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Review Paper

A GARCH-EVT Framework for Premium Calculation under Heavy-Tailed Risks

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ABSTRACT

Insurance pricing under heavy-tailed risk models presents challenges due to extreme losses and market volatility. Traditional premium principles often fail to capture tail risks accurately, while Extreme Value Theory (EVT)-based approaches may lead to excessive pricing. This study proposes a hybrid premium calculation strategy integrating actuarial methods with EVT, GARCH models, Bayesian inference, and machine learning to improve pricing accuracy and risk responsiveness. The study models claim severity using heavy-tailed distributions. Additionally, a regime-switching GARCH-EVT model dynamically adjusts premiums based on market volatility. Our findings suggest that integrating financial risk management techniques with data-driven actuarial modeling enhances premium accuracy, particularly for catastrophe insurance and reinsurance pricing.

Key words: *Heavy tailed risks, GARCH, Machine learning, Premium calculation and MCMC.*

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I. INTRODUCTION

Insurance markets are increasingly exposed to risks that exhibit heavy-tailed behaviours, such as natural catastrophes, pandemics, and cyber incidents. These events are characterized by low frequency but extremely high severity, posing significant challenges for premium calculation and solvency management. The pricing of insurance products in the presence of heavy-tailed risks has attracted significant research interest, especially following the increasing occurrence of catastrophic events. Traditional models, while useful under moderate risk settings, often fail to account for extreme losses and the changing dynamics of financial and insurance markets.

Conventional methods for calculating premiums such as the expected value, variance loading, or exponential principles often prove inadequate in scenarios where extreme or catastrophic losses are prevalent. They typically assume light-tailed or moderately skewed risk profiles, failing to capture the magnitude and dynamics of heavy-tailed claims. Heavy-tailed distributions, such as the Pareto, Fréchet, and Generalized Pareto Distribution (GPD), are widely recognized for their ability to model extreme insurance losses [4]. These models capture the phenomena where rare events, though infrequent, can cause disproportionately large losses. However, while these distributions effectively model the tail behaviour, they are often static and lack the ability to adapt dynamically to changing market conditions. While Extreme Value Theory (EVT) has been widely applied to address these issues, providing asymptotic models for the tail of the loss distribution [9], EVT by itself does not account for the time dynamics of risk exposure. For instance, periods of heightened volatility following a catastrophe are not captured purely through static tail models. Consequently, premiums based solely on EVT may become overly conservative or insensitive to market regime changes. EVT provides a robust framework for tail risk modeling. The Peaks-over-Threshold (POT) approach applies the Generalized Pareto Distribution (GPD) to characterize losses exceeding a defined high threshold. The use of Extreme Value Theory (EVT) in actuarial science has significantly improved insights into risk metrics like Value-at-Risk (VaR) and Conditional Tail Expectation (CTE). Recent work has also highlighted the importance of optimal tail threshold selection in improving EVT-based VaR estimation accuracy [3]**.** Nonetheless, EVT assumes that data points are independent and identically distributed (*i.i.d.*), which may not hold in time-dependent insurance loss processes, particularly after large catastrophic events.

Volatility clustering and regime shifts are critical features of financial and insurance loss data. The GARCH models have been instrumental in modeling conditional heteroskedasticity, recognizing that periods of high losses tend to cluster together. These features are essential for catastrophe and reinsurance pricing, where market behaviour can change dramatically post-event. To address the time-dependent nature of volatility, the GARCH family of models [1] provides a flexible way to model conditional heteroskedasticity. Moreover, incorporating Markov-switching frameworks [6] allows for explicit modeling of regime changes, such as shifts from low-risk to high-risk environments. Extensions of this framework, such as regime-switching GARCH model [5] allow the model parameters to shift based on latent states, such as "calm" versus "turbulent" periods, better capturing real-world market dynamics. In insurance contexts, GARCH models have been used to model claims volatility, especially in dynamic reinsurance pricing.

Beyond capturing volatility, uncertainty in model parameters remains a major concern. Traditional frequentist methods provide point estimates without quantifying the uncertainty. Bayesian methods, particularly Markov Chain Monte Carlo (MCMC) techniques, offer a systematic approach to account for parameter uncertainty, leading to more robust premium estimates under model risk and they have been increasingly applied to derive posterior predictive distributions [10]. Bayesian approaches enable a full probabilistic treatment of the model, accounting for estimation errors and making the premium calculations more robust under uncertainty. Recent studies show that Bayesian EVT models outperform classical methods in predicting tail risks.

Moreover, incorporating machine learning methods introduces fresh insights and innovative approaches. Machine learning models can assist in predicting volatility states, optimizing regime classification, or even improving tail estimation when classical assumptions break down [12]. Combining machine learning with statistical models ensures that the framework is flexible, data-driven, and capable of adapting to changing insurance landscapes. Machine learning techniques have been introduced to insurance modeling for tasks such as risk classification and claims prediction. In the context of premium calculation, machine learning models can improve volatility state prediction, regime identification, and threshold selection for EVT, making the premium estimation more accurate and adaptive to new patterns in the data.

In present paper, we propose a regime-switching GARCH-EVT premium calculation framework augmented with Bayesian inference and machine learning techniques. By unifying these approaches, the model dynamically adjusts to both heavy-tailed claim severities and evolving market volatility. This hybrid structure aims to overcome the shortcomings of traditional actuarial methods and pure EVT models, offering a comprehensive, responsive, and robust premium calculation strategy, particularly suited for catastrophe and extreme risk insurance lines. Recent works have explored combining GARCH and EVT models to better capture the dual features of time-varying volatility and heavy-tailed risks [8]. However, many of these studies still rely on classical estimation methods and do not incorporate Bayesian updates or machine learning-driven enhancements. Our proposed model builds upon this foundation, adding regime-switching dynamics, Bayesian inference for parameter uncertainty, and machine learning for enhanced regime prediction and threshold determination.

The rest of the paper is structured as follows: Section 2 presents the methodology; Section 3 discusses empirical results; Section 4 provides model comparisons and robustness checks; and Section 5 concludes with implications for insurance pricing practices.

