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**Research Paper**

# Ulam-Hyers Stability of Quadratic Functional Equation in Modular and F-Spaces

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***Abstract:*** *This paper investigates the stability of new quadratic functional equations in the context of modular and F-spaces. By considering the behaviour of solutions under perturbations, we establish conditions for the existence and uniqueness of stable solutions. The study extends previous results on stability theory to encompass the specific framework of quadratic functionals in modular and F-spaces, providing insights into the robustness of solutions within these mathematical structures.*
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## I. Introduction

Ulam [15] presented the following stability problem in 1940, given metric group $G(\cdot,\rho)$, number $\varepsilon > 0$ and mapping $f : G \to G$ that fulfils the condition $\rho(f(x \cdot y), f(x) \cdot f(y)) < \varepsilon$ for all $x, y$ in $G$, do we have an automorphism a of $G$ and constant $k > 0$, relying exclusively on $G$, such that $\rho(a(x), f(x)) \leq k\varepsilon$ for all $x$ in $G$? We refer to the automorphism equation $a(x \cdot y) = a(x) \cdot a(y)$ as stable if the response is in the affirmative.

Ulam's dilemma received a positive partial solution from Hyers [5] a year later. Rassias proved a generalized version of Hyers result in 1978 in [13]. Since then, a number of authors have thoroughly examined the stability issues of several functional equations ([3–4], [6], [8], [12], [14], [16–18], [20–22]).

In fact, we also direct readers to the following papers: [1] for new developments on the conditional stability of the homomorphism equation and books; [2] for developments in Ulam's type stability; and ([7], [10]) for a comprehensive knowledge of stability theory.

The theory of modular linear spaces and its supporting theory were developed in 1950 by Nakano [11], and they were later improved upon by Koshi, Shimogaki [9], and Yamamuro [19].

In 2016, Xiuzhong Yang [22] proved the stability of a quadratic functional equation in F-space.
We recall some notions of F-space, $\beta$-normed spaces and modular space, for detailed understanding of the properties of the above spaces, the readers are required to read the book [14].

**Definition 1.1.** Let $X$ be a linear space over $\mathbb{K}$ that denotes either complex or real numbers. A non-negative valued function $\|\cdot\|$ defined on $X$ is called F-norm (or briefly a norm) if it satisfies the following conditions:
$(N_1)$ $\| x \| = 0$ if and only if $x = 0$.
$(N_2)$ $\| ax \| = \| x \|$ for all $a \in \mathbb{K}, |a| = 1$.
$(N_3)$ $\| x + y \| \leq \| x \| + \| y \|$.
$(N_4)$ $\|a_n x\| \to 0$ provided $a_n \to 0$.
$(N_5)$ $\|a x_n\| \to 0$ provided $x_n \to 0$.
$(N_6)$ $\|a_n x_n\| \to 0$ provided $a_n \to 0, x_n \to 0$.
A linear space equipped with F-norm is called $F^*$-space which will be denoted by $(X, \|\cdot\|)$ or $X$. A complete $F^*$-space is called F-space.

**Definition 1.2.** Let $X$ be a linear space over $\mathbb{K}$ that denotes either complex or real numbers and $0 < \beta \leq 1$. A non-negative real valued function $\|\cdot\|$ defined on $X$ is called $\beta$-norm if it satisfies the following conditions:
$(N_1)$ $\| x \| = 0$ if and only if $x = 0$.
$(N_2)$ $\| ax \| = |a|^\beta \| x \|$ for all $a \in \mathbb{K}$.
$(N_3)$ $\| x + y \| \leq \| x \| + \| y \|$.
A linear space equipped with $\beta$-norm is called $\beta$-normed space, which will be denoted by
$(X, \|\cdot\|)$ or briefly $X$.

**Remark 1.3.** It is clear that $\beta$-normed space is a special $F^*$ space, and when $\beta = 1$, $\beta$-normed space become normed space. Comparing with the normed space, $F^*$-space does not possess good metric properties, and the study of the stability of functional equations becomes more difficult.

