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Research Paper

A Study on Neutrosophic Hertzian wave Labeling Graphs and its Application

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Abstract
In this paper, we have devised a new idea of Neutrosophic Hertzian wave Labeling in graphs by combining the concept of Hertzian wave labelling and Neutrosophic labeling. We have defined simple, finite, undirected, connected Neutrosophic Hertzian wave Labeling graphs, where the membership functions are independent and this paper to perform the membership of truth(appearance) which means we are getting higher frequencies, the membership of false (non-appearance) getting lower frequencies and the membership of indeterminate(indeterminacy) getting medium or oscilatting frequencies. In additionally we discussion for Pyramid graph, Spin-top graph and Complete graphs and the concepts of Neutrosophic Hertzian wave labeling in subgraphs have also been investigated and verified in this study.
Mathematics Subject Classification: 03E72,05C72,05C78.

Keywords:
Intuitionistic Hertzianwave Labeling graphs [IHLG],
Intuitionistic Hertzianwave Labeling subgraphs,
Neutrosophic Hertzianwave Labeling graphs[NHLG],
Neutrosophic Hertzianwave Labeling subgraphs.

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I. Introduction

Graph theory[15] provides methods for solving real life problems, originating from Euler's result to the konigsberg bridge problem. Using the graphical structure represent the elements such as vertices and edges. But it gives sometimes uncertainty, vagueness and indeterminate detailed in the final outcome. For the reason that, the fuzzy logic method employed. Then the concept of fuzzy set was initiated by Zadeh [1965][40]. The fuzzy set have varying membership degrees between zero and one. It depicts the membership is vaguely defined set. Thereafter, Kaufmann[24, 23] unveiled the concept of fuzzy in graph theory and illusrated the fuzzy relation. In this idea on fuzzy graphical system gives high accuracy outcome. Rosenfeld[30] developed some concept of fuzzy graph models and described their properties. Yah and bang[38], Bhattacharya[8], Bhuttani[25, 9, 16] has established some connectivity concepts. Rosa[29, 1] was introduced graph labeling, Gani[26, 19, 32, 17, 12] and Shubhashini initiated the fuzzy graph labeling[27]. Hale[21] pushed for the introduction of the radio labelling concept in 1980. The channel assignment of FM radio stations inspired Chartrand, Erwin, Zhang, and Harary to create radio labelling of graph in 2001[13, 7]. Radio labeling[12, 18, 37] of connected graph G in an injection f from the vertices of G to the natural number such that $d(n,m) + |f(n) - f(m)| \geq 1 +$ diam(G) for every two distinct vertices n and m of G. Distance between any two vertices in the graph of G, denoted by $d(n, m)$ is the length of a shortest path between n and m. The eccentricity of vertex $e(n)$ is defined as $e(n) = \max\{d(n,m):n,m \in N\}$ and diam(G) = $\max\{e(n):n \in N\}$. T.Bharathi and Antony vinoth introduced the radio reciprocal membershiph function on cycle related graphs.

In 1983 Krassimir Atannosov's[4, 6, 39] presented the idea of intuitionistic fuzzy set[5] due to fact that the inaccuracy occurs in fuzzy theory, Then Atannaosov[6, 14, 36, 5] modified the version of fuzzy set. It uses the two independent appearance and non-appearance function which sum of these values being less than or equal to one. Shannon and Atonnosov excavated idea of the intuitionistic fuzzy relation and intuitionistic fuzzy graph. Parvathi[28] disscused the different opertions on intuitionistic fuzzy graph and Akram[2], Nagoorgani described the various properties of intuitionistic fuzzy graph[19]. It helps to compare the real world problem with intuitionistic fuzzy structure[3, 31]. A part from this, a new concept is need to deal with the situation due to indeterminate information pervailing the fuzzy or intuitionistic fuzzy graph.
F.Smarandache established the notion of Neutrosophic set[33, 10, 11], which is modifying the concept of the fuzzy and intuitionistic fuzzy set. Neutrosophic set[35, 34] to reduce the uncertainity outcome of intuitionistic fuzzy set. Neutrosophic has a independent components are appearance, non-appearance and indeterminacy always lies between zero and three. Kandasamy[22],[20] provided the concept of neutrosophic graphs, it reveal the neutrosophic graphs in graph theory. In this paper, owing to the appeal of these led to emergence of a new dimension in the neutrosophic hertizan wave labeling graphs. The idea described in the preceding sections was formulated as Neutrosophic hertizan wave labeling graphs notion. The objective is to allocate channels of frequencies in order to avoid interfering between transmitters. Finally, this concept has been related to X-ray Crystallography system analysis the particle and its functions have been found. This paper described the sections: Section 2 demonstrate the fundamental terminology of the neutrosophic hertizan wave labeling graph[NHLG]. Section 3, includes intuitionistic fuzzy hertizan labeling graphs definition, examples and theorem. Section 4, includes Neutrosophic fuzzy hertizan labeling graphs definition, examples and theorem. Section 5, includes algorithm of neutrosophic fuzzy hertizan labeling graphs. Section 5, includes application of neutrosophic hertizan labeling graphs, by analysing the X-ray Crystallography functions. Section 6, conclusion and obtained the results on the labeling work.

