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Research Paper

An Optimal Replenishment Policy for Retailer Inventory System With Refurbishing

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ABSTRACT: *Reverse logistics has been concerned and viewed as a new trend of research. This article investigates how the retailer can optimally determine the purchasing quantity and can optimally determine the return rate. The retailer purchases product to sell and collect used product to refurbish and resell. Total cost is minimized through optimal order quantity and return rate with refurbishing. Finally, some numerical examples are provided to illustrate the features of the proposed problem and the sensitivity analyses of the parameters with respect to the optimal solution are performed.*

Keywords: *Reverse logistics, Retailer, Order quantity, Return rate, Refurbishing*

I. INTRODUCTION

Due to rapid progress of technology, the life cycles of numerous products are shortened. In recent years, the environment is constantly changing, so the concerns of environmental issues are growing. Waste resource is not only an environmental problem, but also an economic loss. Teng et al. (2011) indicated that recycling is one of used-products recovery ways to protect our environment. Recycling of used product becomes more and more important to the industry (Wojanowski, et al., 2007). Industrial applications include photo copiers, tires, personal computers and et cetera. Reverse logistics has been concerned and viewed as a new area of research (Ferrer, 1997a; Ferrer, 1997b; Hart, 1997).

Over the last few decades, there has been an increase in the number of publications on reverse logistics. Ilgin and Gupta (2010) stated that reverse logistics involves all the activities associated with the collection and either recovery or disposal of used products. Reverse logistics has been concerned and viewed as a new area of research. Thierry et al. (1995) suggested five options of product recovery: repair, refurbishing, remanufacturing, cannibalization, and recycling. Repair: recycled used products to “working order.” Refurbishing: brought used products up to specified quality. Remanufacturing: brought used products up to quality standards those as good as new products. Cannibalization: recovered a finite set of reusable parts from used products or components. Recycling: reused materials from used products and components. Wei et al. (2015) indicated that reverse logistics process may offer companies a unique opportunity to improve their profits and to serve social responsibility. As resources are limited on earth, people pay much attention on environmental protection. Due to emerging consciousness for environmental protection, such as over-production resulting in resource waste and inappropriate disposal of used products causing environmental pollution, a large number of countries legislate to regulate that manufacturers must take responsibility for recycling, refurbishing, remanufacturing and disposal for used products. This trend promotes the development of reverse logistics. This paper is organized as follows. In the following section the literature is reviewed. In Section 3, the model development is described and then solution procedure is proposed to find the optimal order quantity and return rate in section 4. Finally, we conclusions are made in Section 5.

II. LITERATURE REVIEW

Schrady (1967) first analyzed the basic economic order quantity (EOQ) model with the concept of repair. He assumed that the demand rate and recovery rate of the product were constant, and the lead time of external purchase and recovery are fixed. A deterministic inventory model with reparable was proposed to decide the optimal procurement and repair quantities for the repairable products.

Several researchers have not only focused on inventory management of the new manufacturing products but also on that of remanufacturing ones. Teunter (2004) determined the optimal lot sizes for new and return products. He assumed remanufacturing used products as good as new and determined the return fraction and demand rate. Kim et al. (2006) proposed a general framework including two alternatives for supplying parts (purchasing from external suppliers and remanufacturing used products as new products), and developed a mathematical model using simulation to obtain the optimal solution.

Inventory systems with remanufacturing are conventionally studied from the perspective of seller. Few researchers examine the interactions between buyer and seller. Savaskan et al. (2004) developed a method with three options to choose the suitable reverse logistics structure for the collection of used products from consumers. The three kinds of options are (1) the seller collects used products directly from the customers, (2) the seller collects used products through the buyer, and (3) the seller collects used products via a third party. It contrasted the results for the three different kinds of collection method with the centrally coordinated system to explain potential sources of inefficiencies in close-loop supply chains. Savaskan and Van Wassenhove (2006) extends the model of Savaskan et al. (2004) considered a single retail change two competing retailers, and discussed the impact of remanufacturing. In contrast to the previous works, we develop a mathematical model for the inventory system with refurbishing from the perspectives of retailer. The retailer purchases product to sell and collect used product to refurbish and resell.

III. MODEL ASSUMPTIONS AND NOTATION DESCRIPTION

The goal of this paper is to develop an optimal replenishment policy for inventory system of retailer with refurbishing. The retailer purchases product to sell and collect used product to refurbish and resell. In order to incorporate collecting used-product activities into the basic EOQ model, the following notation and assumptions are utilized throughout this paper.