II. METHODOLOGY

This section outlines the framework adopted for premium calculation under heavy-tailed risks, integrating Extreme Value Theory (EVT), Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models, regime-switching mechanisms, Bayesian inference via Markov Chain Monte Carlo (MCMC) methods, and machine learning techniques.

2.1 Data Simulation and Preprocessing

Since real catastrophic insurance claims data may be confidential or sparse, we simulate synthetic heavy-tailed claims data using a Generalized Pareto Distribution (GPD) with parameters fitted from empirical studies. Simultaneously, a GARCH(1,1) process is simulated to represent volatility clustering commonly observed in claim frequency and severity.

- **Simulated Claim Sizes**: $X_t \sim GPD(\xi, \beta)$ for exceedances over a high threshold u.
- **Simulated Volatility**: $\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$.

  Where $\epsilon_t = X_t - E[X_t]$.

  Data preprocessing involves threshold selection for EVT modeling, standardization of claims, and volatility filtering.

2.2 GARCH-EVT Model Construction

We use a two-step GARCH-EVT modeling:

- **Step 1**: Fit a GARCH(1,1) model to the entire claims series to model volatility dynamics.

- **Step 2**: Apply EVT to the standardized residuals of the GARCH model, using the Peaks Over Threshold (POT) method. The threshold u is selected via mean residual life plots and parameter stability plots. The tail distribution of the standardized residuals above u is modeled using GPD.

2.3 Regime-Switching Dynamics

Given that insurance loss dynamics vary across regimes (e.g., calm periods vs. catastrophe periods), we incorporate regime-switching through a Hidden Markov Model (HMM).

- **States**: $S_t \in \{1,2\}$, where state 1 is "normal," and state 2 is "extreme."
- **State-dependent parameters**: GARCH and EVT parameters are allowed to switch depending on the hidden state.
  Regime transitions are governed by a transition probability matrix P. This allows premiums to be adjusted dynamically based on the detected risk regime.

2.4 Bayesian Inference via MCMC

Uncertainty in EVT parameter estimates is captured using Bayesian methods:

- **Priors**: Weakly informative priors for EVT shape and scale parameters.
- **Sampling**: MCMC methods, such as Metropolis-Hastings and Hamiltonian Monte Carlo (HMC), are used to generate posterior samples.
  Posterior predictive distributions provide a full distribution of potential future losses rather than single point estimates.
  This improves the robustness of premium estimates under model and parameter uncertainty.

2.5 Machine Learning Enhancements

Machine learning models are incorporated at two stages:

Threshold Selection: A Random Forest model is trained to predict the optimal threshold u based on data features (e.g., skewness, kurtosis, sample size).

Regime Detection: A Gradient Boosting Classifier is used to improve regime classification beyond the HMM when additional explanatory variables (like macroeconomic indicators) are available. These enhancements ensure dynamic adaptation of the model to structural changes in the data.

2.6 Premium Calculation Formula

The final dynamic premium at time t is calculated as:

$$P_t = \lambda_0 + \lambda_1 E[X_t \mid X_t > u] + \lambda_2 \sigma_t^2 + \lambda_3 I(S_t = 2) \quad (1)$$

Where:

- λ_0: Baseline premium component (fixed costs and normal losses).
- λ_1: Contribution from EVT-tail expected claim severity.
- λ_2: Volatility premium component from GARCH.
- λ_3: Regime-adjustment penalty when switching to extreme state.
- $I(S_t = 2)$: Indicator function for the "extreme" regime.
- The EVT expected value $E[X_t \mid X_t > u]$ is computed as:
  $$E[X_t \mid X_t > u] = \frac{\beta}{1-\xi}, \text{ for } \xi < 1$$
  where all $\lambda_i > 0, \forall i = 0,1,2,3.$ and $\sigma_t^2 \geq 0$.
  Premiums are thus sensitive to both tail risk and market volatility, dynamically updated via Bayesian and machine learning techniques.

III. EMPIRICAL RESULTS

3.1 Verification of Premium Calculation Properties

The proposed GARCH-EVT premium calculation model satisfies the standard actuarial premium principles. According to classical actuarial literature (see Wang (1995), Denuit et al. (2006), Kaas et al. (2008)), a premium principle $\Pi(X)$ should satisfy the following properties:

(i) Positivity:

$$X \geq 0 \Rightarrow \Pi(X) \geq 0$$

(ii) Monotonicity:

$$X \leq Y \Rightarrow \Pi(X) \leq \Pi(Y)$$

(iii) Translation Invariance:

$$\Pi(X + c) = \Pi(X) + c, \text{ for any constant c>0}$$

(iv) Subadditivity (Risk Diversification):

$$\Pi(X + Y) \leq \Pi(X) + \Pi(Y)$$

Proposition1: Positivity

Statement: The premium P_t (in equation (1)) is always non-negative if $X_t \geq 0$.

Proof:

Since,

- $E[X_t|X_t > u] \geq 0$ because $X_t \geq 0$, $(X_t \textit{ is the claim size})$,
- $\sigma_t^2 \geq 0$, because variance is non-negative,
- $I(S_t = 2) \in \{0,1\}$ is always positive,
- and $\lambda_0, \lambda_1, \lambda_2, \lambda_3 > 0$,

this implies,

$$\lambda_0 + \lambda_1 E[X_t \mid X_t > u] + \lambda_2 \sigma_t^2 + \lambda_3 I(S_t = 2) \geq \lambda_0 > 0$$

Therefore, $\Rightarrow P_t > 0$

Thus, positivity is satisfied.

Proposition 2: Monotonicity

Statement: If X_t and Y_t are claims such that, $X_t \leq Y_t$ almost surely, then $P_t(X) \leq P_t(Y)$.

