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Research Paper

Profit Analysis of Redundant System with Two Types of Failure Modes

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ABSTRACT: *In this, we study two units redundant system with the concept of two types of failure, partial failure and complete failure. Practical examples of such systems can be seen in the electronics field. A repairman having knowledge of both types of repair with repair time distribution as general is available for repairs. The system stops working only when both of the units are in complete failure mode. Reliability analysis of the system is done to determine various measures by using Semi - Markov process and Regenerative Point Technique. Taking exponential failure time distribution and other general time distribution, the various measures are obtained to determine the effectiveness of the system. The conclusions profit from the system are carried out by Graphical studies using MATLAB.*

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KEYWORDS: *Mean Failure Time, Uptime, Busy Period, Availability, Regenerative Point Method*

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I. INTRODUCTION

In the early fifties i.e. after the world war-II, in several areas mainly life testing electronic and missiles, main attention of both the engineers and statisticians was reliability. Studies, researchers found the reliability of the system and other measures like availability, regenerative point technique.[1] and [4]. Nakagawa and Osaki [5] discussed repair and preventive maintenance of a two-unit standby redundant system. Singh [10],We, generally, use the word "reliable" for a person, equipment or the statement of a person in our daily life. Showing this property of a person or equipment the word 'reliability' is used. This paper is to explain the current applications of Reliability in Daily life. Gupta and Goel [6] explained administrative delay in repair of a two-unit priority standby system. Taneja, G. [7] discussed some reliability models with different types of failure and repair with Stochastic and profit analysis. Tuteja, R.K. and Taneja. G. [8] explained Profit analysis of one-unit system with partial failure with random inspection.

Yakubu et al. [12] explained multi hardware software system with failure interaction. Agnihotri and Satsangi [9] explained the concept of finding optimum change in time of units so that the operative unit can be given rest. In his paper, to increase the reliability of the system, there is interchange of the operative and the standby units after the optimum interchangement time. Yusuf et al. (13) explained Reliability analysis of communication network system with redundant relay station under partial and complete failure. Bhatia and Kumar [11] analysed Centrifuge System with neglected faults in Thermal Power Plant and calculated Performance and Profit measures. In many practical situations, for example dim light concept, partial failure concept is used. This study explains two identical units redundant system. First unit undergoes two types of failure, partial failure and complete failure while second unit has only one type of failure. The following measures are obtained which gives the effectiveness of the system. Mean (Average) Time to System Failure (MTSF), Steady State Availability of the System, Expected Number of Visits by the Repairman, Expected Busy Period of Repairman for both partially failed unit and completely failed unit, Expected Profit obtained. The conclusions about MTSF, Availability and profit from the system are carried out by Graphical studies

II. MODEL AND STATES DESCRIPTION

1. There are two units in this system. Initially one is operative unit and other unit is in cold standby.
2. The first unit of the system has three modes, that is, operative mode, partial failure mode and failure mode.
3. Second unit of the system has only one type of failure.
4. The distribution time of failure of the unit is exponential, although repair time distribution by the repairman is arbitrary.
5. Repairman is always available with the system at all times.
6. After repair, unit functions perfectly.
7. Breakdown of the system occurs if both of the units are under repair.
8. A unit works properly after every repair.
9. Each of the random variable is independent.

1. Transition Diagram

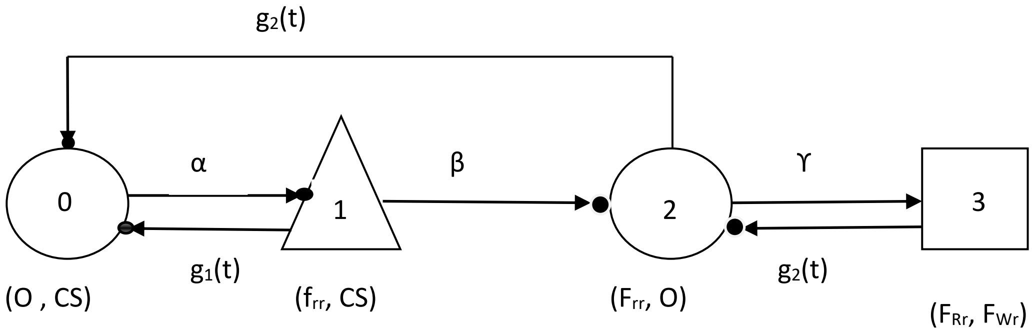

Diagram 1(Transition Diagram – The Model)