**Definition 1.4.** Let $V$ be a vector space over $\mathbb{K}$ ( $C$ or $R$). A generalized functional $\rho : V \to [0,\infty)$ is called a modular if for arbitrary $u, v \in V$, $\rho$ satisfies the following properties:
(a) $\rho(u) = 0$ if and only if $u = 0$
(b) $\rho(\beta u) = \rho(u)$ for every scalar $\beta$ with $|\beta| = 1$
(c) $\rho(\beta u + \gamma v) \le \rho(u) + \rho(v)$, whenever $\beta, \gamma \ge 0$ and $\beta + \gamma = 1$
If we replace (c) by (c') $\rho(\beta u + \gamma v) \le \beta\rho(u) + \gamma\rho(v)$, whenever $\beta, \gamma \ge 0$ and $\beta + \gamma = 1$, then, the modular $\rho$ is called convex. A modular $\rho$ defines a corresponding modular space, i.e., the vector space $V\rho$ given by: $V\rho = \{u \in V; \rho(cu) \to 0$ as $c \to 0\}$.

**Definition 1.5.** If $V\rho$ is a modular space and the sequence $\{V_n\}$ in $V\rho$, then
(i) $v_n \to v$ if $\rho(v_n - v) \to 0$ as $n \to \infty$
(ii) $\{v_n\}$ is known as $\rho$-Cauchy if $\rho(v_m - v_n) \to 0$ as $m, n \to \infty$
(iii) A subset $A \subseteq V\rho$ is known as $\rho$-complete if and only if every $\rho$ - Cauchy sequence is $\rho$ convergent in A.
There are many forms of the quadratic functional equation among them of great interest to us is the following:

$$f(x + y - 2z) + f(x - 2y + z) = f(2y - 2z) + f(x - z) + f(x - y) \quad (1)$$

**Result 1.6.** Let $F$ be a linear space and a mapping $\phi: E \to F$ satisfies the functional equation (1), then the following results are hold:
(1) $\phi(q^t s) = q^{2t}\phi(s)$ foll all $s \in E, q \in Q, t \in Z$
(2) $\phi(s) = s\phi(1)$ foll all $s \in E$ if $\phi$ is continuous.
The purpose of this paper is to study the stability of equations in $F$-space and Modular space. Section 2 is devoted to the study of stability of (1) in $F$-space and section 3 for stability of functional equation in modular space, in more general setting such that the target spaces are $F$-spaces or complete $\beta$-normed spaces and modular space. Throughout section 2, let $X$ be a real $\beta$-normed space and $Y$ be complete $F$-spaces or $\beta$ normed space. Also $\mathbb{R}, \mathbb{K}$, and $\mathbb{N}$ stand for the set of all real numbers, real numbers, or complex numbers and natural numbers, respectively.

## II. Hyers-Ulam-Rassias Stability of (1) in *F*-Space

From now on, let $X$ be a real vector space and let $Y$ be $F$-space in which there exists $1/3 \le c < 1$ such that $\| y/3 \| \le c \| y \|$ for all $x \in Y$, unless we give any specific reference.
We will investigate the Hyers-Ulam-Rassias stability problem for functional equation (1). Thus, we find the condition that there exists a true quadratic function near an approximately quadratic function.

**Theorem 2.1.** Let $X$ be a real vector space and $Y$ be $F$-space in which there exists $1/3 \le c < 1$ such that $\| y/3 \| \le c \| y \|$ for all $x \in Y$, and let $\varphi: X \times X \to \mathbb{R}^+$ be a function such that

$$\sum_{i=0}^{\infty} c^{2i}\varphi(3^i z, 3^i 2z, 3^i 3z) \quad (2)$$

converges and

$$\lim_{n\to\infty} c^{2n}\varphi(3^n x, 3^n y, 3^n z) = 0, \quad (3)$$

for all $x, y, z \in X$. Suppose that $f$ satisfies

$$\|f(x + y - 2z) + f(x - 2y + z) - f(2y - 2z) - f(x - z) - f(x - y)\| \le \varphi(x, y, z), \quad (4)$$

for all $x, y, z \in X$. Then, there exists unique quadratic function $g: X \to Y$ which satisfies (4) and the inequality

$$\| f(z) - g(z) \| \le c^2 \sum_{i=0}^{\infty} c^{2i}\varphi(3^i z, 3^i 2z, 3^i 3z), \quad (5)$$

for all $z \in X$. Function $g$ is given by

$$g(z) = \lim_{n\to\infty} 3^{-2n} f(3^n z), \quad (6)$$

for all $z \in X$.