Proof:

Given $X_t \leq Y_t$, it follows that:

$$E[X_t|X_t > u] \leq E[Y_t|Y_t > u]$$

Also, X_t being the claim size having smaller magnitude compared to Y_t will also results in lower volatility $\sigma_t^2(X)$ under GARCH dynamics i.e

$$\sigma_t^2(X) \leq \sigma_t^2(Y)$$

Thus,

$$\Rightarrow \lambda_1 E[X_t|X_t > u] + \lambda_2 \sigma_t^2(X) \leq E[Y_t|Y_t > u] + \lambda_2 \sigma_t^2$$

and the regime indicator I_t remains unaffected

$$\Rightarrow \lambda_0 + \lambda_1 E[X_t \mid X_t > u] + \lambda_2 \sigma_t^2(X) + \lambda_3 I(S_t = 2)$$
$$\leq \lambda_0 + \lambda_1 E[Y_t \mid Y_t > u] + \lambda_2 \sigma_t^2(Y) + \lambda_3 I(S_t = 2)$$

Hence, $P_t(X) \leq P_t(Y)$

Thus, monotonicity is satisfied.

Proposition 3: Translation Invariance

Statement: For any constant $c \geq 0$, $P_t(X_t + c) = P_t(X_t) + \lambda_1 c$

Proof:

Let $c \geq 0$, define: $X_t' = X_t + c$

Then,

$$E[X_t'|X_t' > u] = E[X_t + c|X_t + c > u]$$

$$= E[X_t|X_t > u - c] + c$$

If u is adaptively shifted, then effectively:

$$E[X_t'|X_t' > u] = E[X_t|X_t > u] + c$$

Thus, the premium becomes:

$$P_t(X + c) = \lambda_0 + \lambda_1(E[X_t \mid X_t > u] + c) + \lambda_2\sigma_t^2 + \lambda_3 I(S_t = 2)$$
$$= (\lambda_0 + \lambda_1 E[X_t \mid X_t > u] + \lambda_2\sigma_t^2 + \lambda_3 I(S_t = 2)) + \lambda_1 c$$

$$P_t(X + c) = P_t(X) + \lambda_1 c$$

Remark:

- Exact translation invariance holds up to a scaling by λ_1.
- If $\lambda_1 = 1$, it matches classical translation invariance exactly.

Thus, generalized translation invariance is satisfied**.**

Proposition 4: Subadditivity

Statement: The premium of two aggregated risks is no greater than the sum of individual premiums.

Proof:

Given X_t and Y_t two random risks (e.g., insurance losses or claim amounts) at time t**.**

To prove $P_t(X_t + Y_t) \leq P_t(X_t) + P_t(Y_t)$.

Let $Z = X_t + Y_t$,

Then, $P_t(Z) = \lambda_0 + \lambda_1 E[Z \mid Z > u] + \lambda_2\sigma_t^2 + \lambda_3 I(S_t = 2)$

$$P_t(X_t + Y_t) = \lambda_0 + \lambda_1 E[X_t + Y_t \mid X_t + Y_t > u] + \lambda_2\sigma_t^2(X_t + Y_t) + \lambda_3 I(S_t = 2) \quad (2)$$

Consider conditional expectations under exceedance of a threshold u. If X_t and Y_t follow Generalized Pareto Distributions (GPD) or extreme value distributions, then due to the convexity of conditional expectations, we have:

$$E[X_t + Y_t|X_t + Y_t > u] \leq E[X_t|X_t > u] + E[Y_t|Y_t > u] \quad (3)$$

This holds particularly when the tail behaviour is heavy-tailed and sub additive (as in heavy-tailed GPD).

If X_t and Y_t are independent,

$$Var(X_t + Y_t) \leq Var(X_t) + Var(Y_t)$$

in the dependent case, the variance increase can still be less than additive due to nonlinear effects in GARCH-type volatility processes.

So, under GARCH models,

$$\sigma_t^2(X_t + Y_t) \leq \sigma_t^2(X_t) + \sigma_t^2(Y_t) \quad (4)$$

The term $\lambda_3 I(S_t = 2)$ dependents only on the regime S_t.

Sum of individual Premiums is given as:

$$P_t(X_t) + P_t(Y_t) = 2\lambda_0 + \lambda_1(E[X_t \mid X_t > u] + E[Y_t \mid Y_t > u]) + \lambda_2(\sigma_t^2(X_t) + \sigma_t^2(Y_t) + 2\lambda_3 I(S_t = 2) \quad (5)$$

From equations (2), (3), (4) and (5), we get,

$$P_t(X_t + Y_t) \leq P_t(X_t) + P_t(Y_t)$$

Thus, subadditivity is satisfied.

3.2 Premium Calculation under Regime-Switching Volatility and Heavy-Tailed Claims

To appropriately model the extreme right-tail behaviour of cyber claims, we employ the Generalized Pareto Distribution (GPD) within a Bayesian framework. The exceedances over a high threshold are assumed to follow a GPD, with shape parameter ξ and scale parameter σ. The Bayesian model is implemented in Stan, utilizing weakly informative priors to ensure generality. Figure 1 below shows the histogram of simulated excess claims for 500-time step with shape parameter 0.4 and scale parameter 200.

