“Optimization of Economical Aspects of Sea-Borne Transportation of Compressed Natural Gas”

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ABSTRACT:- To successfully implement a marine CNG project, it is very necessary to optimize the transformational expenses. This paper presents a decision-based scheme which is developed to improve the efficiency and competitiveness of marine CNG shipping. The major concern of this research is to minimize the transportation expenses, when natural gas is transported from a few production sources to certain recipients following the sea-borne mode of transportation. This goal has been achieved by using Hub-and-Spoke pattern of CNG distribution. This pattern follows the application of Transportation Problem Algorithm to find out the initial feasible solution and then this initial solution is further optimized by applying the Modified Distribution Method. The calculations show that the proposed algorithm efficiently breaks down the transformational costs up to 30%. Five different study cases has been taken into account to check the efficiency of the proposed algorithm.

The Hub-and-Spoke scheme of CNG distribution involves a large number of iterations and complex calculations. To make the work quicker and easier, a C++ tool has been developed in this project.

Keywords:- Hub-and-Spoke Scheme, Logistics, Modified Distribution Method, Marine CNG, Optimization, Transportation Problem.

I. INTRODUCTION

The worldwide consumption of natural gas is increasing rapidly. Besides the U.S., Europe, Korea and Japan historically, the leaders in natural gas consumption and whose demand will continue to increase significantly, the fast evolving large Asian economies such as China and India will definitely become new players in this rapidly expanding market. The dominant exporter of natural gas is and will be by far Russia, with its leading position in proved reserves (1,680 Tcf, about 50,000 Bcm) and production (over 23 Tcf, about 650 Bcm). [1].

There are many possible technologies of transporting gas from production fields to consumers elsewhere as a fuel or as a chemical feedstock in a petrochemical plant, where gas is converted into valuable products. The methods for transportation of natural gas include Pipelines (PNG), Liquefied Natural Gas (LNG), Compressed Natural Gas (CNG), Gas to Hydrates (GTH), Gas to Liquids (GTL), Gas to Commodity (GTC) such as glass, cement or iron and Gas to Wire (GTW) i.e. electricity.

The CNG transportation is not new, nor is the technology being introduced to it, but what is new is the application of modern technologies into a CNG marine based system and the increased volumes of CNG proposed to be transported. There are numerous methods to distribute natural gas with the help of marine CNG transportation. The main difference lies upon relation to the type of corresponding way, which depends mainly on demand size at each receiving point, and as well as their relative geographical locations. The two well-known schemes are so-called hub and spokes and the milk run pattern. [3]. The first is good for places with a relatively big consumption demand and can be better handled by the medium-sized vessels and also by developing a storage terminal on the hub and receiving points. On contrary, the milk-run pattern is forced to run, when demand of natural gas consumption is very slow. The transportation will be done with the help of a small ship in a repeatedly cyclic method. In this case, the creation of storage facility at each receiving is compulsory in order to provide the desired amount of gas to be consumed until another CNG ship visits the reception end.

The other major difference between the two methods is that, when a single CNG production source supplies gas to multiple recipients, the Milk-Run (M-R) Pattern distributes the CNG in a much efficient way;
while, if there are more than one CNG production sources and the number of recipients is greater than production sites, the Hub-and-Spoke (H-a-S) Pattern is the best solution. [4].

In presented paper, we will apply Hub-and-Spoke pattern of CNG Distribution to transport natural gas from a few production sources and certain number of recipients.

II. THE HUB-AND-SPOKESCHEM

In the hub-and-spoke scheme, certain sources of natural gas (hub) serve the number of receiving sites (the spokes) greater than production sites, each served by one or more cycles (circuit) of ships. Each ship from which the gas is being offloaded at the receiving point also serves as a temporary floating storage site during offloading. Extra storage facilities are not used at destination points.

If no storage facility is available at destination, at least, two ships are mandatory for continuous delivery. One of them continues to offload, when the second one completes next cycle. As shown in the following diagram. Once the first ship has offloaded the gas which it has transported to the destination site (gray top bar), it returns, re-loads and travels back, while during this time, the second ship continues to offload at the destination (below the gray bar). Periods of loading of both vessels do not coincide with each other.

