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Review Paper

About Artincokernel of The Group $(Q_2l^2\times C_7)$ Where $l\neq 2$,l is prime number

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I. Abstract:
In this research,westudied cyclic decomposition of the abelian factor group:
$AC(Q_2l^2\times C_7) = \overline{R}(Q_2l^2\times C_7) / T(Q_2l^2\times C_7)$ when $l\neq 2$,l is prime number.The group$(Q_2l^2\times C_7)$ is the direct product group of the quaternion group Q_2l^2,which has an order of ***$4l^2$,*** *and the cyclic group C_7which has an order of7.As a result ,the order of thegroup$(Q_2l^2\times C_7)$ is* ***given by***
$|Q_2l^2\times C_7|$ ***$= 28l^2$****.Thus,cyclic decomposition of $AC(Q_2l^2\times C_7)$ is:*

$$AC(Q_{2l^2}\times C_7) = \bigoplus_{i=1}^{8} C_7 .$$

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II. Introduction:

Representation theory is a mathematical field that explores abstract algebraic structures by illustrating their elements as transformations of vector spaces, thereby investigating their properties in a modular fashion. It provides a tangible framework by representing abstract algebraic objects through matrices and algebraic operations.In 2007, A. H. Mohammed [11] found the Artincokernel of the dihedral group D_n when n is an even number . In 2008, A. H. Abdul-Munem [12] studied ArtinCokernel of The Quaternion group Q_{2m} when m is an odd number.

In 2013, N. A. Rahi [13].studiedThe cyclic decomposition of the factor group $cf(Q_{2m}\times C_5, Z)/\overline{R}(Q_{2m}\times C_5)$when m is an odd number.In thisresearch, we will find the cyclic decomposition of the factor group $AC(Q_2l^2\times C_7)$ when r is an odd number.

III. Preliminars:

In this part of the research, we will present some definitions and theorems that help us in finding Artin characters table for the group $(Q_2l^2\times C_7)$.I will symbolize in this research,principal character with the symbol (pc),prime number with the symbol (pr) , positive integer number with the symbol (pin),Γ-classes with the symbol(Γ-c),odd numberwith the symbol(odn),cyclic subgroup with the symbol(csg)and Artin characters with the symbol(Arc).

Definition (3.1):[2]

The Artin characters table of Gdenoted by Ar(G)it is a table in whichthe third row is the size of the centralized**$|C_G(CL_\alpha)|$** ;The second row is the number of elements in each conjugate class,Thefirst row is Γ-conjugate classesand other rows contains the values of (Arc).

Theorem:(3,2):[2]
The general form of Ar(Cl^s)ofThecyclicgroup$Cl^s$when l is (pr) and s is (pin) is given by :-

Ar(Cl^s)=

Γ-c	[1]	$[x^{r^{s-1}}]$	$[x^{r^{s-2}}]$	$[x^{r^{s-3}}]$	⋯	[x]
$\lvert CL_a\rvert$	1	1	1	1	⋯	1
$\lvert C_{r^s}(CL_\alpha)\rvert$	l^s	l^s	l^s	l^s	⋯	l^s
φ'_1	l^s	0	0	0	⋯	0
φ'_2	l^{s-1}	l^{s-1}	0	0	⋯	0
φ'_3	l^{s-2}	l^{s-2}	l^{s-2}	0	⋯	0
⋮	⋮	⋮	⋮	⋮	⋮	
φ'_s	l^1	l^1	l^1	l^1	⋯	0
φ'_{s+1}	1	1	1	1	⋯	1

Table(3,1)

Corollary (3,3):*[2:]*
Let $m=l_1^{\alpha1}.l_2^{\alpha2}\ldots\ldots l_n^{\alpha n}$where the biggest common denominator(l_i, l_j)=1, if $i\neq j$,
l_i's are (pr)and$\alpha_n$any (pin), then;

$$Ar(C_m)=A(C_{l_1^{\alpha1}})\otimes A(C_{l_2^{\alpha2}})\otimes\ldots\ldots\otimes A(C_{l_n^{\alpha n}})$$

Example:(3,4):
To find Ar(C_{50}) for the cyclic group C_{50} .by using theorem (3.2) to find Ar(C_2) and Ar(C_5^2) are as follows :

Ar(C_2)=Ar(C_5^2)=

Γ- c	[1]	[x]
$\lvert CL_\alpha\rvert$	1	1
$\lvert C_{C_2}(CL_\alpha)\rvert$	2	2
φ'_1	2	0
φ'_2	1	1