Good State   Partially Failed State   Down State   Regenerative Point

S_0 : (O , CS) **:** First unit is operative and Second is cold standby

S_1 : (f_{rr}, CS) **:** First unit is partially failed, under repair and Second unit is cold standby

S_2 : (F_{rr}, O) First unit is failed, under repair of regular repairman and Second unit is operative

S_3 : (F_{Rr}, F_{Wr}) **:** First unit is failed, under repair of regular repairman and Second unit is failed, waiting for repair

III. STEADY STATE EQUATIONS

The direct transition probabilities from the state S_i to S_j are :

$$Q_{0,1}(t) = \int_0^t \alpha e^{-(\alpha)u}\, du \qquad Q_{1,0}(t) = \int_0^t e^{-(\beta)u} g_1(u)\, du$$

$$Q_{1,2}(t) = \int_0^t \beta\, e^{-(\beta)u}\, \overline{G_1(u)}\, du \qquad Q_{2,0}(t) = \int_0^t e^{-(\gamma)u}\, g_2(u)\, du$$

$$Q_{2,3}(t) = \int_0^t \gamma e^{-(\gamma)u}\, \overline{G_2(u)}\, du \qquad Q_{2,2}^{(3)}(t) = \int_0^t e^{-(\gamma)u}\, \overline{G_2(u)}\, \left[\int_u^t \frac{g_2(v)}{\overline{G_2(v)}} dv\right] du$$

(1-6)

The Steady State Transition Probabilities are

$$P_{0,1} = 1 \qquad P_{2,0} + P_{2,3} = 1 \qquad P_{2,0} + P_{2,2}^{(3)} = 1$$

(7-9)

Mean Sojourn Time (μ_i) in regenerative state I is defined as the stay time in that state before transits to any other state.

$$\mu_0 = \frac{1}{\alpha} \qquad \mu_1 = \frac{1 - g_1^*(\beta)}{\beta} \qquad \mu_2 = \frac{1 - g_2^*(\gamma)}{\gamma}$$

(10-12)

The unconditional mean time taken by the system to go from regenerative state j, it is measured from span of entrance into the state i

$$m_{0,1} = \mu_0 \qquad m_{1,0} + m_{1.2} = \mu_1 \qquad m_{2,0} + m_{2.3} = \mu_2$$

$$m_{2,0} + m_{2,2}^{(3)} = K' \quad \text{Where} \quad K' = -g_2^{*'}(0)$$

(13-16)

3.1 ANALYSIS OF SYSTEM PERFORMANCE

Different measures of the system performance are obtained by solving above probabilities and recursive relations obtained.

Mean time to system failure (T_0) = N/D

Where

$N = \mu_1 + P_{1,2}\mu_2 + \mu_0$

$D = P_{1,2}P_{2,3}$

Steady state Availability (A_0) = N_1/D_1

Where

$N_1 = P_{2,0}\mu_0 + P_{2,0}\mu_1 + P_{1,2}\mu_2$

$D_1 = P_{2,0}\mu_0 + P_{2,0}K + P_{1,2}K'$

Busy Period Analysis of the repairman (B_0)= N_2/D_1

Where

$N_2 = P_{2,0}K + P_{1,2}K' \qquad K = -g_1^{*'}(0), \; K' = -g_2^{*'}(0)$

$D_1 = P_{2,0}\mu_0 + P_{2,0}K + P_{1,2}K'$

Expected Number of Visits by the Repairman (V_0) = N_3/D_1

Where

$N_3 = P_{0,1}\, P_{2,0}$

$D_1 = P_{2,0}\mu_0 + P_{2,0}K + P_{1,2}K'$ (17-22)

3.2 COST PROFIT ANALYSIS

The expected profit of the system is

$P = C_0A_0 - C_1B_0 - C_2V_0$ (23)

Where

C_0 = Revenue per unit up time of the system

C_1= Cost per unit time for which repairman is busy

C_2= Cost per visit by the repairman

IV. RESULTS AND DISCUSSION

4.1 PARTICULAR CASE

For graphical analysis, we consider the following particular case

$$g_1(t) = \delta e^{-\delta t}$$

$$g_2(t) = \lambda e^{-\lambda t}$$

Where δ and λ are repair rate of first and second unit respectively.