2 Preliminaries

Some terminologies and definitions related to fuzzy graphs are listed below, serving as the foundation for our proposed labeling technique and the results obtained. The universal set should be R. A fuzzy set in R is represented by S $=\{ (t, \mu_S(t)|\mu_S(t)>0, t\in R\}$ where the function μ_S: R$\rightarrow$[0,1] is the appearance of element t in the fuzzy set S. A fuzzy graph G $= (\mu, \gamma)$ is a pair of function, where μ is a fuzzy subset of vertex set N and γ is a symmetric fuzzy relation on μ.A graph G:(μ, γ) is said to be fuzzy labeling graph, if $\mu : N \rightarrow [0, 1]$ and $\gamma : N \times N \rightarrow [0, 1]$are injective such that the appearance value of nodes and edges are distinct and $\gamma_A(mn) < min(\mu_A(m), \mu_A(n))$ for all $m, n \in N$.

3 Intuitionistic Hertzian wave Labeling Graph

Definition 1 *An intuitionistic fuzzy graph* G$=(N,L)$ *is said to be an intuitionistic fuzzy labeling (IFL). if* μ_A:$N \rightarrow$ *[0,1],*μ_{NA}*:* $N \rightarrow$*[0,1] and* γ_A*:* $N\times N\rightarrow$ *[0,1],* γ_{NA}*:* $N\times N\rightarrow$ *[0,1] are 1-1 function, where* N *denotes vertex set and* L *denotes edge set.*
$\gamma_A(m,n) < min\ \{\ \mu_A(m),\mu_A(n)\}$
$\gamma_{NA}(m,n) < max\ \{\ \mu_{NA}(m),\mu_{NA}(n)\}$
$0\leq \gamma_A(m,n) + \gamma_{NA}(m,n)\leq 1$ *for every edges* $(m,n)\in L$.

Definition 2 *An intuitionistic fuzzy labeling graph* G$=(N,\mu,\gamma)$ *is said to be Intuitionistic hertzian wave Labeling (IHLG) graph. if* μ_A*:* $N \rightarrow$ *[0,1],*μ_{NA}*:* $N\rightarrow$*[0,1] and* γ_A*:* $N\times N\rightarrow$ *[0,1],* γ_{NA}*:* $N\times N\rightarrow$ *[0,1] such that the satisfies following conditions,*

(i) $\delta_A(n_i,n_j) + \frac{1}{|\mu_A(n_i)-\mu_A(n_j)|} > diam_A(G)$

(ii) $\delta_{NA}(n_i,n_j) + \frac{1}{|\mu_{NA}(n_i)-\mu_{NA}(n_j)|} > diam_{NA}(G)$

where $\mu = (\mu_A,\mu_{NA})$, $\gamma = (\gamma_A,\gamma_{NA})$ *are distinct nodes and edges.* δ_A, δ_{NA} *are* γ*-distance of intuitionistic labeling graphs(i,j = 1,2,3...n).*

Note: We denoted Appearance, Indeterminacy and Non-Appearance as A, I, NA respectively.

Theorem 1 *For any complete intuitionistic fuzzy graph n≥3 vertices,* K_m *is satisfying an IHLG.*

Let G = K_m be a graph. Define an intuitionistic fuzzy complete graph G = (N,μ,γ) where $N = (n_1,n_2,n_3,...n_n)$ such that μ_A: $N \to [0,1]$, μ_{NA}:$N \to [0,1]$denote the degree of appearance and non-appearance of the vertices $n_i \in N$ repectively,
$0 \leq \mu_A(n_i) + \mu_{NA}(n_i) \leq 1$ for every $n_i \in n(i = 1,2,3,....,n)$. $L \subseteq N \times N$ where, $\gamma_A : N \times N \to [0,1]$ and $\gamma_{NA}: N \times N \to [0,1]$ are such that
$\gamma_A((n_i),(n_j)) \leq \min[\mu_A(n_i)),\mu_A(n_j)]$
$\gamma_{NA}((n_i),(n_j)) \leq \max[\mu_{NA}(n_i),\mu_{NA}(n_j)] \longrightarrow (1)$
$0 \leq \gamma_A((n_i),(n_j)) + \gamma_{NA}((n_i),(n_j)) \leq 1$ for every $((n_i),(n_j)) \in N \times N$.
Fix n_i, where i varies from 1 to n.The vertex of a graph G that is adjacent to every vertex and has the appearance function, indeterminacy, and non-appearance function to interval [0,1].