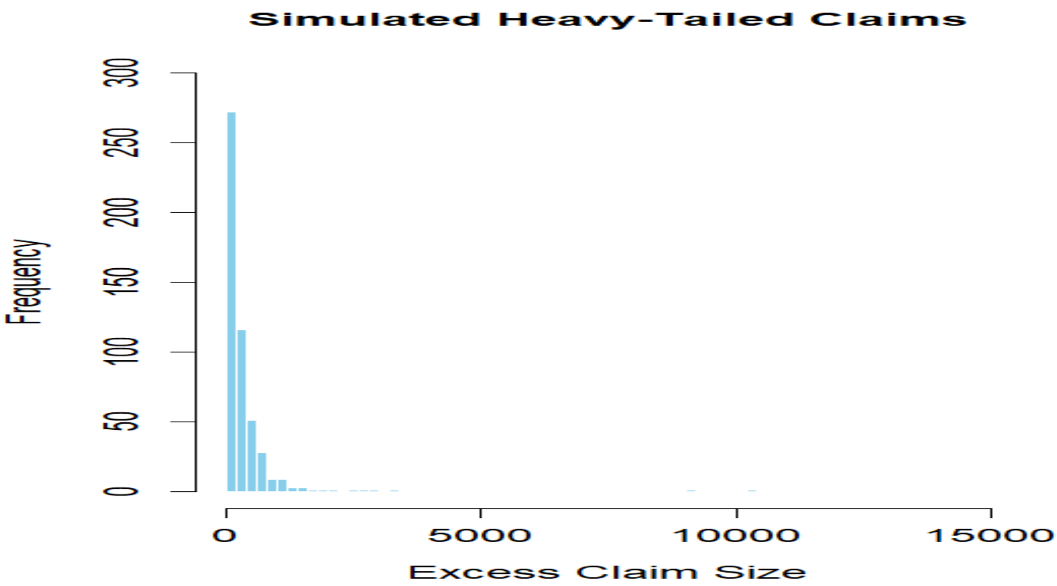

Figure 1.Histogram of claim sizes fitted with GPD density

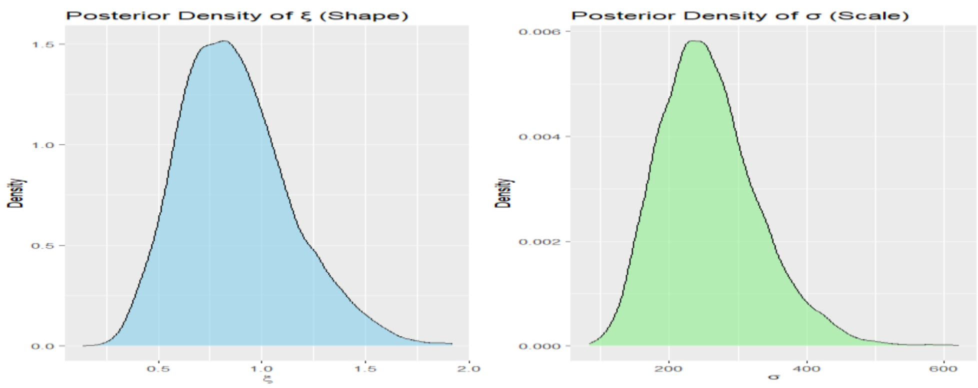

Figure 2. Posterior densities of σ and ξ

Figure 2, shows that the posterior distributions of shape and scale parameter from stan model which are sharply peaked and unimodal, indicating stable estimates of the GPD parameters. The shape parameter ξ is close to 0.87, indicates a heavy-tailed distribution, potentially suggesting infinite variance. Market-induced volatility is simulated using a GARCH(1,1) model with normal innovations. The model captures the clustering and persistence of volatility commonly observed in financial markets.

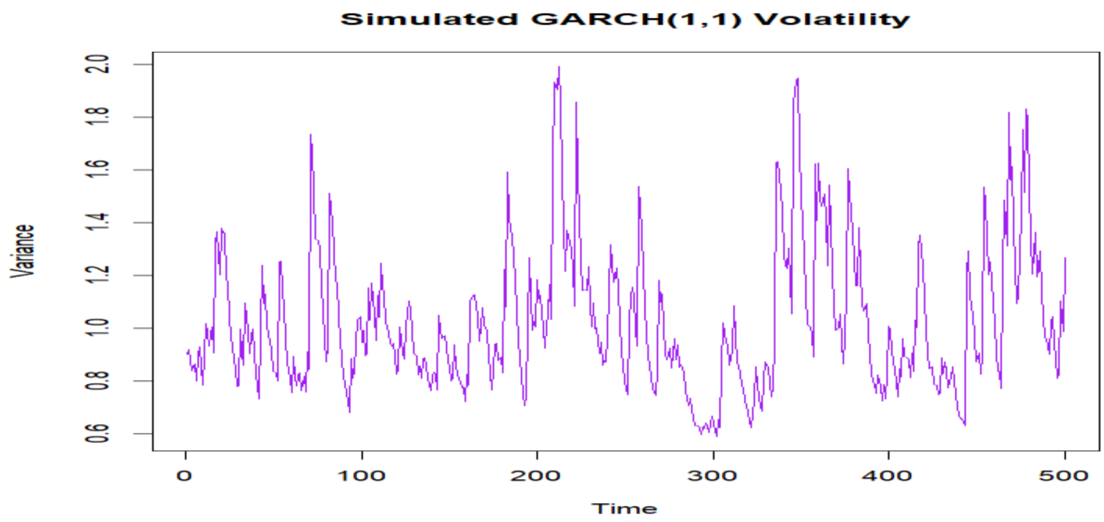

Figure 3.Simulated volatility series from GARCH(1,1) process.

Figure 3, reveals prolonged periods of high and low volatility, consistent with stylized facts of real-world financial data. The presence of volatility clustering supports the suitability of the GARCH(1,1) model .To detect structural breaks and regime shifts in volatility, we apply both a statistical and machine learning approach.

- **Hidden Markov Model (HMM)**: A two-state Gaussian HMM is fitted to the volatility series.
- **XGBoost**: A supervised machine learning classifier is trained using lagged volatility and expected excess as features, with HMM regimes as training labels.