Γ- c	[1]	$[x^5]$	[x]
$\lvert CL_\alpha\rvert$	1	1	1
$\lvert C_{C_{5^2}}(CL_\alpha)\rvert$	5^2	5^2	5^2
φ'_1	5^2	0	0
φ'_2	5	5	0
φ'_3	1	1	1

Table(3,2)Table(3,3)

NowAr(C_{50})=Ar(C_2)⊗Ar(C_5^2)as the following:

Ar(C_{50})=

Γ- c	[1]	$[x^6]$	$[x^2]$	$[x^9]$	$[x^3]$	[x]
$\lvert CL_\alpha\rvert$	1	1	1	1	1	1
$\lvert C_{C_{50}}(CL_\alpha)\rvert$	50	50	50	50	50	50
φ'_1	50	0	0	0	0	0
φ'_2	10	10	0	0	0	0
φ'_3	2	2	2	0	0	0

φ'_4	25	0	0	25	0	0
φ'_5	5	5	0	5	5	0
φ'_6	1	1	1	1	1	1

Table(3,5)

Theorem(3.5):[3]

Ar(Q_{2m})of the Quaternion group Q_{2m} when m is an (odn) is given as follows :

	Γ-C of C_{2m}								
Γ-C	x^{2p}				x^{2p+1}				[y]
| CL_α|	1	2	…	2	1	2	…	2	m
|$C_{Q_{2m}}$ (CL_α)|	4m	2m	…	2m	4m	2m	…	2m	4
Φ_1	2.Ar(C_{2m})								0
Φ_2									0
:									:
Φ_l									0
Φ_{l+1}	m	0	…	0	m	0	…	0	1

Table (3.6)where $0 \leq p \leq m-1$, l is the number of Γ-c of C_{2m} and Φ_j are the (Arc) of group Q_{2m} , for all $1 \leq j \leq l+1$.

Example (3.6):

To findAr(Q_{50}) by usingexample(3,4)andtheorem(3.5).

Ar(Q_{50}) =

Γ-C	Γ-Classe of C_{50}						[y]
| CL_α|	1	2	2	1	2	2	25
|$C_{Q_{50}}$(CL_α)|	100	50	50	100	25	25	4
Φ_1	2Ar(C_{50})						0
Φ_2							0
Φ_3							0
Φ_4							0
Φ_5							0
Φ_6							0
Φ_7	25	0	0	25	0	0	1

=

Γ-C	[1]	[x^{10}]	[x^2]	[x^{25}]	[x^5]	[x]	[y]
| CL_α|	1	2	2	1	2	2	25
|$C_{Q_{50}}$ (CL_α)|	100	25	25	100	25	25	4
Φ_1	100	0	0	0	0	0	0
Φ_2	20	20	0	0	0	0	0
Φ_3	4	4	4	0	0	0	0
Φ_4	50	0	0	50	0	0	0
Φ_5	10	10	0	10	10	0	0
Φ_6	2	2	2	2	2	2	0
Φ_7	25	0	0	25	0	0	1

Table (3.7)

Theorem (3,7): [1]
The Ar($Q_2l^2\times C_7$)of the Group($Q_2l^2\times C_7$) where l≠2 and r is (pr); is given as follows:
Ar($Q_2l^2\times C_7$)=

Γ- c	[1,I]	[x^{2l},I]	[x^2 ,I]	[x^{l^2},I]	[x^l ,I]	[x ,I]	[y,I]	[1,z]	[x^{2l},z]	[x^2,z]	[x^{l^2},z]	[x^l,z]	[x,z]	[y,z]
$\|CL_\alpha\|$	1	2	2	1	2	2	l^2	1	2	2	1	2	2	l^2
$\|Q_2l^2\times C_7(CL_\alpha)\|$	$28l^2$	$14l^2$	$14l^2$	$28l^2$	$14l^2$	$14l^2$	28	$28l^2$	$14l^2$	$14l^2$	$28l^2$	$14l^2$	$14l^2$	28
$\Phi_{(1,1)}$	**7Ar(Q_{2l^2})**							**0**						
$\Phi_{(2,1)}$														
$\Phi_{(3,1)}$														
$\Phi_{(4,1)}$														
$\Phi_{(5,1)}$														
$\Phi_{(6,1)}$														
$\Phi_{(7,1)}$														
$\Phi_{(1,2)}$	**Ar(Q_{2l^2})**							**Ar(Q_{2l^2})**						
$\Phi_{(2,2)}$														
$\Phi_{(3,2)}$														
$\Phi_{(4,2)}$														
$\Phi_{(5,2)}$														
$\Phi_{(6,2)}$														
$\Phi_{(7,2)}$														

Table (3.8)
Which is [14]×[14] matrix square.