**Proof.** Replacing $(x, y, z)$ by $(z, 2z, 3z)$ in (4), we have

$$\| f(3z) - 9f(z) \| \le \varphi(z, 2z, 3z). \quad (7)$$

It follows that

$$\|f(z) - 9^{-1}f(3z)\| \le c^2\varphi(z, 2z, 3z), \quad (8)$$

for all $z \in X$. Replacing $z$ by $3z$ in (8) and by the assumption on norm, we get

$$\|9^{-1}f(3z) - 9^{-2}f(3^2z)\| \leq c^2 \cdot c^2\varphi(3z, 2.3z, 3.3z). \quad (9)$$

Hence,

$$\|f(z) - 9^{-2}f(3^2x)\| \leq c^2[\varphi(z, 2z, 3z) + c^2\varphi(3.z, 3.2z, 3.3z)] \quad (10)$$

for all $z \in X$. Using the induction on positive integer $n$, we obtain that

$$\|f(z) - 9^{-n}f(3^nz)\| \leq c^2 \sum_{i=0}^{n-1} c^{2i}\varphi(3^iz, 3^i2z, 3^i3z), \quad (11)$$

for all $z \in X$. In order to prove convergence of sequence $\{9^{-n}f(3^nx)\}$, replace $x$ by $3^mx$ to find that, for $n, m > 0$,

$$\begin{aligned}\left\|9^{-(n+m)}f(3^{n+m}z) - 9^{-m}f(3^mz)\right\| &= \|9^{-m}[9^{-n}f(3^{n+m}x) - f(3^mx)]\| \\ &\leq c^2c^2 \sum_{i=0}^{n} c^{2(m+i)}\varphi(3^{m+i}z, 3^{m+i}2z, 3^{m+i}3z) \\ &= c^2c^2 \sum_{i=m}^{n} c^{2i}\varphi(3^{m+i}z, 3^{m+i}2z, 3^{m+i}3z). \quad (12)\end{aligned}$$

Since the right hand side of the inequality tends to 0 as $m$ tends to infinity, sequence $\{9^{-n}f(3^nx)\}$ is a Cauchy sequence.

Therefore, we may define $g(z) = lim_{n\to\infty}3^{-2n}f(3^nz)$ for all $z \in X$. By letting $n \to \infty$ in (11), we arrive at formula (5). To show that $T$ satisfies (1), replace $x, y, z$ by $3^nx, 3^ny, 3^nz$, respectively; then, it follows tha

$$\begin{aligned}\| 9^{-n}[f\left(3^n(x + y - 2z)\right) + f(3^n(x - 2y + z) - f\left(3^n(2y - 2z)\right) \\ -f\left(3^n(x - z)\right) - f(3^n(x - y))] \| \\ \leq c^{2n}\varphi(3^nx, 3^ny, 3^nz). \quad (13)\end{aligned}$$

Taking the limit as $n \to \infty$, we find that $g$ satisfies (1) for all $x, y, z \in X$. To prove the uniqueness of quadratic function $T$ subject to (5), let us assume that there exists quadratic function $S: X \to Y$ which satisfies (1) and inequality (5). Obviously, we have $S(3^nz) = 9^nS(z)$ and $T(3^nz) = 9^nT(z)$ for all $z \in X$ and $n \in N$. Hence, it follows from (5) that

$$\begin{aligned}\| S(z) - T(z) \| = \|9^{-n}[S(3^nz) - T(3^nz)]\| &\leq c^{2n}\|S(3^nz) - f(3^nz)\| + \|f(3^nz) - T(3^nz)\| \\ &\leq c^2 \sum_{i=0}^{\infty} c^{2(n+i)}\varphi(3^i3^nz, 3^i3^n2z, 3^i3^n3z), \quad (14)\end{aligned}$$

for all $z \in X$. By letting $n \to \infty$ in the preceding inequality, we immediately find the uniqueness of $g$. This completes the proof of the theorem.

**Corollary 2.2.** Let $X$ be a real vector space and $Y$ be a complete $\beta$-normed space $(0 < \beta \leq 1)$, and let $\varphi: X \times X \to R^+$be a function such that

$$\sum_{i=0}^{\infty} 9^{-\beta i}\varphi(3^iz, 3^i2z, 3^i3z) \quad (15)$$

converges and

$$\lim_{n\to\infty} 9^{-\beta n}\varphi(3^nx, 2^ny, 3^nz) = 0, \quad (16)$$

for all $x, y, z \in X$. Suppose that $f$ satisfies

$$\| f(x + y - 2z) + f(x - 2y + z) - f(2y - 2z) - f(x - z) - f(x - y) \| \leq \varphi(x, y, z), \quad (17)$$

for all $x, y, z \in X$. Then, there exists unique quadratic function $g: X \to Y$ which satisfies (1) and inequality

$$\| f(z) - g(z) \| \leq 27^{-\beta} \sum_{i=0}^{\infty} 9^{-\beta i}\varphi(3^iz, 3^i2z, 3^i3z), \quad (18)$$

for all $z \in X$. Function $g$ is given by

$$g(z) = \lim_{n\to\infty} 3^{-2n}f(3^nz), \quad (19)$$

for all $z \in X$.