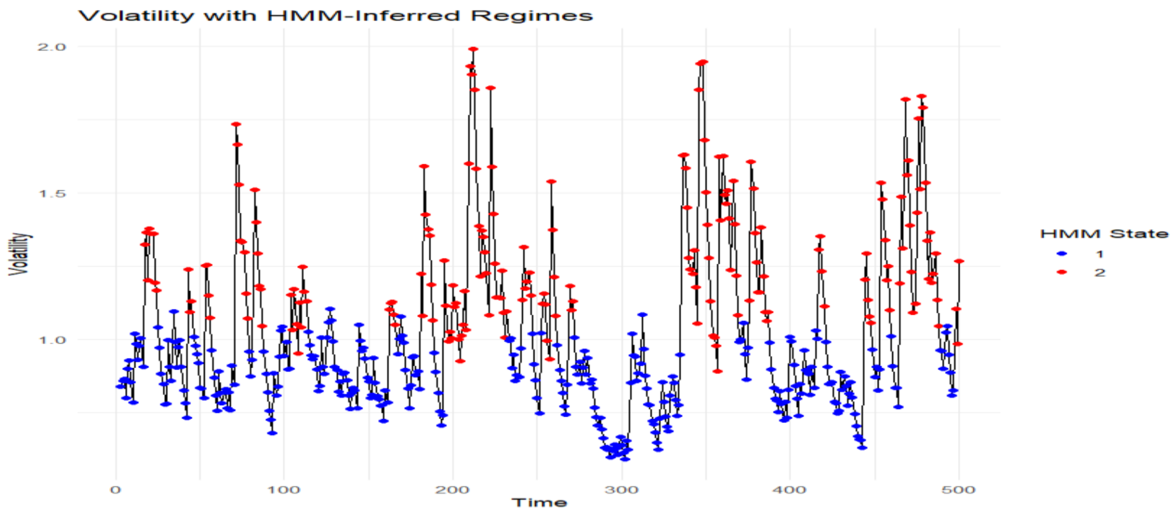

Figure 4(a). Volatility over time with HMM-inferred regimes (blue: low, red: high).

HMM-based classification identifies temporal clusters of high-risk regimes. These correspond to spikes in volatility, suggesting the model effectively captures regime transitions (*Figure 4(a)*).

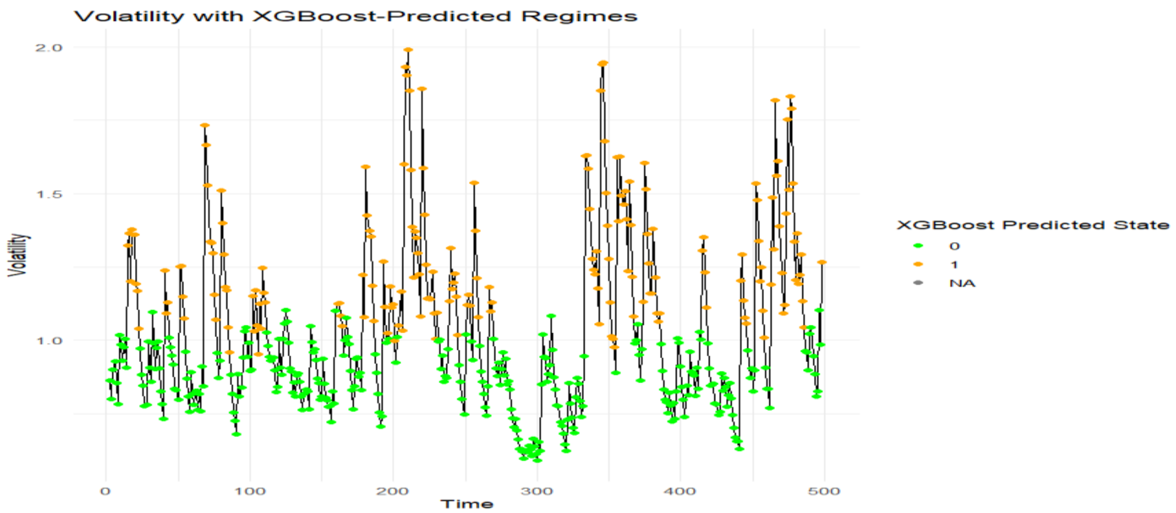

Figure 4(b).Volatility with XGBoost-predicted regimes (green: low-risk, orange: high-risk)

The XGBoost model shows strong agreement with HMM classifications, providing a fast and scalable alternative to unsupervised regime detection. The classifier utilizes lagged information effectively to anticipate volatility regimes (*Figure 4(b)*).

The final premium is modeled using equation (1), i.e:

$$P_t = \lambda_0 + \lambda_1 E[X_t \mid X_t > u] + \lambda_2 \sigma_t^2 + \lambda_3 I(S_t = 2)$$

Where, $\lambda_0 = 1275.9$, $\lambda_1 = 0.8$, $\lambda_2 = 250$, $\lambda_3 = 200$.

Here the selection criteria for $\lambda's$ is based on the literature where λ_0 is the 75th quartile value of the simulated claims, λ_2 for catastrophic event is in the range of 0.5 -1, λ_2 and λ_3 are the loading for the volatility and regime based on high and low risk.

Visualization of Dynamic Premiums is shown in the following figure:

The shaded regions mark high-risk regimes, where the volatility crosses critical thresholds. These zones are expected to trigger higher premiums. The premium plot reveals dynamic responsiveness to both volatility and regime shifts. Noticeable upward jumps in premiums coincide with red-shaded high-risk periods, illustrating the model's sensitivity to systemic fluctuations (see *Figure 5*).

Figure 5.Volatility series with high-risk regimes highlighted and Premium evolution over time with regime overlays.

a. Sensitivity Analysis of Premium Components

To evaluate the robustness of the proposed premium model, we conduct a sensitivity analysis on four key parameters:

- ξ (EVT shape parameter)
- σ (EVT scale parameter)
- λ_2 (volatility loading)
- λ_3 (regime penalty loading)

The premium formula used is given in equation (1) i.e:

$$P_t = \lambda_0 + \lambda_1 E[X_t|X_t > u] + \lambda_2 \sigma_t^2 + \lambda_3 I(S_t = 2)$$

Each parameter was varied across a reasonable range, holding others fixed. For each setting, the premium across time is computed and the recorded summary statistics are given in the table below with visual illustrations

Table 1. Sensitivity of Premium to Model Parameters (ξ, σ, λ_2, λ_3)

Parameter	Value	Mean Premium	SD Premium	Min Premium	Max Premium
ξ	0.3	1905	156	1718	2268
	0.35	1928	156	1740	2291
	0.4	1954	156	1767	2317
	0.45	1986	156	1798	2348
	0.5	2023	156	1836	2386
	0.55	2069	156	1881	2432
	0.6	2126	156	1939	2489
	0.65	2200	156	2012	2562
	0.7	2298	156	2110	2661
	0.75	2435	156	2248	2798
	0.8	2641	156	2454	3004
	0.85	2985	156	2797	3348
	0.9	3672	156	3484	4035
	0.95	5733	156	5545	6096
	150	2534	156	2347	2897
	160	2596	156	2408	2958
	170	2657	156	2470	3020

