGS of all cities can be very helpful for a country to maintain the assets in order to facilitate the people. It would highlight those cities which need some enhancement. The basic idea used in this application is illustrated by a flowchart on Figure (6).

VII. Conclusion

The graph theory has numerous implementations to address diverse problems in different fields, such as networking, connectivity, data analysis, cluster analysis, signal processing, image optimization, scheduling and planning. In order to deal with the uncertainty and vagueness of the graphical system, the use of fuzzy-graphical methods is very natural. Fuzzy Graph Theory has a wide range of uses in the simulation of various real-time processes. The level of work contained in the structure differs with divergent degrees of accuracy. In this paper, we presented a number of various concepts relating to fuzzy-graphic structures, such as the weighted mean product of two fuzzy-graphic structures and regular fuzzy-graphic structures, and examined several related attributes. In addition, we have suggested the execution of fuzzy-graphic frame works to compare five major cities of Pakistan based on four criteria, such as Hospitals, Education, Industry and Roads. Social networks are an important component of communication in modern life. Their analysis is necessary both to understand the current situation and to improve some aspects of life. Using the concepts of graph theory "centrality" and "intermediate centrality", a method of analyzing connections in cyberspace was proposed. This method was used to build a prototype for social network analysis. The RabbitMQ, NetworkX, and Matplotlib ties algorithm include depth setting, data sending, response retrieval, and optimization.

References

[1]. A. Kaufmann, Introduction a la Theorie des Sour-ensembles Flous. Paris France Masson et Cie, 1973.

[2]. A. Rosenfeld, Fuzzy Graph, L. A. Zadeh, K. S. Fu and M. Shimura, Eds. New York, NY, USA: Academic, 1975, pp. 77–95

[3]. A.N. Gani and K. Radha, "On regular fuzzy graphs" J. Phys. Sci., vol. 12, pp. 33–44, May 2008.

[4]. A. N. Gani and K. Radha, "The degree of a vertex in some fuzzy graphs," Int. J. Algorithms, Compute. Math., vol. 2, no. 3, pp. 107–116, 2009.

[5]. ATIQ UR REHMAN MUHAMMADHUSSAIN 1, WOJCIECH SAŁABUN 2, (Member, IEEE), SHAHZAD FAIZI3, ANDJAROSŁAW WA¸TRÓBSKI, On Graph Structures in Fuzzy Environment Using Optimization Parameter, May 2021.

[6]. F. Long, N. Ning, C. Song, B. Wu, Strengthening Social Networks Analysis by Networks Fusion, In: Proceedings of the ACM International Conference on Advances in Social Networks Analysis aand Mining, ASONAM '2019, Vancouver, BC, Canada, 2019, pp. 460-463. doi: 10.1145/3341161.3342939.

[7]. M. Pruthvi, S. Karthika, N. Bhalaji, "SMART COLLEGE"- Study of Social Network and IoT Convergence, in: Proceedings of the 2nd International Conference on IoT in Social, Mobile, Analytics and Cloud, I-SMAC '2018, Palladam, India, 2018, pp. 100-103. doi: 10.1109/I SMAC.2018.8653787.

[8]. N. Salia, C. Tompkins, O. Zamora, An Erdős-Gallai type theorem for vertex-colored graphs. Graphs and Combinatorics, volume 35, 2019, pp. 689–694. doi:10.1007/s00373-019-02026-1.

[9]. J. Mena, Investigative Data Mining for Security and Criminal Detection. Butterworth Heinmann, 2003.

[10]. J. N. Mordeson and P. Chang-Shyh, "Operations on fuzzy graphs," Inf. Sci., vol. 79, nos. 3–4, pp. 159–170, Jul. 1994.

[11]. Svitlana Popereshnyak, Iryna Yurchuk Taras Shevchenko National University of Kyiv, Bohdan Hawrylyshyn str. 24, Kyiv, UA-04116, April 2021, Ukraine Social Networks: Analysis, Algorithms and Their Implementation

[12]. S. A. Bitaghsir, S. Kashipazha, A. Dadlani, A. Khonsari, Social-aware Mobile Road Side Unit forContent Distribution in Vehicular Social Networks, in: Proceedings of the 2019 IEEE Symposium on Computers and Communications, ISCC '2019, Barcelona, Spain, 2019, pp. 1-6, doi: 10.1109/ISCC47284.2019.8969669.
[13]. U. Brandes, A Faster, Algorithm for Betweenness Centrality, The Journal of Mathematical Sociology, volume 25 (2004). doi: 10.1080/0022250X .2001. 9990249.
[14]. R. V. Ramakrishnan and T. Dinesh, "On generalized fuzzy graph structure" Appl. Math. Sci., vol. 5, no. 4, pp. 173–180, 2011.
[15]. Lili Zhang[1]. Shu Sun[1]. Ruping Wang[2] .Chunfeng Suo[1] . Graph-based multi-attribute decision-making method with new fuzzy information measures, 2025.