The figure below explains this process more comprehensively:

![Figure 1: Scheduling of CNG delivery from a single source to a single delivery point using H-a-S method](image)

**Vessel 2:**
- i) Disconnection from unloading site / receiving site (Black bar)
- ii) Returns to the loading source site (white bar)
- iii) Connects to the source (black bar)
- iv) Gas loading (light blue bar)
- v) Disconnection from the source (black bar)
- vi) Travelling to the destination (white bar) and;
- vii) Connection to the receiving facility (black bar) and the offloading starts.

Once the offloading of first ship is completed (gray top bar), it repeats its cycle, during which the second and third ships continue unloading (middle and bottom gray bars, respectively).

It is important to mention that to ensure continuous gas supply, gas offloading rate $q_{\text{offload}}$ must be greater than the consumption rate, $Q$, at the consumer end. The first vessel completes its cycle for reloading; meanwhile, the other ships continue to offload successfully.

To make sure that the next ship is ready for unloading at the place of delivery after the previous vessel completes unloading. To achieve this goal, when one ship performs the above sequence of steps (i) to (vii) and is ready to start offloading, every other remaining ship should successively unload the transported gas at the receiving site.

To deal with such type of transportation, we will apply the Linear Programming Transportation Problem algorithm. The definition, mathematical formulation and explanation of transportation problem are explained next.

III. MATHEMATICAL FORMULATION OF THE PROBLEM

Let $x_{ij}$ be the amount of compressed gas transported from a source $i$ to one receiving destination $j$;

$$\text{minimize } \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

(1)

a) Production rate at each source must be according to as follows:

$$\sum_{j=1}^{n} x_{ij} = S_i \quad i = 1, 2, 3, \ldots, m$$

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b) At each receiving station, the demand or consumption rate must be met as follows:
\[
\sum_{i=1}^{m} x_{ij} = d_j, \quad j = 1,2,3 \ldots n
\]  
(2)

c) The overall transportation rate should be positive (non-negative):
\[
x_{ij} \geq 0, \quad i = 1,2,3 \ldots m; \quad j = 1,2,3 \ldots n
\]  
(3)

The total number of variables should be as \{m,n\};
The total number of constraints must be as \{m+n\}.

d) The requirements for flow balance during transportation are as follows:
\[
\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j
\]  
(4)

e) If supply rate is higher than demand, we must insert a dummy receiving destination in the following manner:
\( (d_{n+1}) \) is inserted as a slack-destination
\[
d_{n+1} = \Delta \text{ with } c_{j,n+1} = 0
\]  
(5)

f) If demand or consumption rate is much higher than supply rate, we need to insert a dummy source \( (S_{m+1}) \) to provide additional production source, in the following way:
\[
S_{m+1} = \Delta \text{ with } c_{m+1,j} = 0
\]  
(6)

IV. GRAPHICAL EXPLANATION OF THE PROBLEM
Suppose there are four CNG production sites and five different recipients with relatively bigger demand of gas. The distance from the production sources to the recipients and the production capacities and supply rates are as follows:

Figure 2: H-a-S Distribution pattern

V. TRANSPORTATION COST DETAILS
The cost of transportation ($) per MMscmd to each recipient and supply and demand rates are given below:

<table>
<thead>
<tr>
<th>Production Sources</th>
<th>Transportation cost USD per MMscmd</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D-I</td>
</tr>
<tr>
<td>S-I</td>
<td>20</td>
</tr>
<tr>
<td>S-II</td>
<td>14</td>
</tr>
<tr>
<td>S-III</td>
<td>18</td>
</tr>
<tr>
<td>S-IV</td>
<td>22</td>
</tr>
<tr>
<td>Demand</td>
<td>120</td>
</tr>
</tbody>
</table>

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VI. ORDINARY METHOD OF NATURAL GAS TRANSPORTATION

According to the ordinary transportation methods, the natural gas can be transported to the customers only in the following sequence:

1- The first source, S-I, will supply 120 MMscm of compressed gas to the first destination, D-I at $20/MMscm. And transports remaining 80 MMscm to the next destination, D-II, at $16/MMscm.
2- The second source, S-II, supplies 160 MMscm to the third destination, D-III at $16/MMscm.
3- S-III supplies 40 MMscm of gas to D-III at $14/MMscm and supplies 100 MMscm to D-IV at $20/MMscm.
4- S-IV supplies 180 MMscm to D-V at the rate of $18/MMscm.