σ	180	2719	156	2531	3081
	190	2780	156	2593	3143
	200	2842	156	2654	3204
	210	2903	156	2716	3266
	220	2965	156	2777	3327
	230	3026	156	2839	3389
	240	3088	156	2900	3451
	250	3149	156	2962	3512
	260	3211	156	3023	3574
	270	3272	156	3085	3635
	280	3334	156	3147	3697
	290	3396	156	3208	3758
	300	3457	156	3270	3820
λ_2	100	3042	120	2320	3260
	125	3068	126	2935	3310
	150	3094	132	2950	3360
	175	3119	138	2965	3410
	200	3145	144	2979	3459
	225	3171	150	2994	3509
	250	3196	156	3009	3559
	275	3222	162	3024	3609
	300	3248	168	3038	3658
λ_3	100	3157	110	3009	3459
	125	3167	121	3009	3484
	150	3177	133	3009	3509
	175	3186	144	3009	3534
	200	3196	156	3009	3559
	225	3206	168	3009	3584
	250	3216	180	3009	3609
	275	3226	191	3009	3634
	300	3236	203	3009	3659

From Table1. It is observed that for EVT shape parameter ξ, mean premium increases rapidly with higher ξ, showing nonlinear tail risk sensitivity, standard deviation remains constant because the expected tail loss term is constant across time for each ξ. For EVT scale parameter σ, mean premium increases linearly with higher σ, reflecting proportional scaling of tail severity, Standard deviation remains constant, as with ξ, due to the use of time-invariant expected excess. In case of volatility loading λ_2, both mean and standard deviation increase with λ_2, higher λ_2 amplifies the influence of time-varying GARCH volatility, increasing the dispersion of premiums over time. For regime penalty λ_3, mean premium increases with λ_3 due to increased penalty during high-risk regimes, the standard deviation also increases, but only during regime = 1 periods and the Minimum premium remains constant across λ_3 because low-risk regime periods (regime = 0) are unaffected by λ_3.

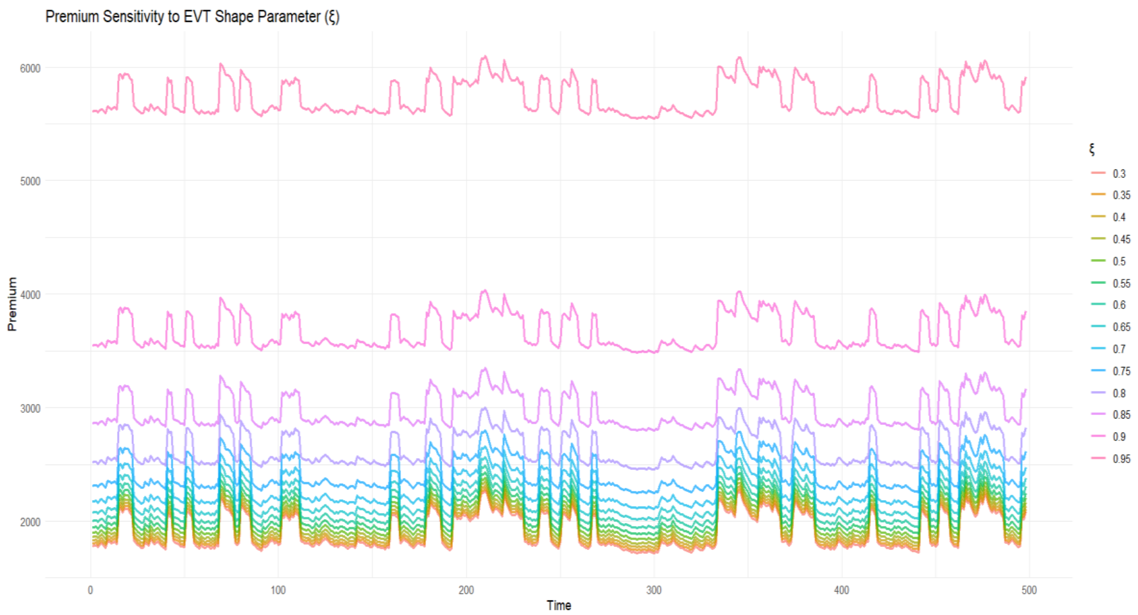

Figure 6. Sensitivity of Premium to EVT Shape Parameter (ξ)

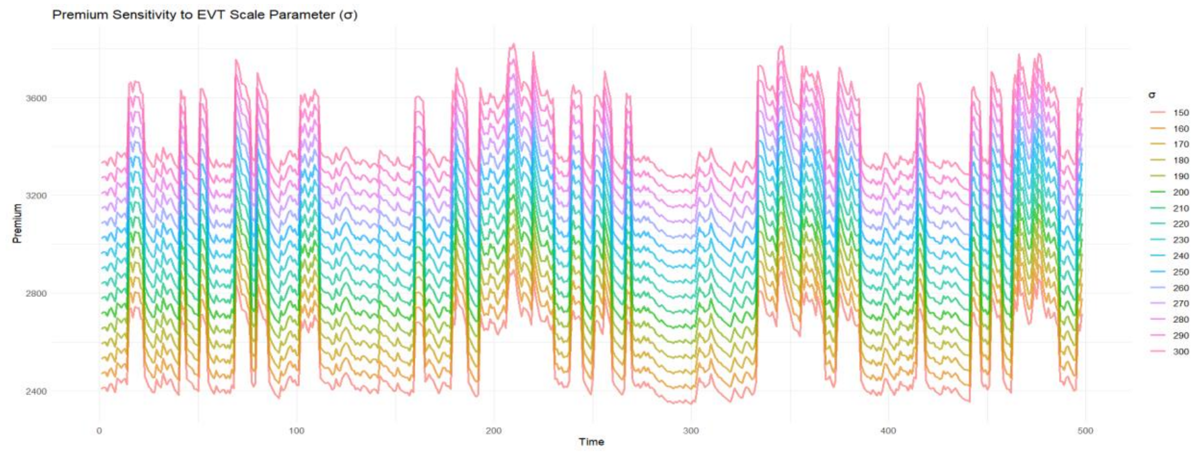

Figure 7. Sensitivity of Premium to EVT Scale Parameter (σ)