Example (3.8):
Ar($Q_{50}\times C_7$)=Ar($Q_{2.5^2}\times C_7$),l=5,usingexample (3.6) as the following :-
Ar(Q_{50})=

Γ-C	[1]	[x^{10}]	[x^2]	[x^{25}]	[x^5]	[x]	[y]
$\|CL_\alpha\|$	1	2	2	1	2	2	25
$\|C_{Q_{50}}(CL_\alpha)\|$	100	25	25	100	25	25	4
Φ_1	100	0	0	0	0	0	0
Φ_2	20	20	0	0	0	0	0
Φ_3	4	4	4	0	0	0	0
Φ_4	50	0	0	50	0	0	0
Φ_5	10	10	0	10	10	0	0
Φ_6	2	2	2	2	2	2	0
Φ_7	25	0	0	25	0	0	1

Table(3,9)

Then by using theorem (3,7), Ar($Q_{50}\times C_7$)is:-
Ar($Q_{50}\times C_7$)=

Γ-C	[1,I]	$[x^{10},I]$	$[x^2,I]$	$[x^{50},I]$	$[x^5,I]$	[x,I]	[y,z]	[1,z]	$[x^{10},z]$	$[x^2,z]$	$[x^{25},z]$	$[x^5,z]$	[x,z]	[x,z]
$\mid CL_\alpha \mid$	1	2	2	1	2	2	25	1	2	2	1	2	2	25
$\mid C_G(CL_\alpha) \mid$	700	350	350	700	350	350	28	700	350	350	700	350	350	28
$\Phi_{(1,1)}$	700	0	0		0	0	0	0	0	0	0	0	0	0
$\Phi_{(2,1)}$	140	140	0	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(3,1)}$	28	28	28	0	0	0	0	0	0	0	0	0	0	0
$\Phi_{(4,1)}$	350	0	0	350	0	0	0	0	0	0	0	0	0	0
$\Phi_{(5,1)}$	70	70	0	70	70	0	0	0	0	0	0	0	0	0
$\Phi_{(6,1)}$	14	14	14	14	14	14	0	0	0	0	0	0	0	0
$\Phi_{(7,1)}$	175	0	0	175	0	0	7	0	0	0	0	0	0	0
$\Phi_{(1,2)}$	100	0	0	0	0	0	0	100	0	0	0	0	0	0
$\Phi_{(2,2)}$	20	20	0	0	0	0	0	20	20	0	0	0	0	0
$\Phi_{(3,2)}$	4	4	4	0	0	0	0	4	4	4	0	0	0	0
$\Phi_{(4,2)}$	50	0	0	50	0	0	0	50	0	0	50	0	0	0
$\Phi_{(5,2)}$	10	10	0	10	10	0	0	10	10	0	10	10	0	0
$\Phi_{(6,2)}$	2	2	2	2	2	2	0	2	2	2	2	2	2	0
$\Phi_{(7,2)}$	25	0	0	25	0	0	1	25	0	0	25	0	0	1

Table(3,10)

4. Factor Group AC(G):

In this part, we aim to explain the basic definitions and important theories of the group AC(G).

Definition (4.1):[6]

The finite factor abelian group AC(G)= $\overline{R}(G)/T(G)$ is called **Artincokernel of G** ,in which T(G) is the subgroup of $\overline{R}$(G) generated by (Atc) and T(G) is a normal subgroup of $\overline{R}$(G).

Definition (4.2.): [7]

Let A be a matrix with entries in a principal ideal domain R . **A k – minor of A** is the determinate of k×k sub-matrix preserving row and column order .

Definition (4.3):[7]

A k-th determinant divisor of A is the greatest common divisor of all the k – minor, this is denoted by $D_K(A)$.

Theorem (4.4):[7]

Let Abe ann×n matrix with entries in a principal ideal domain R,
then there exist matrices B and C such that :

1 - B andC are invertible.
2 - BAC = D.
3 - D is a diagonal matrix.
4 -If we denote D_{ii} by d_i then there exists a natural number m ; 0 ≤ m ≤ n such that j > m implies d_j =0 and j ≤ m implies $d_j \neq 0$ and 1 ≤ j ≤ m implies $d_j \mid d_{j+1}$.