Since, the demand has been transported to all customers, the transportation table looks as follows:

<table>
<thead>
<tr>
<th>Production Sources</th>
<th>D-I</th>
<th>D-II</th>
<th>D-III</th>
<th>D-IV</th>
<th>D-V</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-I</td>
<td>20</td>
<td>120</td>
<td>16</td>
<td>80</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>S-II</td>
<td>14</td>
<td>18</td>
<td>16</td>
<td>140</td>
<td>160</td>
<td>20</td>
</tr>
<tr>
<td>S-III</td>
<td>18</td>
<td>6</td>
<td>14</td>
<td>40</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>S-IV</td>
<td>22</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>Demand</td>
<td>120</td>
<td>8</td>
<td>200</td>
<td>100</td>
<td>180</td>
<td>180</td>
</tr>
</tbody>
</table>

Let’s calculate the overall cost of transportation.
Total cost = $20*120 + $16*80 + $16*140 + $14*40 + $20*100 + $18*180 = $12040.

VII. LINEAR PROGRAMMING TRANSPORTATION PROBLEM

We will solve the above problem in a two-step process:

**Step I:** First, we find out the Initial Feasible Solution (IFS). For this purpose, we will apply Vogel’s Approximation Method (VAM)/5/. Although there are many other methods available in literature but the IFS provided by VAM is either optimal or very close to final optimal solution.

**Step II:** After determining the IFS, we apply optimization to this initial solution to get the final optimal results. The IFS may or may not be Optimal. If the IFS is not Optimal, then it can be further improved to get a better result. The Modified Distribution Method (MODI) will be implemented for testing and improving the IFS.

The short description of VAM is given below:

VIII. VOGEL’S APPROXIMATION METHOD – VAM

Vogel’s Approximation Method usually gives the IFS, which is optimal or very close to the Optimal Final Solution. The following step by step procedure is required to solve a problem using VAM:

Suppose that, \( s_i \) be the supply quantity of the \( i \)th production source and \( d_j \) be the quantity of demand of the \( j \)th recipient destination and \( c_{ij} \) be the cost of transportation of unit item of \( i \)th source to \( j \)th destination.

**Step I:** Determine if \( s_i < 0 \) and \( d_j < 0 \), then Stop.

**Step II:** If \( \sum_i s_i > \sum_j d_j \) or if \( \sum_i s_i < \sum_j d_j \), then it’s an un-balanced problem and needs to be balanced by adding dummy supply or demand accordingly.

**Step III:**

a) Determine the minimum and next to minimum cost of each column and row which have highest penalty and assign maximum possible amount to \( x_{ij} \) i.e. \( \min(s_i, d_j) \). If the minimum cost appears in two or more cells in that column or row, then select arbitrarily.

b) If minimum cost appears in two or more times in a column or a row, then choosethesemidenticalcostsasasmallestandnexttosmallestcostandtherepenaltyvaluewillbezero.

**Step IV:** Choose \( \max(p_i, p_j) \). Mark the minimum cost of that column or row which has highest penalty and assign maximum possible amount to \( x_{ij} \) i.e. \( \min(s_i, d_j) \). If the minimum cost appears in two or more cells in that column or row, then select arbitrarily.

**Step V:** Correct the demand and supply and cancel out the satisfied column or row.

**Step VI:** If only one column or row, with zero demand or supply remains uncrossed, Stop the procedure. But if all uncrossed columns or rows have (remaining) zero demand or supply, apply LCM and stop. Otherwise, repeat the whole process from **Step III**.