Thus, we have

$$P_{0,1} = 1 \qquad P_{1,0} = \frac{\delta}{\beta+\delta}$$

$$P_{1,2} = \frac{\beta}{\beta+\delta}$$

$$P_{2,0} = \frac{\lambda}{\gamma+\lambda} \qquad P_{2,3} = \frac{\gamma}{\gamma+\lambda}$$

$$P_{2,2}^{(3)} = \frac{\gamma}{\gamma+\lambda}$$

$$\mu_0 = \frac{1}{\alpha} \qquad \mu_1 = \frac{1}{\beta+\delta} \qquad \mu_2 = \frac{1}{\gamma+\lambda}$$

4.2 Numerical Study and Graphical Analysis

Considering the following numerical values:

α = 0.005, β = 0.005, γ = 0.005, δ = 0.03,λ = 0.02

Different measures of system effectiveness are

Mean time to system failure (MTSF) (T_0) = 490

Steady State Availability is (A_0) = 0.9290

Busy Period Analysis of the repairman (B_0)= 0.1913

Expected Number of Visits by the Repairman (V_0) = 0.0045

Figure 1 shows the pattern of profit w.r.t failure rate (α) for different fixed values of repair rate (δ). The profit decreases with increase in the values of failure rate (α).

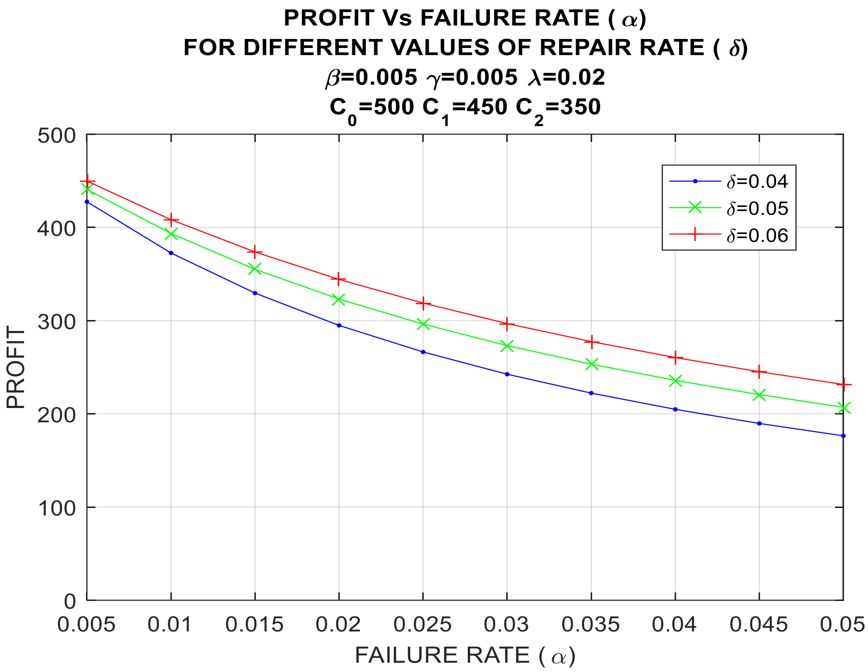

Figure 1 (Profit Versus Failure Rate)

Figure 2 shows the pattern of profit with respect to repair rate (δ) for different values of failure rate (α). The profit increases with increase in the values of repair rate (δ).

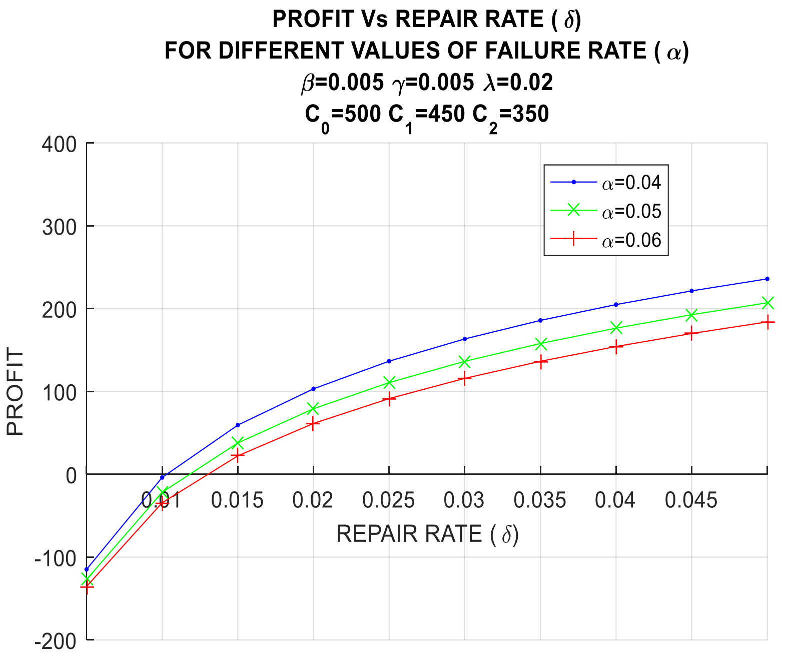

Figure 2 (Profit Versus Repair Rate)