Definition (4.5): [7]

Let A be matrix with entries in a principal ideal domain R such that A is equivalent to matrixD = diag $\{d_1, d_2, \ldots, d_m, 0, 0, \ldots, 0\}$ where $d_j \mid d_{j+1}$ for 1 ≤ j < m.We call D **the invariant factor matrix of**A and $d_1, d_2, \ldots, d_m$ the invariant factors of A.

Remark (4.6): [6]

Let l be the number of all distinct Γ- Cof G then Ar(G) and $\overset{*}{\equiv}$(G)are of rank l. According to the Artin's theorem there exists an invertible matrix A^{-1}(G) with entries in Q such that :

$$(G) = A^{-1}(G) .Ar (G)\overset{*}{\equiv}$$

$$A (G) = Ar(G) . (^{-1}(G))^{-1}$$

and this implies

A (G) is the matrix expressing the T(G) basis in terms of the $\overline{R}$(G) basis. By Theorem (3. 4), there exist two matrices B(G) and C(G) with determinant ± 1 such that:

B(G). A(G).C(G) = diag $\{d_1, d_2, \dots, d_j\}$= D (G) where $d_i = \pm D_i(G)/D_{i-1}(G)$ This process yields a new basis for T(G) and $\overline{R(G)}$,$\{v_1, v_2, \dots, v_l\}$ and $\{u_1, u_2, \dots, u_l\}$ respectively, with the property $v_j = d_j u_j$.

Theorem (4.7):[6]

AC(G) = $\bigoplus_{i=1}^{l} C_{d_i}$ in which $d_i = \pm D_i(G) / D_{i-1}(G)$ and l is the number of all distincΓ-c ofG.

Corollary (4.8):[6]

|AC(G) |=| det (A(G))| .

Lemma (4.9):[6]

Let Sof rankℓ and Y of rank ℓ be two invertible matrices , over a principal ideal domain R and let: $B_1 S C_1$ =

D(S) = diag $\{d_1(S), d_2(S), \dots, d_l(S)\}$ And

$B_2 Y C_2$ = D(Y) = diag $\{d_1(Y), d_2(Y), \dots, d_m(Y)\}$ the invariant factor matrices of S and Y then: $(B_1 \otimes B_2)(S\otimes Y)(C_1 \otimes C_2) = D(S) \otimes D(Y)$ and from this the invariant factor matrices of

S⊗ Y can be written dig (S⊗Y)=$d_1(S).d_1(Y), d_2(S).d_2(Y), \dots\dots\dots, d_l(S).d_m(Y)$

Lemma (4.10):[6]

If H_1 and H_2 are two matrices of degree m and t respectively, then:

det $(H_1\otimes H_2) = (\det(H_1))^t . (\det(H_2))^m$.

Proposition(4.11):[7]

Let H_1 and H_2 be two p-groups then the matrix which expresses the $T(H_1\times H_2)$ basis of $\overline{R}(H_1\times H_2)$ basis is $A_1\otimes A_2$.

5. The Cyclic Decomposition of AC($Q_{2l^2}\times C_7$) where ,l≠ 2 and l is (pr)

In this section we will study of the cyclic decomposition of AC($Q_{2l}\times C_7$) where ,l≠ 2 and l is (pr).

Proposition (5. 1): [8]

If l is a (pr) and s is a (pin), then

$$A(C_{l^s}) = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 1 & \cdots & 1 \\ 0 & 0 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}$$

Capacity square matrix of (s+1)×(s+1).

Proposition(5.2):[9]

In general, the general formula for the matrices $B(C_{l^s})$ and $C(C_{l^s})$ are :

$$B(C_{l^s}) = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots \\ & & & & & \ddots & & \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

Capacity square matrix of (s+1)×(s+1).