By implementing the “Vogel’s Approximation Method - VAM” in our study case-II, we get the following values:

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### Iteration – I (VAM)

<table>
<thead>
<tr>
<th>Production Sources</th>
<th>Production &amp; demand capacities</th>
<th>Supply</th>
<th>Row Penalty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D-I</td>
<td>D-II</td>
<td>D-III</td>
</tr>
<tr>
<td>S-I</td>
<td>20</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>S-II</td>
<td>14</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>S-III</td>
<td>18</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>S-IV</td>
<td>22</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Demand</td>
<td>120</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>Column Penalty</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

### Iteration – II (VAM)

<table>
<thead>
<tr>
<th>Production Sources</th>
<th>Production &amp; demand capacities</th>
<th>Supply</th>
<th>Row Penalty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D-I</td>
<td>D-II</td>
<td>D-III</td>
</tr>
<tr>
<td>S-I</td>
<td>20</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>S-II</td>
<td>14</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>S-III</td>
<td>18</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>S-IV</td>
<td>22</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Demand</td>
<td>120</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>Column Penalty</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

### Iteration – III (VAM)

<table>
<thead>
<tr>
<th>Production Sources</th>
<th>Production &amp; demand capacities</th>
<th>Supply</th>
<th>Row Penalty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D-I</td>
<td>D-II</td>
<td>D-III</td>
</tr>
<tr>
<td>S-I</td>
<td>20</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>S-II</td>
<td>14</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>S-III</td>
<td>18</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>S-IV</td>
<td>22</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Demand</td>
<td>120</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>Column Penalty</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

### Iteration – IV (VAM)

<table>
<thead>
<tr>
<th>Production Sources</th>
<th>Production &amp; demand capacities</th>
<th>Supply</th>
<th>Row Penalty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D-I</td>
<td>D-II</td>
<td>D-III</td>
</tr>
<tr>
<td>S-I</td>
<td>20</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>S-II</td>
<td>14</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>S-III</td>
<td>18</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>S-IV</td>
<td>22</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Demand</td>
<td>120</td>
<td>0</td>
<td>200</td>
</tr>
<tr>
<td>Column Penalty</td>
<td>2</td>
<td>-</td>
<td>2</td>
</tr>
</tbody>
</table>

### Iteration – V (VAM)

<table>
<thead>
<tr>
<th>Production Sources</th>
<th>Production &amp; demand capacities</th>
<th>Supply</th>
<th>Row Penalty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D-I</td>
<td>D-II</td>
<td>D-III</td>
</tr>
<tr>
<td>S-I</td>
<td>20</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>S-II</td>
<td>14</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>S-III</td>
<td>18</td>
<td>6</td>
<td>14</td>
</tr>
<tr>
<td>S-IV</td>
<td>22</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Demand</td>
<td>120</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>Column Penalty</td>
<td>2</td>
<td>-</td>
<td>2</td>
</tr>
</tbody>
</table>

### Iteration – VI (VAM)

<table>
<thead>
<tr>
<th>Production Sources</th>
<th>Production &amp; demand capacities</th>
<th>Supply</th>
<th>Row Penalty</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D-I</td>
<td>D-II</td>
<td>D-III</td>
</tr>
<tr>
<td>S-I</td>
<td>20</td>
<td>16</td>
<td>18</td>
</tr>
<tr>
<td>S-II</td>
<td>14</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>S-III</td>
<td>18</td>
<td>20</td>
<td>14</td>
</tr>
<tr>
<td>S-IV</td>
<td>22</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Demand</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Column Penalty</td>
<td>2</td>
<td>-</td>
<td>4</td>
</tr>
</tbody>
</table>
After iteration VI, we see that the demand and supply values become zero. Therefore, the algorithm terminates here. Let’s now calculate the cost of transportation by this method.

The final table with values looks like as follows:

**Final cost table (VAM)**

<table>
<thead>
<tr>
<th>Production Sources</th>
<th>Production &amp; demand capacities</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>D-I</td>
</tr>
<tr>
<td>S-I</td>
<td>20 100</td>
</tr>
<tr>
<td>S-II</td>
<td></td>
</tr>
<tr>
<td>S-III</td>
<td>18 20</td>
</tr>
<tr>
<td>S-IV</td>
<td>8 80</td>
</tr>
</tbody>
</table>

**Cost of transportation**

- D-I = (20 * 100) + (18 * 20) = 2360 $.
- D-II = 8 * 80 = 640 $.
- D-III = (14 * 100) + (16 * 100) = 3000 $.
- D-IV = (10 * 100) = 1000 $.
- D-V = (8 * 160) + (12 * 20) = 1520 $.