Let P be the path of the graph G. consider $K(p) = \sum \frac{1}{\gamma(n_{i-1},n_i)}$, where K(p)is γ - length of G.Which must satisfy the following conditions:
An intuitionistic fuzzy labeling complete graph G = (N,γ,μ) if $\mu_A : N \to [0,1]$,$\mu_{NA} : N \to [0,1]$ and $\gamma_A : N \times N \to [0,1]$, $\gamma_{NA}: N \times N \to [0,1]$ are 1-1 function where $\mu = (\mu_A,\mu_{NA})$, $\gamma = (\gamma_A,\gamma_{NA})$ edges and vertices are distinct such that,
$\gamma_A((n_i),(n_j)) < \min[\mu_A(n_i)),\mu_A(n_j)]$
$\gamma_{NA}((n_i),(n_j)) < \max[\mu_{NA}(n_i),\mu_{NA}(n_j)] \longrightarrow (2)$
$0 \leq \gamma_A((n_i),(n_j)) + \gamma_{NA}((n_i),(n_j)) \leq 1$
where i,j varies 1 to n, which also and intuitionistic satisfies the following conditions:

(i) $\delta_A(n_i,n_j) + \frac{1}{|\mu_A(n_i)-\mu_A(n_j)|} > \text{diam}_A(G)$

(ii) $\delta_{NA}(n_i,n_j) + \frac{1}{|\mu_{NA}(n_i)-\mu_A(n_j)|} > \text{diam}_{NA}(G) \longrightarrow (3)$

Therefore, satisfies all the above equation (1),(2),(3). Hence any complete intuitionistic fuzzy graph n ≥ 3 vertices that satisfies an Intuitionistic Hertzian Labeling Graph.

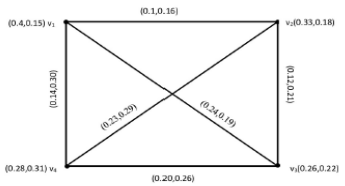


Figure 1: K_m(Complete Intuitionistic Hertzian wave Labeling Graph)

Example:1

$\delta_A(v_1,v_2) = 10;\delta_A(v_1,v_3) = 4.16;\delta_A(v_1,v_4) = 7.14;\delta_A(v_2,v_1) = 10;$
$\delta_A(v_2,v_3) = 8.3;\delta_A(v_2,v_4) = 4.3;\delta_A(v_3,v_1) = 4,16;\delta_A(v_3,v_2) = 8.3;$
$\delta_A(v_3,v_4) = 5$
$e(v_1) = 10;e(v_2) = 10;e(v_3) = 8.3;\ e(v_4) = 7.14$
$\text{diam}_A(G) = 10$
$10 + 14.28 > 10$; when i=1, j=2
$\delta_{NA}(v_1,v_2) = 6.25;\delta_{NA}(v_1,v_3) = 5.26;\delta_{NA}(v_1,v_4) = 9.1;\delta_{NA}(v_2,v_1) = 6.25;$
$\delta_{NA}(v_2,v_3) = 4.76;\delta_{NA}(v_2,v_4) = 3.44;\delta_{NA}(v_3,v_1) = 5.26;\delta_{NA}(v_3,v_2) = 4.76;$
$\delta_{NA}(v_3,v_4) = 3.84$
$e(v_1) = 9.1;e(v_2) = 6.25;e(v_3) = 5.26;\ e(v_4) = 9.1$
$\text{diam}_{NA}(G) = 9.1$
$6.25 + 33.33 > 9.1$; when i=1, j=2.

Definition 3 *An IHL graph* H = (*N*, ψ,τ) *is called an IHL subgraph of* G = (*N*,μ,γ). *if* $\psi_A(n_i) \leq \mu_A(n_i)$, $\psi_{NA}(n_i) \geq \mu_{NA}(n_i)$ *for all* $n_i \in N$ *and* $\tau_A(n_i,n_j) \leq \gamma_A(n_i,n_j)$, $\tau_{NA}(n_i,n_j) \geq \gamma_{NA}(n_i,n_j)$ *for all edges* (n_i,n_j) *then the satisfies the following conditions,*

(i) $\delta_A(n_i,n_j) + \frac{1}{|\psi_A(n_i)-\psi_A(n_j)|} > diam_A(H)$

(ii) $\delta_{NA}(n_i,n_j) + \frac{1}{|\psi_{NA}(n_i)-\psi_{NA}(n_j)|} > diam_{NA}(H)$, *where (i,j=1,2,3...n).*

Theorem 2 *If* H = (*N*, ψ,τ) *is an IHL subgraph of* G = (*N*,μ,γ) *then* $\tau^{\infty}{}_A(n_i,n_j) \leq \gamma^{\infty}{}_A(n_i,n_j)$, $\tau^{\infty}{}_{NA}(n_i,n_j) \geq \gamma^{\infty}{}_{NA}(n_i,n_j)$ *for all edges* (n_i,n_j).