D(C_{l^s})= diag $\underbrace{\{1,1,.......,1\}}_{s+1}$ and C(C_{l^s})= I_{s+1} , I_{s+1} is an identity matrix

Remark (5.3):[9]

In general if so $m = l_1^{\alpha 1} \cdot l_2^{\alpha 2} \cdot \cdots \cdot l_n^{\alpha n}$ where the biggest common denominator

(l_i , l_j)=1, if i ≠ j and for all i ,1≤ i ≤ n , $l_i \neq 2$ are (Pr) and α_i any (pin) is all i = 1, 2, … , n ;then :

$C_m = C_{l_1^{\alpha_1}} \times C_{l_2^{\alpha_2}} \times \dots \times C_{p_l^{\alpha_n}}$,

1- By Proposition (4.11) we get:

$A(C_m) = A(C_{l_1^{\alpha_1}}) \otimes A(C_{l_2^{\alpha_2}}) \otimes \dots \otimes A(C_{l_n^{\alpha_n}})$.

Now you can writeA (C_m)as :

$$A(C_m) = \begin{bmatrix} & & & & & 1 \\ & & R(C_m) & & & 1 \\ & & & & & \vdots \\ & & & & & 1 \\ 0 & 0 & 0 & \dots & 0 & 1 \end{bmatrix}$$

So that R(C_m)is a matrix we obtained by deleting the last column {1,1,...,1} and the last row

{0, 0,...,0, 1}fromthe tensor product,$A(C_{l_1^{\alpha_1}}) \otimes A(C_{l_2^{\alpha_2}}) \otimes \dots \otimes A(C_{l_n^{\alpha_n}})$,Where A ($C_m$) is of order,

[(α_1 +1)(α_2 +1)… (α_n +1) × (α_1 +1)(α_2 +1)… (α_n +1)]square matrix.

2- Using Lemma (4.9) we get

a- $B(C_m) = B(C_{l_1^{\alpha_1}}) \otimes B(C_{l_2^{\alpha_2}}) \otimes \dots \otimes B(C_{l_n^{\alpha_n}})$.

b- $C(C_m) = C(C_{l_1^{\alpha_1}}) \otimes C(C_{l_2^{\alpha_2}}) \otimes \dots \otimes C(C_{l_n^{\alpha_n}})$.

Proposition (5.4):[10]

If $m = l_1^{\alpha 1} \cdot l_2^{\alpha 2} \cdot \cdots \cdot l_n^{\alpha n}$ where the biggest common denominator

(l_i , l_j)=1, if i ≠ j, l_i 's they are (pr), and α_i(pin), then

$$A(Q_{2m}) = \left[\begin{array}{ccccc|ccccc|c} & & & & 1 & & & & & 1 & 1 \\ & 2R(C_m) & & & 1 & & 2R(C_m) & & & 1 & 1 \\ & & & & \vdots & & & & & \vdots & \vdots \\ & & & & 1 & & & & & 1 & 1 \\ 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 1 & 1 \\ \hline & & & & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\ & 2R(C_m) & & & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\ & & & & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ & & & & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 1 & 0 \\ \hline 1 & 1 & \cdots & 1 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \end{array}\right]$$

Which is $\left[2(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_n+1)+1\right]\times\left[2(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_n+1)+1\right]$ square matrix , $R(C_m)$ It's like a matrix in observations (5.3).

Proposition (5.5):[10]

If $m = l_1^{\alpha 1}\cdot l_2^{\alpha 2}\cdot\cdots\cdot l_n^{\alpha n}$ where the biggest common denominator (l_i, l_j)=1, if i ≠ j and l_i 's be(pr) and α_i(pic), then:

1-The matrixB(Q_{2m})taking the form

$$B(Q_{2m}) = \left[\begin{array}{ccccc|ccccc|c}
 & B(C_m) & & & & & -B(C_m) & & & & 0 \\
 & & & & & & & & & & 0 \\
 & & & & & & & & & & \vdots \\
 & & & & & & & & & & 0 \\
 & & & & & & & & & & 0 \\
\hline
0 & 0 & \cdots & 0 & 0 & & & & & & 0 \\
0 & 0 & \cdots & 0 & 0 & & B(C_m) & & & & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & & & & & & \vdots \\
0 & 0 & \cdots & 0 & 0 & & & & & & 0 \\
0 & 0 & \cdots & 0 & 0 & & & & & & 0 \\
\hline
0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1
\end{array}\right]$$

2-The matrixC(Q_{2m})taking the form

$$C(Q_{2m}) = \left[\begin{array}{ccccc|ccccc|c}
0 & 0 & \cdots & 0 & 0 & & & & & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & & I_k & & & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & & & & & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 0 & & & & & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & -1 & -1 & -1 & -1 & 0 & 1 \\
\hline
 & & & & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
 & I_k & & & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
 & & & & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
 & & & & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & \cdots & 0 & 0 & 1 & 1 & \cdots & 1 & 1 & -1 \\
\hline
0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 & 0 & 0
\end{array}\right]$$

Where $k = (\alpha_1+1)\cdot(\alpha_2+1)\cdot(\alpha_3+1)\cdots(\alpha_n+1)-1$.