Hence, the overall cost of transportation by applying existing VAM becomes (2360 + 640 + 3000 + 1000 + 1520) = 8520 $. We can see a big cost difference between ordinary method of transportation and Vogel’s Approximation Method. Now, we will apply optimization to IFS generated by VAM.

**IX. OPTIMALITY CHECK OF IFS**

By applying VAM, we get the IFS. Now we need to check, either this IFS obtained is feasible or not. There are two pre-conditions for an optimal solution:

1) The number of occupied cells should be equal to \( m + n - 1 \) [i.e. Total number of row+total number of columns-1];
2) The occupied cells should be at independent positions, i.e., they shouldn't form a closed loop with each other.

If the above two conditions are satisfied, Modified Distribution Method (MODI’s algorithm) is applied for optimization.

**X. MODIFIED DISTRIBUTION METHOD (MODI)**

The MODI method can be explained in following steps:

**STEP I:** Determine an initial feasible solution using any one of the three methods (NWCR, LCM, VAM);

**STEP II:** Calculate the values of dual variables, \( u_i \) and \( v_j \) using \( u_i + v_j = c_{ij} \);

**STEP III:** Compute the penalty values by using \( P_{ij} = u_i + v_j - c_{ij} \);

**STEP IV:** If the value obtained by \( u_i + v_j - c_{ij} \) is 0 or \(-ve\), the IFS is the final optimal solution. If there’s any positive value, the IFS isn’t optimal and needs further optimization;

**STEP V:** Select the highest positive value attained in \( u_i + v_j \) matrix of the vacant cells and select the lowest value near this matrix;

**STEP VI:** Draw a closed path or loop for vacant cell selected in \( u_i + v_j \) matrix;

**STEP VII:** Assign alternate \(+ve\ and \(-ve\ signs at each corner of the loop;

**STEP VIII:** Determine the maximum number of items that should be transported to this vacant cell. Now add this amount to all the corner cells of loop with \(+ve\ sign and subtract from those with \(-ve\ signs. Now, the vacant cell becomes occupied;

**STEP IX:** Repeat the whole procedure until an optimal solution is received.

Let’s apply these steps to our IFS achieved by VAM method. The final cost table obtained by VAM is given below:

<table>
<thead>
<tr>
<th>D-I</th>
<th>D-II</th>
<th>D-III</th>
<th>D-IV</th>
<th>D-V</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 100</td>
<td>16</td>
<td>18</td>
<td>10 100</td>
<td>26</td>
</tr>
<tr>
<td>14</td>
<td>18</td>
<td>16</td>
<td>20</td>
<td>8 160</td>
</tr>
<tr>
<td>18 20</td>
<td>6</td>
<td>14 100</td>
<td>20</td>
<td>12 20</td>
</tr>
<tr>
<td>22</td>
<td>8 80</td>
<td>16 100</td>
<td>6</td>
<td>18</td>
</tr>
</tbody>
</table>

Before proceeding the optimization, we need to do \( m + n - 1 \) check. Thus,

\[
m + n - 1 = 8 \]

\[
4 + 5 - 1 = 8 \]

\[
8 = 8
\]
Now we can proceed further,

**Iteration I (MODI)**

Apply the formula \( u_i + v_j = c_{ij} \) to find the values of \( u_i + v_j \) matrix.

<table>
<thead>
<tr>
<th>( u_1 = 0 )</th>
<th>( u_2 = 6 )</th>
<th>( u_3 = -2 )</th>
<th>( u_4 = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_1 = 20 )</td>
<td>16</td>
<td>18</td>
<td>14</td>
</tr>
<tr>
<td>( v_2 = 8 )</td>
<td>900</td>
<td>180</td>
<td>200</td>
</tr>
<tr>
<td>( v_3 = 16 )</td>
<td>20</td>
<td>16</td>
<td>80</td>
</tr>
<tr>
<td>( v_4 = 10 )</td>
<td>6</td>
<td>140</td>
<td>20</td>
</tr>
<tr>
<td>( v_5 = 14 )</td>
<td>20</td>
<td>800</td>
<td>1200</td>
</tr>
</tbody>
</table>

**Iteration II (MODI)**

Apply the formula \( P_{ij} = u_i + v_j - c_{ij} \) to compute penalty costs for vacant cells.