Table 1. Notation for retailer

q	Order quantity
t	The inventory system cycle length (decision variable)
D	Demand rate per unit per unit time, $q = Dt$
K	Fixed replenishment cost for retailer
h_n	Holding cost of new product per unit per unit time
h_c	Holding cost of collected and refurbished product per unit per unit time
t_r	The duration time to refurbishing returned products, $t_r=\tau t$
C_L	A scaling parameter for investment on collecting used product
c_n	Unit cost for manufacturing a new product
c_r	Unit cost for refurbishing a returned product
τ	Return rate of used product from customers (decision variable)

The following assumptions are made in developing the proposal model.

1. An infinite planning horizon.
2. No deterioration of production.
3. Shortages are not allowed.
4. Demand rate and return rate are known and constant.
5. No space, capacity or capital constraints.
6. No quantity discount.
7. No disposal.
8. The production rate is infinite.
9. The setup cost per run and inventory holding cost per unit per unit time are known and constant.
10. The inventory holding cost for return product is less than new one.
11. The refurbished products are as good as new products.

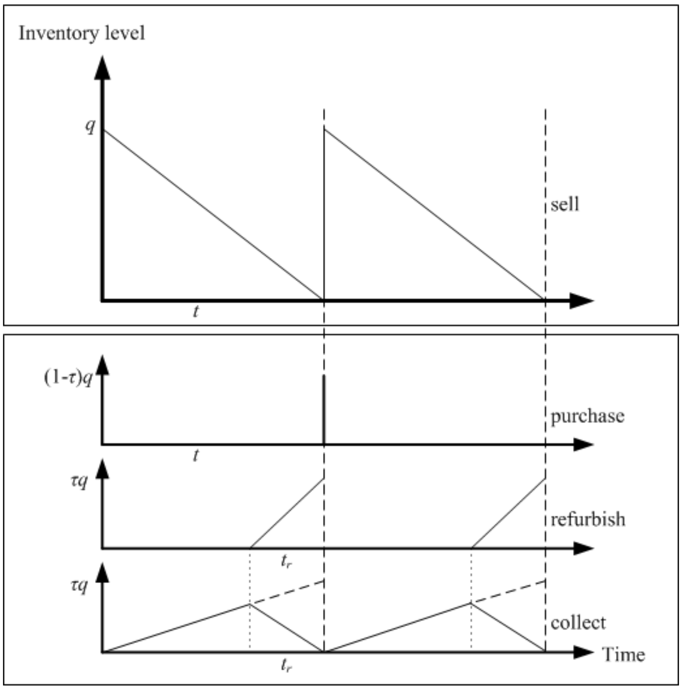

Figure 1. Inventory behaviors of the retailer

Figure 1 shows the inventory behavior of retailer. We note that replenishment policy of the buyer follows traditional EOQ model and places an order of quantity q periodically with cycle length t. We also note the replenishment policy of the seller follows lot-for-lot policy and places an order with quantity $(1-\tau)q$, where τ is the return percentage. Besides, for the quantity τq, the seller collect used products between two successive orders of the buyer. Once the time reaches $t-t_r$, the seller starts to remanufacture used products where t_r is remanufacturing time span derived from the remanufacturing rate and return rate.

IV. THE COORDINATED INVENTORY SYSTEM MODEL

In this model, retailer decides the order quantity q and the return rate $\boldsymbol{\tau}$ simultaneously. Under these definitions and assumptions, the relevant costs incurred in an inventory system cycle are: (1) the fixed replenishment cost K; (2) the purchasing cost of the products $c_n(1-\tau)q$; (3) the inventory holding cost $\frac{1}{2}qth_n$, $\frac{1}{2}\tau qt_rh_c$ and $\frac{1}{2}\tau qth_c-\frac{1}{2}\tau qt_rh_c$; (4) the investment cost of collecting used products per unit time $C_L\tau^2$; and (5) the refurbishing cost of the products $c_r\tau q$. The corresponding cost per unit time is given by

$$\text{Minimize}\prod(q,\tau)=\frac{KD}{q}+c_n(1-\tau)D+c_r\tau D+\frac{h_nq+\tau h_cq}{2}+C_L\tau^2 \tag{1}$$

$$\text{subject to: } 0<\tau\le 1 \tag{2}$$

The fixed point method is employed to obtain order quantity of the coordinated model. From equation (1), in order to obtain the optimal return rate τ^*, $\prod(q,\tau)$ is differentiated with respect to τ and set equal to zero.

$$\frac{\partial\prod(q,\tau)}{\partial\tau}=-c_nD+c_rD+\frac{h_cq}{2}+2C_L\tau=0 \tag{3}$$

Therefore, we can obtain $\hat{\tau}$ as follows

$$\hat{\tau}=\frac{2(c_n-c_r)D-h_cq}{4C_L} \tag{4}$$

From equation (1), in order to obtain the optimal order quantity q^*, $\prod(q,\tau)$ is differentiated with respect to q and set equal to zero.