They are $\left[2(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_n+1)+1\right]\times\left[2(\alpha_1+1)\cdot(\alpha_2+1)\cdots(\alpha_n+1)+1\right]$square matrix .

6.The Main Results

In this section we findingthe $A(Q_{2l^2}\times C_7)$, the matrix $B(Q_{2l^2}\times C_7)$,the matrix $C(Q_{2l^2}\times C_7)$ and give the cyclic decomposition of the factor group $AC(Q_{2l^2}\times C_7)$ Whenl≠2 andl is (pr)**.**

Theorem (6.1):

If m =l^2 ; l≠2 ,l is (pr),then the matrix $A(Q_{2l^2}\times C_7)$ of the group$Q_{2l^2}\times C_7$ is:

$$A(Q_{2l^2}\times C_7) = \left[\begin{array}{c|c}
A(Q_{2l^2}) & A(Q_{2l^2}) \\
\hline
0 & A(Q_{2l^2})
\end{array}\right]$$

Which is [14]× [14] square matrix ,$A(Q_{2l^2})$ is similar to the matrix in Proposition (5.4).

Proof :

By definition of A(G) we find the matrix $A(Q_{2l^2}\times C_7)$:

$A(Q_{2l^2}\times C_7) = Ar(Q_{2l^2}\times C_7)\,.(\equiv^*(Q_{2l^2}\times C_7))^{-1}=$

$$\left[\begin{array}{ccccccc|cccccc}
2 & 2 & 1 & 2 & 2 & 1 & 12 & 2 & 1 & 2 & 2 & 1 & 1\\
0 & 2 & 1 & 0 & 2 & 1 & 10 & 2 & 1 & 0 & 2 & 1 & 1\\
0 & 0 & 1 & 0 & 0 & 1 & 10 & 0 & 1 & 0 & 0 & 1 & 1\\
2 & 2 & 1 & 0 & 0 & 1 & 02 & 2 & 1 & 0 & 0 & 1 & 0\\
0 & 2 & 1 & 0 & 0 & 1 & 00 & 2 & 1 & 0 & 0 & 1 & 0\\
0 & 0 & 1 & 0 & 0 & 1 & 00 & 0 & 1 & 0 & 0 & 1 & 0\\
1 & 1 & 1 & 0 & 0 & 0 & 01 & 1 & 1 & 0 & 0 & 0 & 0\\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 02 & 2 & 1 & 2 & 2 & 1 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 00 & 2 & 1 & 0 & 2 & 1 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 00 & 0 & 1 & 0 & 0 & 1 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 02 & 2 & 1 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 00 & 2 & 1 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 00 & 0 & 1 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 01 & 1 & 1 & 0 & 0 & 0 & 0
\end{array}\right]$$

Which is [14]× [14] square matrix

$$= \left[\begin{array}{c|c} A(Q_{2l^2}) & A(Q_{2l^2}) \\ \hline 0 & A(Q_{2l^2}) \end{array}\right]$$

Which is [14]× [14] square matrix

Proposition (6.2)

If m =l^2 ; and l is (pr) , then the matrix $B(Q_{2l^2}\times C_7)$ and the matrix $C(Q_{2l^2}\times C_7)$ of the group $Q_{2l^2}\times C_7$ are :

$$B(Q_{2l^2}\times C_7)= N\,.\left[\begin{array}{c|c} 0 & B(Q_{2l^2}) \\ \hline B(Q_{2l^2}) & -B(Q_{2l^2}) \end{array}\right]$$

Which is14×14square matrix.

$$C(Q_{2l^2}\times C_7)= \left[\begin{array}{c|c} 0 & C(C_{2l^2}) \\ \hline C(Q_{2l^2}) & 0 \end{array}\,.\,N\right]$$

Which is14×14square matrix.

$$\text{WhereN=}\left[\begin{array}{ccc|ccccccccccc}
1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{array}\right]$$

Which is14×14square matrix.