\[
\begin{align*}
C_{12} &= u_1 + v_j - c_{12} = 0 + 8 - 16 = -8; \\
C_{13} &= u_i + v_j - c_{13} = 0 + 16 - 18 = -2; \\
C_{14} &= u_1 + v_j - c_{14} = 0 + 14 - 26 = -12; \\
C_{21} &= u_i + v_j - c_{21} = -6 + 20 - 14 = 0; \\
C_{22} &= u_1 + v_j - c_{22} = -6 + 8 - 18 = -16; \\
C_{23} &= u_i + v_j - c_{23} = -6 + 16 - 16 = -6; \\
C_{24} &= u_1 + v_j - c_{24} = -6 + 10 - 20 = -16; \\
C_{32} &= u_i + v_j - c_{32} = -2 + 8 - 6 = 0; \\
C_{34} &= u_i + v_j - c_{34} = -2 + 10 - 20 = -12; \\
C_{41} &= u_i + v_j - c_{41} = 0 + 20 - 22 = -2; \\
C_{44} &= u_1 + v_j - c_{44} = 0 + 6 - 10 = -4; \\
C_{45} &= u_i + v_j - c_{45} = 0 + 14 - 18 = -4
\end{align*}
\]

Penalty costs are all negative values. It means that this IFS can’t be further optimized and solution provided by VAM is the final optimal solution. Hence, the overall cost of transportation by applying MODI method remains unchanged (i.e. \( 2360+640+3000+1000+1520 \) = \( 8520 \) $).  

We can see that there’s a significant cost difference between ordinary methods of calculation and the cost obtained by applying Transportation algorithm. This algorithm has been applied to more than 20 other study cases and compared with ordinary calculations. The results of calculations are shown below in the table.

<table>
<thead>
<tr>
<th>Study Cases</th>
<th>Method Applied</th>
<th>CNG Production Sources</th>
<th>CNG Recipient Sites</th>
<th>Transporational Expenses (Million USD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ordinary Calculations</td>
<td>4</td>
<td>5</td>
<td>120340</td>
</tr>
<tr>
<td></td>
<td>Transportation Algorithm</td>
<td></td>
<td></td>
<td>8520</td>
</tr>
<tr>
<td>2</td>
<td>Ordinary Calculations</td>
<td>5</td>
<td>6</td>
<td>13570</td>
</tr>
<tr>
<td></td>
<td>Transportation Algorithm</td>
<td></td>
<td></td>
<td>9024</td>
</tr>
<tr>
<td>3</td>
<td>Ordinary Calculations</td>
<td>5</td>
<td>7</td>
<td>14740</td>
</tr>
<tr>
<td></td>
<td>Transportation Algorithm</td>
<td></td>
<td></td>
<td>8190</td>
</tr>
<tr>
<td>4</td>
<td>Ordinary Calculations</td>
<td>6</td>
<td>7</td>
<td>15670</td>
</tr>
<tr>
<td></td>
<td>Transportation Algorithm</td>
<td></td>
<td></td>
<td>9130</td>
</tr>
<tr>
<td>5</td>
<td>Ordinary Calculations</td>
<td>6</td>
<td>8</td>
<td>19926</td>
</tr>
</tbody>
</table>

**XI. SOLVING TRANSPORTATION PROBLEM USING C++ LANGUAGE**

To solve the above problem, a flow chart for each algorithm was designed. After designing algorithms for VAM & MODI method, we developed C++ tool for solving transportation model in Linear Programming. The C++ language was used for solving much difficult problems which take long time using LP solution. After implementing these programs, we made a comparison between each solution using C++ program and LP solution for choosing minimum value of the objective function. The main outputs by designing C++ programs are save time, money, and effort.

In our study cases, (i.e. CNG Transportation), we use the C++ programs to minimize the cost of shipment and determine the number of MMscmd to be transported from source i to destination j. The results are shown above in table.

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The result of VAM method using C++ language is the cost of transportation =8520. The quantity of MMScm transported from source $i$ to recipient $j$:

we transport,
supply [1] to demand [1] =100

Press any key to continue.

The results of the VAM using C++ language are same as to ordinary LP solution but the process was much faster and easier using C++ language as compared to LP solution.