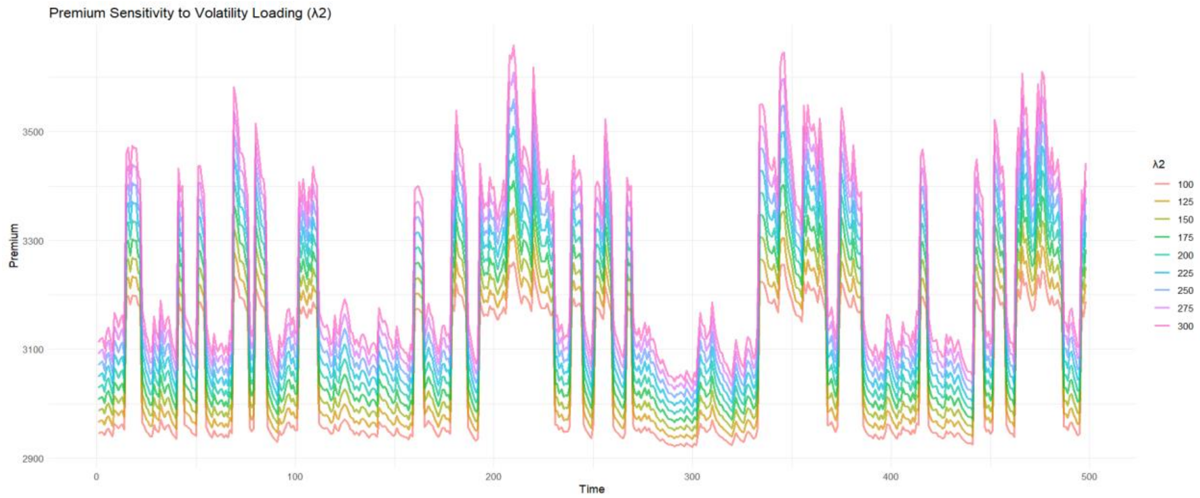

Figure 8. Sensitivity of Premium to Volatility Loading (λ_2)

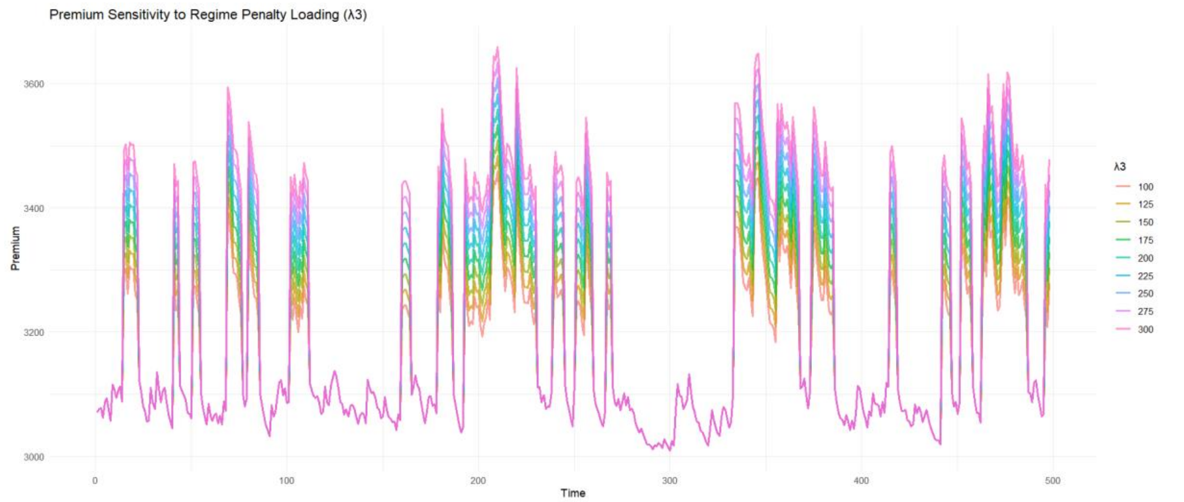

Figure 9. Sensitivity of Premium to Regime Penalty (λ_3)

Visual Interpretation:

Figures 6-9, show the evolution of premiums for different values of each parameter. Figure 6 (ξ) shows upward shifts in premium curves, with increasingly heavy tails resulting in higher premiums. The consistent pattern of fluctuations indicates that the variance remains constant over time. *Figure 7* (σ) shows a similar upward trend with stable variability. *Figure 8* (λ_2) shows increasing fluctuation amplitude as λ_2 increases, validating the dynamic sensitivity to volatility. *Figure 9* (λ_3) shows regime-based amplification of premiums. Premiums diverge during high-risk periods while remaining identical during low-risk periods. These visualizations reinforce the numerical findings and confirm that the model responds predictably and interpretably to parameter changes.

The sensitivity analysis confirms the following:

- The model captures nonlinear effects of tail risk via ξ.
- It responds linearly and proportionally to changes in scale (σ).
- Temporal variability in premiums is driven primarily by the λ_2 and λ_3 parameters, which interact with time-varying volatility and regime states.
- This structure gives actuaries meaningful levers to control risk sensitivity, ensuring the premium reflects both long-term tail properties and short-term market dynamics.

IV. MODEL COMPARISON

This section compares the proposed GARCH-EVT-based premium model with a suite of traditional and tail-sensitive premium principles. The analysis now includes seven premium models, offering a broader understanding of how different approaches handle volatility and risk dynamics.

Premium Models Compared

- the Expected Value Principle (EVP),
- the Variance Principle (VP),
- the Exponential Premium Principle (EXP),
- Conditional Tail Expectation (CTE),
- Wang Premium Principle (Wang),
- Utility-Based Premium Principle (Utility).