Figure 3 shows the pattern of profit P w.r.t revenue per unit up time of the system (C_0) for different values of cost per unit time for which repairman is busy(C_1). It is clear from the graph that there is increase in profit as revenue up time of the system (C_0) increases for fixed values of (C_1). From the figure it may be observed that for $C_1 = Rs.300$, the profit is >(greater than) or =(equal to) or (less than) < 0 according as C_0 is > or = or < Rs. 650. Hence the system is profitable to the company whenever $C_0 \geq Rs.650$. Similarly for $C_1 = Rs.400$ and $C_1 = Rs.500$ resp. the profit is > or = or < 0 according as C_0 is > or = or < Rs. 860. and Rs. 1075 resp. Hence the company will be in profit with this system whenever $C_0 \geq$ Rs. 860. and Rs. 1075 resp.

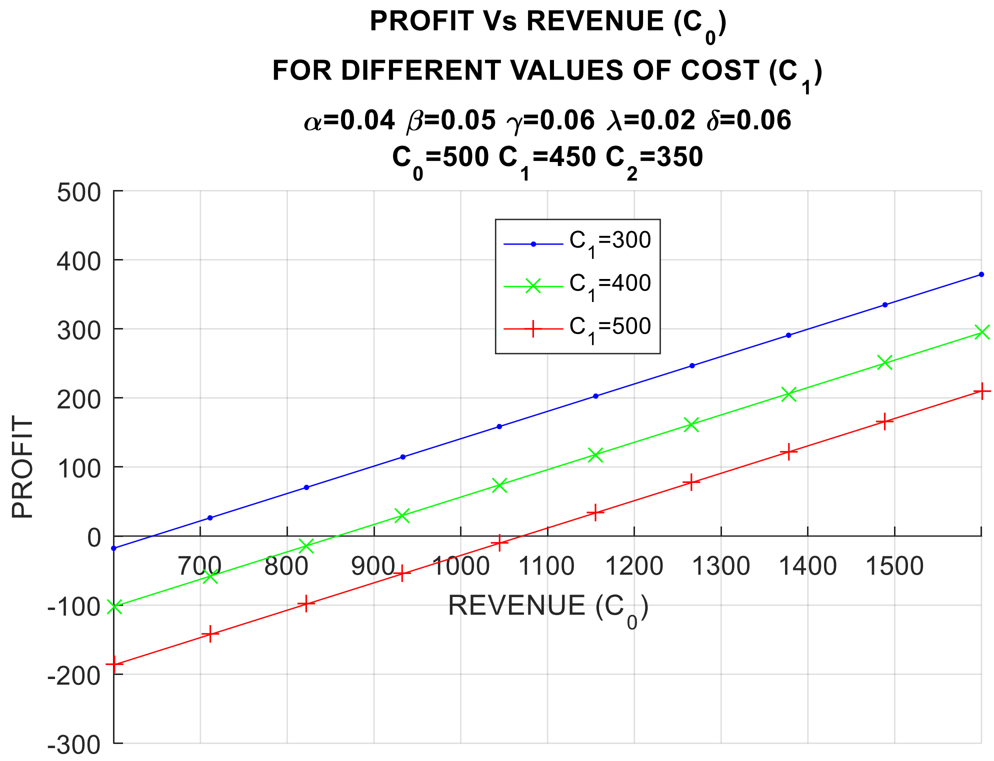

Figure 3 (Profit Versus Revenue

V. CONCLUSION

From the analysis of graphs, it can be observed that there is decrease in MTSF & Profit as the values of failure rate increases. Thus it can be concluded that lesser the rate of failure, higher the reliability and profit of the system. The cut off points for the profit of the system w.r.t revenue per unit up time for cost per unit time for which repairman is busy are also obtained. This would help to decide how much be the revenue per unit up time to have positive profit from the system.

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