$$\frac{\partial \prod(q,\tau)}{\partial q} = -\frac{KD}{q^2} + \frac{h_n + \tau h_c}{2} = 0 \qquad (5)$$

Therefore, we can obtain $\hat{q}$ as follows

$$\hat{q} = \sqrt{\frac{2KD}{h_n + \tau h_c}} \qquad (6)$$

By examining equation (4) and equation (6), it is very difficult to obtain closed-form solutions for the optimal (τ^*, q^*). However, by examining the second order sufficient conditions, it is easy to show that $\prod(q,\tau)$ is not a concave function of (τ , q). The main aim is to determine the order quantity q and the return rate τ such that $\prod(q,\tau)$ is minimized. The approaches used to prove the optimality of the solution and the convergence property of the proposed algorithm is very similar to those employed by Maihami and Karimi (2014).

Proposition 1. *For any given* τ *, the second-order derivative of* $\prod(q,\tau)$ *with respect to* q *is positive.*

Proof.

$$\frac{\partial^2 \prod(q,\tau)}{\partial q^2} = \frac{2KD}{q^2} >= 0 \qquad (7)$$

Therefore, $\prod(q,\tau)$ is a concave function with respect to q for any given τ , which implies that there a unique optimal order quantity $\hat{q}$ exists that satisfies equation (1).

Proposition 2. *For any given* q *, the second-order derivative of* $\prod(q,\tau)$ *with respect to* τ *is positive.*

Proof.

$$\frac{\partial^2 \prod(q,\tau)}{\partial \tau^2} = 2C_L\tau > 0 \qquad (8)$$

Therefore, $\prod(q,\tau)$ is a concave function with respect to τ for any given q , which implies that there a unique optimal return rate $\hat{\tau}$ exists that satisfies equation (1).

According to the concavity properties shown in equations (7) and (8) as well as the equations shown in equations (4) and (6) for $\hat{q}$ and $\hat{\tau}$, an efficient algorithm can be developed to search for the optimal q^* and τ^* which minimizes $\prod(q,\tau)$. While performing the search algorithm, if the return rate $\hat{\tau}$ obtained from equation (4) does not fall into the interval (0, 1], then the return rate $\hat{\tau}$ is set to be equal to 1. In such a case, the initial value for the order quantity is set to be $q_0 = \sqrt{\frac{2KD}{h_n + h_c}}$. The proposed iterative procedure to search for the optimal order quantity and return rate is outlined as follows.

__**Algorithm**__

Initial Step:

Input the values of relevant parameters c_n , c_r , D , K , h_c , h_n and C_L .

Let *n*=0, $\prod_n(q,\tau) = 0$ and $q_0 = \sqrt{\frac{2KD}{h_n + h_c}}$.

Iteration Step:

Do {

Let *n*=*n*+1.

Compute $\tau_n = \frac{2(c_n - c_r)D - h_c q_{n-1}}{4C_L}$.

If $\tau_n > 1$, then $\tau_n = 1$.

Compute $q_n = \sqrt{\dfrac{2KD}{h_n + \tau h_c}}$.

Calculate $\prod_n (q_n, \tau_n) = \dfrac{KD}{q_n} + c_n (1 - \tau_n) D + c_r \tau_n D + \dfrac{h_n q_n + \tau_n h_c q_n}{2} + C_L \tau_n^{\ 2}$.

} Until $\left| \prod_{n-1}(q_{n-1}, \tau_{n-1}) - \prod_n (q_n, \tau_n) \right| < \varepsilon$

Output Step:

If $\prod_{n-1}(q_{n-1}, \tau_{n-1}) > \prod_n (q_n, \tau_n)$

Output $(Q^*, \tau^*) = (Q_n, \tau_n)$ and $\prod^* = \prod_n (q_n, \tau_n)$.

Else

Output $(Q^*, \tau^*) = (Q_{n-1}, \tau_{n-1})$ and $\prod^* = \prod_{n-1}(q_{n-1}, \tau_{n-1})$.

The following numerical example is provided to illustrate the features of the proposed model and the iterative algorithm to search for the optimal q^* and τ^*.

Example 1. The values of parameters are c_n = 10, c_r = 5, K = 1500, D = 5000, C_L = 50000, h_n = 0.8, and h_c = 0.3. By employing the algorithm developed in this section, this numerical example can be solved very efficiently. Assuming $\varepsilon = 10^{-8}$, the outcome of each iteration for proposed algorithm to search for the optimal q^* and τ^* is shown in Table 2.

Table 2. Iterative procedure to search for the optimal solution at interior points.