Let G =(*N*,μ,γ) be any IHL graph and H = (*N*, ψ,τ)subgraph. Let (n_i,n_j) be any path in G. then strength $(\tau^{\infty}{}_A(n_i,n_j),\tau^{\infty}{}_{NA}(n_i,n_j))$. since, H is an IHL subgraph of G, then $\psi_A(n_i) \leq \mu_A(n_i)$,$\psi_{NA}(n_i) \geq \mu_{NA}(n_i)$ for all $n_i \in N$ and $\tau_A(n_i,n_j) \leq \gamma_A(n_i,n_j)$, $\tau_{NA}(n_i,n_j) \geq \gamma_{NA}(n_i,n_j)$ for all edges (n_i,n_j) where i,j varies from 1 to n which implies that $\tau^{\infty}{}_A(n_i,n_j) \leq \gamma^{\infty}{}_A(n_i,n_j)$, $\tau^{\infty}{}_{NA}(n_i,n_j) \geq \gamma^{\infty}{}_{NA}(n_i,n_j)$.

Example:2

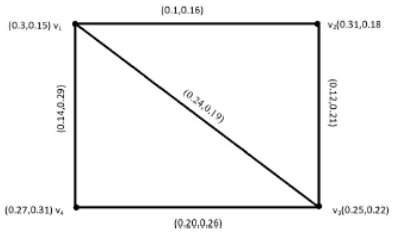


Figure 2: subgraph K_m

$\delta_A(v_1,v_2)$= 12.46;$\delta_A(v_1,v_3)$= 12.14;$\delta_A(v_1,v_4)$= 9.16;$\delta_A(v_2,v_1)$= 12.46;
$\delta_A(v_2,v_3)$= 14.16;$\delta_A(v_2,v_4)$= 13.3;$\delta_A(v_3,v_1)$= 12.14;$\delta_A(v_3,v_2)$= 14.16;
$\delta_A(v_4,v_1)$= 9.16;$\delta_A(v_4,v_2)$= 13.3;$\delta_A(v_3,v_4)$= 16.30
$e(v_1)$= 12.46;$e(v_2)$= 14.16;$e(v_3)$= 16.30; $e(v_4)$= 16.30
$diam_A(H)$= 16.30
12.46 + 100 > 16.30; when i=1, j=2
$\delta_{NA}(v_1,v_2)$= 10.02;$\delta_{NA}(v_1,v_3)$= 7.28;$\delta_{NA}(v_1,v_4)$= 8.70;$\delta_{NA}(v_2,v_1)$= 10.02;
$\delta_{NA}(v_2,v_3)$= 11.51;$\delta_{NA}(v_2,v_4)$= 8.6;$\delta_{NA}(v_3,v_1)$= 7.28;$\delta_{NA}(v_3,v_2)$= 11.51;
$\delta_{NA}(v_4,v_1)$= 8.70;$\delta_{NA}(v_4,v_2)$= 8.6;$\delta_{NA}(v_3,v_4)$= 8.70
$e(v_1)$= 10.2;$e(v_2)$= 11.51;$e(v_3)$= 11.51; $e(v_4)$= 8.70
$diam_{NA}(H)$= 11.51
10.02 + 33.33 > 11.51; when i=1, j=2.

4 Neutrosophic Hertzian wave Labeling Graphs

Definition 4 *A neutrosophic graph is the form* G = (*N*,*L*) *where* $N = (n_1,n_2,n_3,...n_n)$ *such that* μ_A:$N \to [0,1]$,μ_I:$N \to [0,1]$, μ_{NA}:$N \to [0,1]$ *denote appearance, Indeterminacy and non-appearance of the vertices* $n_i \in N$ *repectively,* $0 \leq \mu_A(n_i) + \mu_I(n_i) + \mu_{NA}(n_i) \leq 3$ *for every* $n_i \in N (i = 1,2,3,....,n)$. $L \subseteq N \times N$ *where,* $\gamma_A : N \times N \to [0,1]$, $\gamma_I : N \times N \to [0,1]$*and* γ_{NA}: $N \times N \to [0,1]$ *are such that,*
$\gamma_A((n_i),(n_j)) \leq min[\mu_A(n_i)),\mu_A(n_j)]$
$\gamma_I((n_i),(n_j)) \leq min [\mu_I(n_i),\mu_I(n_j)]$
$\gamma_{NA}((n_i),(n_j)) \leq max [\mu_{NA}(n_i),\mu_{NA}(n_j)]$

$0 \le \gamma_A((n_i),(n_j)) + \gamma_I((n_i),(n_j)) + \gamma_{NA}((n_i),(n_j)) \le 3$ *for every edges in* L.