## III. Hyers-Ulam Stability of (1) in Modular Space

In this section, we take $V$ to be a real vector space and mapping $W\rho$ to be complete modular space, we present Hyers-Ulam Stability of the functional equation (1). Let denote mapping

$\phi : V \to W\rho$:

$$Df(x, y, z) = f(x + y - 2z) + f(x - 2y + z) - f(2y - 2z) - f(x - z) - f(x - y) \quad (20)$$

for all $x, y, z \in V$.

**Theorem 3.1.** Let $\psi: V^n \to [0, \infty)$ be a function such that

$$\lim_{m\to\infty} \frac{1}{3^{2m}}\psi(3^mx, 3^my, 3^nz) = 0 \quad (21)$$

and

$$\sum_{i=0}^{\infty} \frac{1}{3^{2i}}\psi(3^iz, 3^i2z, 3^i3z) < \infty \quad (22)$$

for all $x, y, z \in V$. If a mapping $\phi: V \to W\rho$ with $\phi(0) = 0$ and such that

$$\rho(D\phi(x, y, z) \leq \psi(x, y, z). \quad (23)$$

for all $x, y, z \in V$, then, there exists a unique quadratic mapping $Q_2: V \to W\rho$ satisfying

$$\rho(f(z) - Q_2(z)) \leq \frac{1}{3^2} \sum_{i=0}^{\infty} \frac{1}{3^{2i}}\psi(3^iz, 3^i2z, 3^i3z) \quad (24)$$

for all $z \in V$.

**Proof:** Replace $(x,y,z)$ by $(z,2z,3z)$ in (23), we get

$$\rho(f(3z)-3^2\phi(z))\leq\psi(z,2z,3z)$$
$$\rho\left(f(z)-\frac{\phi(3z)}{3^2}\right)\leq\frac{1}{3^2}\psi(z,2z,3z)$$

and then semi-convexity of $\rho$ and $\sum_{i=0}^{m-1}\frac{1}{3^{2(i+1)}}\leq 1$, we have

$$\rho\left(f(z)-\frac{\phi(3^m z)}{3^{2m}}\right)\leq\rho\left(\sum_{i=0}^{m-1}\left(\frac{f(3^i z)}{3^{2i}}-\frac{f(3^{i+1}z)}{3^{2(i+1)}}\right)\right)$$
$$\leq\sum_{i=0}^{m-1}\rho\left(\frac{f(3^i z)}{3^{2i}}-\frac{f(3^{i+1}z)}{3^{2(i+1)}}\right)$$
$$\leq\frac{1}{3^2}\sum_{i=0}^{m-1}\frac{1}{3^{2i}}\psi(z,2z,3z) \tag{25}$$

for all $z$ in V. So,

$$\rho\left(\frac{\phi(3^m z)}{3^{2m}}-\frac{\phi(3^t z)}{3^{2t}}\right)=\frac{1}{3^{2t}}\rho\left(\frac{\phi(3^{m-t}3^t z)}{3^{2(m-t)}}-f(3^t z)\right)$$
$$\leq\frac{1}{3^2}\sum_{i=0}^{m-t-1}\frac{1}{3^{2i}}\psi(3^i3^t z,3^i3^t 2z,3^i3^t 3z)$$
$$\leq\frac{1}{3^2}\sum_{i=t}^{m-1}\frac{1}{3^{2i}}\psi(3^i z,3^i 2z,3^i 3z) \tag{26}$$

for all $z$ in $V$ and all non negative integers $m,t$ with $m>t$. Thus $\left\{\frac{\phi(3^m z)}{3^{2m}}\right\}$ is a Cauchy sequence in complete modular space $W\rho$, so there exists a $Q_2: V\to W\rho$ as

$$\lim_{m\to\infty}\frac{\phi(3^m z)}{3^m}=Q_2(z) \tag{27}$$

for all $z$ in $V$. Now

$$\rho\left(\frac{3^2Q_2(z)-Q_2(3z)}{9^2}\right)=\rho\left(\frac{1}{9^2}\left(\frac{f(3^{m+1}z)}{3^{2m}}-Q_2(3z)\right)+\frac{1}{3^2}\left(Q_2(z)-\frac{f(3^{m+1}z)}{3^{2(m+1)}}\right)\right)$$
$$\leq\frac{1}{9^2}\rho\left(\frac{f(3^{m+1}z)}{3^{2m}}-Q_2(3z)\right)+\frac{1}{3^2}\rho\left(Q_2(z)-\frac{f(2^{m+1}z)}{3^{2(m+1)}}\right) \tag{28}$$