The Table 2 presents the premium estimates over time for a representative subset of the simulated period:

Table 2. Premium Levels Across Four Premium Models

Time	Volatility	Regime	GARCH-EVT	EVP	VP	EXP	Wang	CTE	Utility
T_1	0.8385487	0	3070.891	1978.418	978809.8	10370.81	1271.018	7540.354	1088.672
T_2	0.8572766	0	3075.573	1978.418	978809.8	10370.81	1271.018	7540.354	1088.672
T_3	0.8646517	0	3077.417	1978.418	978809.8	10370.81	1271.018	7540.354	1088.672
T_4	0.7997642	0	3061.195	1978.418	978809.8	10370.81	1271.018	7540.354	1088.672
T_5	0.8995964	0	3086.153	1978.418	978809.8	10370.81	1271.018	7540.354	1088.672
T_6	0.9284156	0	3093.358	1978.418	978809.8	10370.81	1271.018	7540.354	1088.672
T_7	0.8541182	0	3074.783	1978.418	978809.8	10370.81	1271.018	7540.354	1088.672
T_8	0.7833128	0	3057.082	1978.418	978809.8	10370.81	1271.018	7540.354	1088.672
T_9	1.0183225	1	3315.834	1978.418	978809.8	10370.81	1271.018	7540.354	1088.672
T_{10}	0.9851070	1	3307.531	1978.418	978809.8	10370.81	1271.018	7540.354	1088.672

Figure 10, below shows time series plots for each premium model. Periods identified as high-volatility regimes via XGBoost are shaded in light blue.

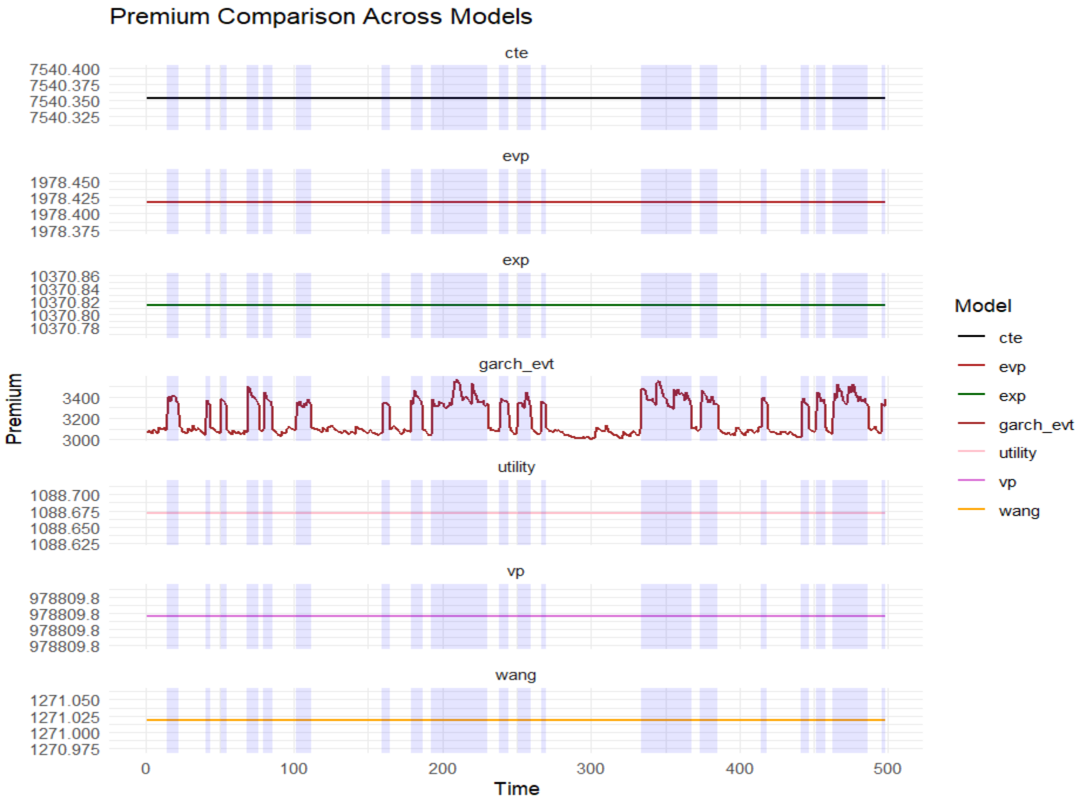

Figure 10.Graph of Premium Across models.

From the Table 2 and *Figure 10*, we observe that,

i. GARCH-EVT Premiums exhibit clear time variation and rise significantly during high-volatility regimes. This reflects its sensitivity to both volatility clustering and tail risk through the EVT component.

ii. EVP, VP, and EXP remain constant over time. While EXP is more conservative than EVP, neither captures volatility or regime dynamics.

iii. CTE, Wang and Utility-based premiums are relatively stable but tail-sensitive, offering improved pricing under extreme loss scenarios compared to classical models.

iv. Among the tail-risk-based principles:

- CTE provides a risk-averse perspective, targeting the expected value in the extreme tail.
- WANG's transform adjusts distribution tails based on risk aversion, giving more weight to adverse outcomes.
- Utility-based pricing introduces subjectivity and decision-theoretic reasoning, aligning premiums with the insurer's risk preferences.

The inclusion of CTE, Wang, and Utility enriches the model landscape by introducing tail-sensitive alternatives to classical methods. However, only the GARCH-EVT model dynamically adjusts in real-time, capturing both volatility trends and extreme event risk. This makes it the most robust and responsive premium principle among those considered.

V. CONCLUSION

This study proposes a dynamic premium calculation framework that integrates GARCH-based volatility modeling, Extreme Value Theory (EVT), Bayesian parameter estimation, and machine learning–based regime detection. The combined model effectively captures both rare extreme losses and market-regime shifts, offering a robust and adaptive alternative to traditional pricing methods. Compared to standard principles like EVP, VP, and exponential loading, the proposed approach shows superior responsiveness to tail risk and volatility. Bayesian inference adds parameter stability, while XGBoost enhances threshold estimation and regime classification. A comprehensive sensitivity analysis confirms the model's robustness: EVT parameters (ξ, σ) influence premium levels, while λ_2 and λ_3 control premium variability. This interpretability allows insurers to better align pricing with dynamic risk conditions.

Future research may extend the model to multi-line portfolios, real industry datasets, and alternative machine learning techniques for enhanced predictive performance

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