Proof:

By Theorem (6.1)we get in the form of A ($Q_{2l^2}\times C_7$)and above form B($Q_{2l^2}\times C_7$) and C($Q_{2l^2}\times C_7$) then: B($Q_{2l^2}\times C_7$)·A($Q_{2l^2}\times C_7$)·C($Q_{2l^2}\times C_7$)=diag{2, 2, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1}=D($Q_{2l^2}\times C_7$) Which is 14×14 square matrix.

Example(6.3)

To find the matrices B($Q_{2.3^2}\times C_7$) and C($Q_{2.3^2}\times C_7$) by Proposition (5.5) to find B($Q_{2.3^2}$) and C($Q_{2.3^2}$) :

$$B(Q_{2.3^2}) = \begin{bmatrix} 1 & -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad C(Q_{2.3^2}) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Then by Proposition (6.2):

$$B(Q_{2.3^2}\times C_7) = N \cdot \left[\begin{array}{c|c} 0 & B(Q_{2.3^2}) \\ \hline B(Q_{2.3^2}) & -B(Q_{2.3^2}) \end{array}\right] =$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 & -1 & 1 & 0 & 0 & -1 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & -1 & 1 & 0 & 0 & -1 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$C(Q_{2.3^2}\times C_7) = \left[\begin{array}{c|c} 0 & C(C_{2.3^2}) \\ \hline C(C_{2.3^2}) & 0 \end{array}\right] N =$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Theorem(6.4):

If m =l^2 ; l≠2 and l is (pr),then the cyclic decomposition of the factor group of

AC($Q_{2l^2}\times C_7$) is :

$$AC(Q_{2l^2}\times C_7))=\bigoplus_{i=1}^{8} C_2$$

Proof:

By Theorem (6.1),we find matrix $A(Q_{2l^2}\times C_7)$ and by proposition (6.2) ,we find $B(Q_{2l^2}\times C_7)$ and $C(Q_{2l^2}\times C_7)$:

$B(Q_{2l^2}\times C_7)\cdot A(Q_{2l^2}\times C_7)\cdot C(Q_{2l^2}\times C_7)=diag\{2,2,2,2,2,2,2,2,1,1,1,1,1,1\}$

Then, by theorem (4.7) we have:

$$AC(Q_{2l^2}\times C_7)=\bigoplus_{i=1}^{8} C_2$$

Example(7.5)

To find $D(Q_{18}\times C_7)$ and the cyclic decomposition of the factor group $AC(Q_{18}\times C_7)$.
By Proposition (5.4)we get in the form of $A(Q_{18})$

$$A(Q_{18})=\left[\begin{array}{ccccc|cc}
2 & 2 & 1 & 2 & 2 & 1 & 1\\
0 & 2 & 1 & 0 & 2 & 1 & 1\\
0 & 0 & 1 & 0 & 0 & 1 & 1\\
2 & 2 & 1 & 0 & 0 & 1 & 0\\
0 & 2 & 1 & 0 & 0 & 1 & 0\\
0 & 0 & 1 & 0 & 0 & 1 & 0\\
1 & 1 & 1 & 0 & 0 & 0 & 0
\end{array}\right]$$

By Theorem (6.1)we get in the form of $A(Q_{18}\times C_7)$

$$A(Q_{18}\times C_7)=\left[\begin{array}{c|ccccccc|cccccc}
2 & 2 & 1 & 2 & 2 & 1 & 1 & 2 & 2 & 1 & 2 & 2 & 1 & 1\\
0 & 2 & 1 & 0 & 2 & 1 & 1 & 0 & 2 & 1 & 0 & 2 & 1 & 1\\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1\\
2 & 2 & 1 & 0 & 0 & 1 & 0 & 2 & 2 & 1 & 0 & 0 & 1 & 0\\
0 & 2 & 1 & 0 & 0 & 1 & 0 & 0 & 2 & 1 & 0 & 0 & 1 & 0\\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0\\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0\\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 2 & 2 & 1 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 2 & 1 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0
\end{array}\right]$$

And we find the matrices $B(Q_{18}\times C_7)$ and $C(Q_{18}\times C_7)$ as in Example (6.3),then
$B(Q_{18}\times C_7)\cdot A(Q_{18}\times C_7)\cdot C(Q_{18}\times C_7)=diag\{2,2,2,2,2,2,2,2,1,1,1,1,1,1\}=D(Q_{18}\times C_7)$,

and by Theorem(6.4), we have $AC(Q_{18}\times C_7)=\bigoplus_{i=1}^{8} C_2$

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