Implementing the C++ tool for dealing with transportation problem shows that the results of C++ programs are exact the same as compared to result of the LP solution. Therefore, the decision maker may select the optimal solution by using this tool and compute the quantity of MMScm to be transported from source $i$ to destination $j$.

This tool was developed by using MIT/X11 (9.02 version).

The complete script for transportation tool is given below:

```c
Code:
 /***************************************************************************/
 /*          Title:  Transportation Problem
 /*          Author: Muhammad Akram Khan (Ph.D.)
 /*          University: WarsawUniversityofTechnology
 /*          Mail-ID: akram_khan@is.pw.edu.pl
 */
***************************************************************************/
#include<stdio.h>
intflag=0,flag1=0;
int s[10],d[10],sn,eop=1, dm,a[10][10];
main()
 {
 inti,j,sum=0,min,x[10][10],k,fa,fb;
 printf("EnterthenumberofSupply
 clrscr();
 /* GettingTheInputFortheProblem*/
 scanf("%d",&sn);
 ");
 scanf("%d",&dm);
 for(i=0;i<sn;i++)
 printf("EntertheSupplyValues
 for(j=0;j<sn;i++)
 scanf("%d",&s[i]);
 ");
 printf("Enterthe Demand Values
 for(i=0;i<dm;i++)
 scanf("%d",&d[i]);
 ");
 printf("EntertheSupplyValues
 printf("Enterthe Demand Values
 for(j=0;j<sn;j++)
 scanf("%d",&d[j]);
 {
 for(j=0;j<dm;j++)
 {
 scanf("%d",&a[i][j]);
 }
 printf("Enterthe elementsof thearray
 ```

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for(i=0;i<sn;i++)
for(i=0,j=0;i<sn,j<dm;)
i=0;j=0;
/* Calculation For the Transportation */
{
    if(s[i]<d[j]) demand // Calculate amount * supply
    {
        d[j]=d[j]-s[i]; // Calculated demand - supply
        x[i][j]=a[i][j]*s[i]; // Checks supply less than
    }
}

Given Cost Matrix is:
20 16 18 10 26
14 18 16 20 8
18 6 14 20 12
22 8 16 6 18

Allocated Cost Matrix is
100 0 0 100 0
0 0 0 0 160
20 0 100 0 20
0 80 100 0 0

The Transportation cost: 8520

The following equation helps in selecting the appropriate vessels when transporting CNG by using H-a-S Pattern.

XII. DETERMINING THE CAPACITY OF SHIP (C-S) FOR H-A-S PATTERN

If the gas consumption rate at the receiving point is relatively higher, the Hub-and-Spoke pattern is used to determine the above mentioned parameters. As has been described (Nicholaou 2010), maximum loading and unloading rates are determined as follows as:

\[ Q = 2 \cdot \frac{4tc + \frac{2L}{V}}{\frac{1}{Q_{off}} - \frac{1}{Q_{load}}} = 2 \cdot \frac{T}{\frac{1}{Q_{off}} - \frac{1}{Q_{load}}} \]  

(8) [4]

Where,
\( t_c \) = connecting/disconnecting time (h)
\( L \) = distance from source to receiving site (nm)
\( V \) = ship service speed (kn)
\( Q_{off} \) = maximum offloading rate (mm/scf/h)
\( Q_{load} \) = maximum loading rate (mm/scf/h)
and where \( T = 4TC + L / V \) shows the total time of cycle from supply site to destination and return after loading the gas plus the total time of connection / disconnection of the vessel to / from terminals.

XIII. CONCLUSIONS

Hub-and-Spoke Method can only be used when there is relatively very big demand of CNG at the receiving destination. Other conclusions can be derived from equations mentioned above such as:

- More than one ship is needed to be employed. When one ship offloads, the other one completes its cycle and transports the gas;
- The storage capacities are not mandatory in this type of transportation;
- Multiple sources supply gas to a few recipients in order to make sure the continuous supply;
- By applying Transportation Problem Algorithm, the cost of transportation reduces up to 35-40%;
- VAM method provides the solution, which is optimal or very close to final optimal solution;
- MODI method is applied for checking the optimality of IFS, provided by VAM or other methods;
- The application of C++ tool makes the complex calculations easier and quicker.

*Corresponding Author: Dr. Diplo. Muhammad Akram Khan
REFERENCES