Definition 5 *A neutrosophic graph* $G = (N,L)$ *is said to be a neutrosophic labeling graph. If* $\mu_A : N \to [0,1]$, $\mu_I : N \to [0,1]$, $\mu_{NA} : N \to [0,1]$ *and* $\gamma_A : N \times N \to [0,1]$, $\gamma_I : N \times N \to [0,1]$, $\gamma_{NA}: N \times N \to [0,1]$ *are 1-1 function where,* $\mu = (\mu_A, \mu_I, \mu_{NA})$, $\gamma = (\gamma_A, \gamma_I, \gamma_{NA})$ *edges and vertices are distinct such that,*
$\gamma_A((n_i),(n_j)) < min[\mu_A(n_i)), \mu_A(n_j)]$
$\gamma_I((n_i),(n_j)) < min[\mu_A(n_i)), \mu_I(n_j)]$
$\gamma_{NA}((n_i),(n_j)) < max\ [\mu_{NA}(n_i), \mu_{NA}(n_j)]$
$0 \le \gamma_A((n_i),(n_j)) + \gamma_I((n_i),(n_j)) + \gamma_{NA}((n_i),(n_j)) \le 3$ *for every edges* $(n_i,n_j) \in L$ $(i,j = 1,2,3,\ldots,n)$.

Example:3



Figure 3: Neutrosophic Fuzzy Graph

Definition 6 *A neutrosophic labeling graph is said to be a Netrosophic Hertzian Labeling (NHL) graph* $G = (N,\mu,\gamma)$. *if* $\mu_A : N \to [0,1]$, $\mu_I : N \to [0,1]$, $\mu_{NA} : N \to [0,1]$ *and* $\gamma_A : N \times N \to [0,1]$, $\gamma_I : N \times N \to [0,1]$, $\gamma_{NA}: N \times N \to [0,1]$ *are 1-1 such that the following conditions are satisfied,*

(i) $\delta_A(n_i,n_j) + \frac{1}{|\mu_A(n_i) - \mu_A(n_j)|} > diam_A(G)$

(ii) $\delta_I(n_i,n_j) + \frac{1}{|\mu_I(n_i) - \mu_I(n_j)|} > diam_I(G)$

(iii) $\delta_{NA}(n_i,n_j) + \frac{1}{|\mu_{NA}(n_i) - \mu_{NA}(n_j)|} > diam_{NA}(G)$, *where i,j varies from 1 to n.*

Definition 7 *A Pyramid* Py_n *is a graph has* $\frac{n(n+1)}{2}$ *vertices and* $\frac{3n(n-1)}{2}$ *edges where* $n \ge 3$ *obtained by descending order on the bottom of n copies.*

Theorem 3 *For any Pyramid neutrosophic fuzzy graph* $n \ge 3$ *vertices,* Py_n *is satisfying NHLG.*

Let $G = Py_n$ be a Pyramid graph. Define a neutrosophic fuzzy graph $G = (N,\mu,\gamma)$, where $N = (n_1,n_2,n_3,\ldots n_n)$ such that $\mu_A : N \to [0,1]$, $\mu_I : N \to [0,1]$, $\mu_{NA} : N \to [0,1]$ denote the degree of appearance, Indeterminacy and non-appearance of the vertices $n_i \in N$ repectively, $0 \le \mu_A(n_i) + \mu_I(n_i) + \mu_{NA}(n_i) \le 3$ for every $n_i \in N (i = 1,2,3,\ldots,n)$. $L \subseteq N \times N$ where $\gamma_A : N \times N \to [0,1]$, $\gamma_I : N \times N \to [0,1]$ and $\gamma_{NA}: N \times N \to [0,1]$ are such that
$\gamma_A((n_i),(n_j)) \le \min[\mu_A(n_i)), \mu_A(n_j)]$
$\gamma_I((n_i),(n_j)) \le \min[\mu_I(n_i)), \mu_I(n_j)]$
$\gamma_{NA}((n_i),(n_j)) \le \max\ [\mu_{NA}(n_i), \mu_{NA}(n_j)] \longrightarrow (1)$
$0 \le \gamma_A((n_i),(v_j)) + \gamma_I((n_i),(n_j)) + \gamma_{NA}((n_i),(n_j)) \le 3$ for every $((n_i),(n_j)) \in N \times N$.
fix n_i, where i various from 1 to n. The vertex of a graph PY_n is adjacent to every other vertex and has the appearance function, indeterminacy, and non-appearance function to interval [0,1].
Let P be the path of the graph G. consider $K(p) = \sum \frac{1}{\gamma(n_{i-1},n_i)}$, where K(p) is γ - length of G. Which must satisfy the following conditions:
A neutrosophic fuzzy labeling graph $Py_n = (N,\gamma,\mu)$ if $\mu_A : N \to [0,1]$, $\mu_I : N \to [0,1]$, $\mu_{NA} : N \to [0,1]$ and $\gamma_A : N \times N \to [0,1]$, $\gamma_I : N \times N \to [0,1]$, $\gamma_{NA}: N \times N \to [0,1]$ are 1-1 function, where $\mu = (\mu_A, \mu_I, \mu_{NA})$, $\gamma = (\gamma_A, \gamma_I, \gamma_{NA})$ edges and vertices are distinct such that,
$\gamma_A((n_i),(n_j)) < \min[\mu_A(n_i)), \mu_A(n_j)]$
$\gamma_I((n_i),(n_j)) < \min[\mu_I(n_i)), \mu_I(n_j)]$
$\gamma_{NA}((n_i),(n_j)) < \max\ [\mu_{NA}(n_i), \mu_{NA}(n_j)] \longrightarrow (2)$
$0 \le \gamma_A((n_i),(n_j)) + \gamma_I((n_i),(n_j)) + \gamma_{NA}((n_i),(n_j)) \le 3$
where i,j varies 1 to n and neutrosophic fuzzy labeling Pyramid graph admits NHL of graph, which also satisfies the following conditions:

(i) $\delta_A(n_i,n_j) + \frac{1}{|\mu_A(n_i)-\mu_A(n_j)|} > \text{diam}_A(G)$

(ii) $\delta_I(n_i,n_j) + \frac{1}{|\mu_I(n_i)-\mu_I(n_j)|} > \text{diam}_I(G)$

(iii) $\delta_{NA}(n_i,n_j + \frac{1}{|\mu_{NA}(n_i)-\mu_{NA}(n_j)|} > \text{diam}_{NA}(G) \longrightarrow (3)$

Therefore, satisfies all the above equation (1),(2),(3). Hence any Pyramid neutrosophic fuzzy graph n≥3 vertices that satisfies NHLG.

Example:4

$\delta_A(v_1,v_2)$= 9.09;$\delta_A(v_1,v_3)$= 4.54;$\delta_A(v_1,v_4)$= 11.61;$\delta_A(v_1,v_5)$= 10.79;
$\delta_A(v_1,v_6)$= 8.54;$\delta_A(v_2,v_1)$= 9.09;$\delta_A(v_2,v_3)$= 7.69;$\delta_A(v_2,v_4)$= 2.32;



Figure 4: Neutrosophic Hertzian wave labeling Pyramid Graph

$\delta_A(v_2,v_5)$= 10;$\delta_A(v_2,v_6)$= 11.69;$\delta_A(v_3,v_1)$= 4.54;$\delta_A(v_3,v_2)$= 7.69;
$\delta_A(v_3,v_4)$= 10.21;$\delta_A(v_3,v_5)$= 6.25;$\delta_A(v_3,v_6)$= 4;$\delta_A(v_4,v_1)$= 11.61;
$\delta_A(v_4,v_2)$= 2.32;$\delta_A(v_4,v_3)$= 10.21;$\delta_A(v_4,v_5)$= 12.52 $\delta_A(v_4,v_6)$= 11.64;$\delta_A(v_5,v_1)$= 10.79
$\delta_A(v_5,v_2)$= 10;$\delta_A(v_5,v_3)$= 6.25;$\delta_A(v_5,v_4)$= 12.52;$\delta_A(v_5,v_6)$= 3.57;$\delta_A(v_6,v_2)$= 11.69;
$\delta_A(v_6,v_3)$= 4;$\delta_A(v_6,v_4)$= 11.64;$\delta_A(v_6,v_5)$= 3.57
$e(v_1)$= 11.61;$e(v_2)$= 11.69;$e(v_3)$= 10.21;$e(v_4)$= 12.52;$e(v_5)$= 12.52;$e(v_6)$= 11.69
$\text{diam}_A(G)$= 12.52.
9.09 + 5.55 > 12.52; when i=1, j=2
$\delta_I(v_1,v_2)$= 3.03;$\delta_I(v_1,v_3)$= 3.33;$\delta_I(v_1,v_4)$= 5.32;$\delta_I(v_1,v_5)$= 7.49;
$\delta_I(v_1,v_6)$= 6.45;$\delta_I(v_2,v_1)$= 3.03;$\delta_I(v_2,v_3)$= 3.57;$\delta_I(v_2,v_4)$= 2.32;
$\delta_I(v_2,v_5)$= 4.54;$\delta_I(v_2,v_6)$= 6.69;$\delta_I(v_3,v_1)$= 3.33;$\delta_I(v_3,v_2)$= 3.57;
$\delta_I(v_3,v_4)$= 5.89;$\delta_I(v_3,v_5)$= 4.16;$\delta_I(v_3,v_6)$= 3.12;$\delta_I(v_4,v_1)$= 5.32;
$\delta_I(v_4,v_2)$= 2.32;$\delta_I(v_4,v_3)$= 5.89;$\delta_I(v_4,v_5)$= 1.81 $\delta_I(v_4,v_6)$= 5.81 ;
$\delta_I(v_5,v_1)$= 7.49;$\delta_I(v_5,v_2)$= 4.54;$\delta_I(v_5,v_3)$= 4.16;$\delta_I(v_5,v_4)$= 1.81;
$\delta_I(v_5,v_6)$= 4;$\delta_I(v_6,v_2)$= 6.69;
$\delta_I(v_6,v_3)$= 3.12;$\delta_I(v_6,v_4)$= 5.81;$\delta_I(v_6,v_5)$= 4