for all $z$ in $V$, then by (27), right hand side of (28) tends to 0 as $m\to\infty$. Thus

$$Q_2(3z)=3^2Q_2(z) \tag{29}$$

Now,

$$\rho(f(z)-Q_2(z))\leq\rho\left(\sum_{i=0}^{m-1}\left(\frac{f(3^i z)}{3^{2i}}-\frac{f(3^{i+1}z)}{3^{2(i+1)}}+\frac{f(3^m z)}{3^{2m}}-\frac{Q_2(3z)}{3^2}\right)\right.$$
$$\leq\frac{1}{3^2}\sum_{i=0}^{m-1}\frac{1}{3^{2i}}\psi(3^i z,3^i 2z,3^i 3z)+\frac{1}{3^2}\rho\left(\frac{f(3^{m-1}3z)}{3^{2(m-1)}}-Q_2(3z)\right)$$
$$\leq\frac{1}{3^2}\sum_{i=0}^{\infty}\frac{1}{3^{2i}}\psi(3^i z,3^i 2z,3^i z)+\frac{1}{3^2}\rho\left(\frac{f(3^{m-1}3v)}{3^{2(m-1)}}-Q_2(3z)\right) \tag{30}$$

for all integer $m>1$ and for all $z$ in $V$. Applying $m\to\infty$, we get the required result.
Replacing $(x,y,z)$ by $(3^m z,3^m y,3^m z)$ in (23), we get

$$\rho(D\phi(3^m x,3^m y,3^m z))\leq\psi(2^m x,2^m y,3^m z).$$

for all $x,y,z\in V$.
Therefore

$$\rho\left(\frac{1}{3^{2m}}D\phi(3^m x,3^m y,3^m z)\right)\leq\frac{1}{3^{2m}}\psi(3^m x,3^m y,3^m z). \tag{31}$$

Taking the limit $m\to\infty$, we get

$$DQ_2(x,y,z)=0 \tag{32}$$

for all $x,y,z\in V$.
To prove the uniqueness of $Q_2$, let $T_2: V\to W\rho$ be another quadartic mapping satisfying (24).

$$\rho\left(\frac{1}{3}Q_2(z)-\frac{1}{3}T_2(z)\right)\leq\frac{1}{3}\rho\left(\frac{Q_2(3^m z)}{3^{2m}}-\frac{f(3^m z)}{3^{2m}}\right)+\frac{1}{3}\rho\left(\frac{f(3^m z)}{3^{2m}}-\frac{T_2(3^m z)}{3^{2m}}\right)$$
$$\leq\frac{1}{3^{2m}}\sum_{i=0}^{\infty}\frac{1}{3^{2i}}\psi(3^i3^m z,3^i3^m 2z,3^i3^m 3z)$$
$$=\sum_{i=m}^{\infty}\frac{1}{3^{2i}}\psi(3^i z,3^i 2z,3^i 3z)$$

By taking $m\to\infty$, we have $Q_2=T_2$
Therefore, the function $Q_2$ is unique. This completes the proof.

**Corollary 3.2.** If a mapping $\phi: V\to W\rho$ with $\phi(0)=0$ and such that

$$\rho(D\phi(x,y,z)\leq(||x||^p+||y||^p+||z||^p). \tag{33}$$

for all $x,y,z\in V$, then, there exists a unique quadratic mapping $Q_2: V\to W\rho$ satisfying

$$\rho(f(z) - Q_2(z)) \le \frac{\|z\|^p}{3^2 - 3^p} \tag{34}$$

for all $z \in V$.

**Corollary 3.3.** If a mapping $\phi: V \to W\rho$ with $\phi(0) = 0$ and such that

$$||D\phi(x, y, z)|| \le (||x||^p + ||y||^p + ||z||^p) \tag{35}$$

for all $x, y, z \in V$, then there exists a unique quadratic mapping $Q_2: V \to W\rho$ satisfying

$$||Q_2(z) - \phi(z)|| \le \frac{||z||^p}{3^2 - 2^p} \tag{36}$$

for all $Z \in V$.

## IV. Conclusion

In this work we introduced a new quadratic functional equation of three variables and discussed about its Hyers-Ulam stability in two different spaces,F-space and Modular space respectively. In future reseachers can discuss about the stability of this functional equation in other spaces.

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