Iteration	q	τ	$\prod$	$\Delta\prod$
1	4144.3305	0.24446088	50495.9365240	---
2	4144.8128	0.24378350	50495.9135575	0.0229665
3	4144.8133	0.24378278	50495.9135574	0.0000001
4	4144.8133	0.24378278	50495.9135574	0.0000000

From Table 1, it is noted that the optimal q^*, and τ^* converges within 4 iterations. Specifically, the optimal solution q^* = 4144.8133, and τ^* = 0.24378278, and the corresponding total cost per unit time is 50495.9135574. In this example, the return rates are all interior points, which fall in the interval of (0, 1).

Example 2. The values of parameters are c_n = 10, c_r = 5, K = 1500, D = 5000, C_L = 50000, h_n = 0.8, and h_c = 0.3. The *Cost* with respect to the return rate τ from 0 to 1 and order quantity q from 0 to 10000 can be depicted as shown in Figure 2.

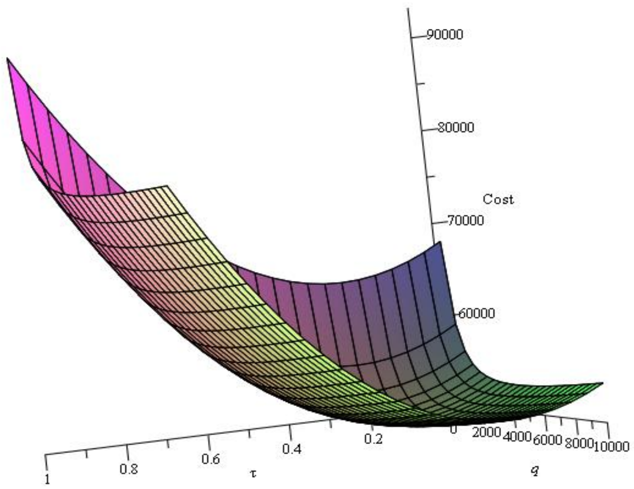

Figure 2. The cost of retailer with respect to the return rate and order quantity

V. CONCLUSION

In this article, assuming the retailer not only sells products but also collects those sold used products to refurbish for reselling, a mathematical model is formulated to minimize the total relevant costs for the retailer. The optimal order quantity of new products and return rate of used products can be searched from an efficient algorithm. Several numerical examples are provided to illustrate the features of the proposed models and the solution procedures.

REFERENCES

[1]. Ferrer, G. (1997a). The economics of tire remanufacturing. *Resources, conservation and recycling*, 19(4), 221-255.

[2]. Ferrer, G. (1997b). The economics of personal computer remanufacturing. *Resources, Conservation and Recycling*, 21(2), 79-108.

[3]. Hart, S. L. (1997). Beyond greening: strategies for a sustainable world. *Harvard business review*, *75*(1), 66-77.

[4]. Ilgin, M. A., & Gupta, S. M. (2010). Environmentally conscious manufacturing and product recovery (ECMPRO): a review of the state of the art. *Journal of environmental management*, 91(3), 563-591.

[5]. Kim, K., Song, I., Kim, J., & Jeong, B. (2006). Supply planning model for remanufacturing system in reverse logistics environment. *Computers & Industrial Engineering*, 51(2), 279-287.

[6]. Maihami, R., & Karimi, B. (2014). Optimizing the pricing and replenishment policy for non-instantaneous deteriorating items with stochastic demand and promotional efforts. *Computers & Operations Research*, 51, 302-312.

[7]. Savaskan, R. C., Bhattacharya, S., & Van Wassenhove, L. N. (2004). Closed-loop supply chain models with product remanufacturing. *Management science*, *50*(2), 239-252.

[8]. Savaskan, R. C., & Van Wassenhove, L. N. (2006). Reverse channel design: the case of competing retailers. *Management science*, *52*(1), 1-14.

[9]. Schrady, D. A. (1967). *A deterministic inventory model for reparable items*. Naval Research Logistics, 14(3), 391-398.

[10]. Teng, H. M., Hsu, P. H., Chiu, Y., & Wee, H. M. (2011). Optimal ordering decisions with returns and excess inventory. *Applied Mathematics and Computation*, 217(22), 9009-9018.

[11]. Teunter, R. (2004). Lot-sizing for inventory systems with product recovery. *Computers & Industrial Engineering*, *46*(3), 431-441.

[12]. Thierry, M., Salomon, M., Van Nunen, J., & Van Wassenhove, L. (1995). Strategie issues in product recovery management. *California management review*, 37(2), 114-135.

[13]. Wei, J., Govindan, K., Li, Y., & Zhao, J. (2015). Pricing and collecting decisions in a closed-loop supply chain with symmetric and asymmetric information. *Computers & Operations Research*, *54*, 257-265.

[14]. Wojanowski, R., Verter, V., & Boyaci, T. (2007). Retail–collection network design under deposit–refund. *Computers & Operations Research*, *34*(2), 324-345.

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