$e(v_1)$= 7.49;$e(v_2)$= 6.69;$e(v_3)$= 5.89 ;$e(v_4)$= 5.89;$e(v_5)$= 7.49;$e(v_6)$= 6.69
$\text{diam}_I(G)$= 7.49.
3.03 + 5.88 > 7.49; when i=1, j=2
$\delta_{NA}(v_1,v_2)$= 5.88;$\delta_{NA}(v_1,v_3)$= 3.84;$\delta_A(v_1,v_4)$= 6.34;$\delta_{NA}(v_1,v_5)$= 6.06;
$\delta_{NA}(v_1,v_6)$= 6.61;$\delta_{NA}(v_2,v_1)$= 5.88;$\delta_{NA}(v_2,v_3)$= 4;$\delta_{NA}(v_2,v_4)$= 5.57;
$\delta_{NA}(v_2,v_5)$= 2.63;$\delta_{NA}(v_2,v_6)$= 4.8;$\delta_{NA}(v_3,v_1)$= 3.84;$\delta_{NA}(v_3,v_2)$= 4;
$\delta_{NA}(v_3,v_4)$= 5.16;$\delta_{NA}(v_3,v_5)$= 2.85;$\delta_{NA}(v_3,v_6)$= 2.77;$\delta_{NA}(v_4,v_1)$= 6.34;
$\delta_{NA}(v_4,v_2)$= 5.57;$\delta_{NA}(v_4,v_3)$= 5.16;$\delta_{NA}(v_4,v_5)$= 1.75 $\delta_{NA}(v_4,v_6)$= 3.92;
$\delta_{NA}(v_5,v_1)$= 6.06;$\delta_{NA}(v_5,v_2)$= 2.63;$\delta_{NA}(v_5,v_3)$= 2.85;$\delta_{NA}(v_5,v_4)$= 1.75;
$\delta_{NA}(v_5,v_6)$= 2.17;$\delta_{NA}(v_6,v_2)$= 4.8;
$\delta_{NA}(v_6,v_3)$= 2.77;$\delta_{NA}(v_6,v_4)$= 3.92;$\delta_{NA}(v_6,v_5)$= 2.17
$e(v_1)$= 6.61;$e(v_2)$= 5.88;$e(v_3)$= 5.16;$e(v_4)$= 6.34;$e(v_5)$= 6.06;$e(v_6)$= 6.61
$\text{diam}_{NA}(G)$= 6.61.
5.88 + 9.09 > 16.30; when i=1, j=2.

Definition 8 *A NHL graph* H = *(V, ψ,τ̇) is called a NHL subgraph of* G= *(N,μ,γ). if* $\psi_A(n_i) \leq \mu_A(n_i)$, $\psi_I(n_i) \leq \mu_I(n_i)$, $\psi_{NA}(n_i) \geq \mu_{NA}(n_i)$ *for all* $n_i \in N$ *and* $\dot{\tau}_A(n_i,n_j) \leq \gamma_A(n_i,n_j)$, $\dot{\tau}_I(n_i,n_j) \leq \gamma_I(n_i,n_j)$, $\dot{\tau}_{NA}(n_i,n_j) \geq \gamma_{NA}(n_i,n_j)$ *for all edges* (n_i,n_j) *then satisfies the following conditions*

(i) $\delta_A(n_i,n_j) + \frac{1}{|\psi_A(n_i)-\psi_A n_j)|} > diam_A(\mathrm{H})$

(ii) $\delta_I(n_i,n_j) + \frac{1}{|\psi_I(n_i)-\psi_A n_j)|} > diam_I(\mathrm{H})$

(iii) $\delta_{NA}(n_i,n_j) + \frac{1}{|\psi_{NA}(n_i)-\psi_A n_j)|} > diam_{NA}(\mathrm{H})$ *where i,j varies from 1 to n.*

Theorem 4 *If* H = *(N, ψ,τ̇) is a Neutrosophic Hertzian wave labeling subgraph of* G = *(N,μ,γ) then* $\dot{\tau}^{\infty}{}_A(n_i,n_j) \leq \gamma^{\infty}{}_A(n_i,n_j)$, $\dot{\tau}^{\infty}{}_I(n_i,n_j) \leq \gamma^{\infty}{}_I(n_i,n_j)$, $\dot{\tau}^{\infty}{}_{NA}(n_i,n_j) \geq \gamma^{\infty}{}_{NA}(n_i,n_j)$ *for all edges* (n_i,n_j).

Let G = (N,μ,γ) be any NHL graph and H = $(N, \psi,\dot{\tau})$ subgraph. Let(n_i,n_j) be any path in G, then strength $(\dot{\tau}^{\infty}{}_A(n_i,n_j), \dot{\tau}^{\infty}{}_I(n_i,n_j), \dot{\tau}^{\infty}{}_{NA}(n_i,n_j))$. since H is a NHL subgraph of G, then$\psi_A(n_i) \leq \mu_A(n_i)$,$\psi_I(n_i) \leq \mu_I(n_i)$, $\psi_{NA}(n_i) \geq \mu_{NA}(n_i)$ for all $n_i \in N$ and $\dot{\tau}_A(n_i,n_j) \leq \gamma_A(n_i,n_j)$, $\dot{\tau}_I(n_i,n_j) \leq \gamma_I(n_i,n_j)$, $\dot{\tau}_{NA}(n_i,n_j) \geq \gamma_{NA}(n_i,n_j)$ for all edges (n_i,n_j), where i,j varies from 1 to n. which implies that $\dot{\tau}^{\infty}{}_A(n_i,n_j) \leq \gamma^{\infty}{}_A(n_i,n_j)$, $\dot{\tau}^{\infty}{}_I(n_i,n_j) \leq \gamma^{\infty}{}_I(n_i,n_j)$, $\dot{\tau}^{\infty}{}_{NA}(n_i,n_j) \geq \gamma^{\infty}{}_{NA}(n_i,n_j)$.

5 Algorithm of Neutrosophic Hertzian wave Labeling Graph

Step 1: To assign the appearance,indeterminacy, non-appearance values for the vertices from the functions $\mu_A(n_i)$,$\mu_I(n_i)$,$\mu_{NA}(n_i)$,where i = 1,2,3...n-1.
Step 2: To label the each edges of the graph from step 1.
$(\gamma_A((n_i),(n_j))< \min[\mu_A(n_i)),\mu_A(n_j)]$
$\gamma_I((n_i),(n_j))< \min[\mu_I(n_i))),\mu_I(n_j)]$
$\gamma_{NA}((n_i),(n_j))< \max[\mu_{NA}(n_i),\mu_{NA}(n_j)]$, where i,j = 1,2,3...n-1.)
Step 3: If $0\leq \gamma_A((n_i),(n_j)) + \gamma_I((n_i),(n_j))+ \gamma_{NA}((n_i),(n_j)) \leq 3$, then go to next step. otherwise go to step 1.
Step 4: Must satisfies the following conditions:

(i) $\delta_A(n_i,n_j) + \frac{1}{|\mu_A(n_i)-\mu_A(n_j)|} > \mathrm{diam}_A(\mathrm{G})$

(ii) $\delta_I(n_i,n_j) + \frac{1}{|\mu_I(n_i)-\mu_I(n_j)|} > \mathrm{diam}_I(\mathrm{G})$

(iii) $\delta_{NA}(n_i,n_j + \frac{1}{|\mu_{NA}(n_i)-\mu_{NA}(n_j)|} > \mathrm{diam}_{NA}(\mathrm{G})$

If not, go to step 1.

6 Application of Neutrosophic Hertzian Labeling Graph in X-ray Crystallography Analysis

A group of two or more atoms are combined the chemical bond is called molecule. These can be made of same atoms or different atoms. Molecules can be small or large like water,proteins,DNA. May be try to discover the new molecules easily finding molecule structure. since, graph theory is useful tool in chemistry, when it comes to analyze and representing molecules and their structure. Here we define a neutrosophic hertzian wave labeling graphical to deal with X-ray Crystallography analysis, owing to the function of particular position of atoms structure can be consider the three membership functions and their labeling.
Consider the X-ray Crystallography G = (N,μ,γ) where the vertex set denotes objects of X-ray Crystallography such as X-ray beam(form of radiation with a very short wave lenth),crystal, boundary of scattered rays and the detector,computing image respectively. Edge set denotes connectivity between the vertices. Since, we dealing with NHLG in X-ray Crystallography. Each vertex and edge has membership function separately the appearance,indeterminacy,non-appearance of vertices denotes the objects getting high,medium,low frequency respectively.



Figure 5: Spin top

The working step of X-ray Crystallography, which to obtain the crystal and placed in the path of a beam. The crystal allow the rays to interact with atom and passing through the crystal. Some rays penetrate and deflect, by analysing the diffraction patterns the emerge when the rays collide with atoms. We can de-



Figure 6: NHL - Spin top

termine the which types of molecules are located by using neutrosophic hertzian wave labeling graph.

7 Conclusion

This study highlights the introduction of an intuitionistic Hertzian wave labeling and neutrosophic Hertzian wave labeling graphs. It has been rapidly progressing and gaining wide spread applications in several sectors. Additionaly, we will carry out investigation on neutrosophic labeling graphs, for application in the fields of mathematics, science and technology.

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