Dense sets in Soft biminimal Spaces

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ABSTRACT: The aim of this paper is to introduce the concept of dense sets in soft biminimal spaces and some of their simple properties.

Keywords: soft set, soft biminimal spaces, dense sets

I. INTRODUCTION

Molodtsov (1999) initiated the theory of soft sets as a new mathematical tool for dealing uncertainty, which is completely a new approach for modeling vagueness and uncertainties. Soft set theory has a rich potential for application in solving practical problems in Economics, Social sciences, Medical sciences etc. Molodtsov successfully applied soft theory into several directions, such as Smoothness of functions, Game theory, Computer research, Riemann integration, Perron integration, Theory of probability, Theory of measurement and so on. The concept of minimal structure space (briefly m-structure) was introduced by V. Popa and T. Noiri (2000). They also introduced the concepts of mX-open set and mX-closed set and characterize those sets using mX-closure and mX-interior operators, respectively. C. Boonpok (2010) introduced the concept of biminimal structure space and studied m1X-m2X-open sets and m1X-m2X-closed sets in biminimal structure spaces. R. Gowri and S. Vembu (2015) introduced the concept of Soft minimal and soft biminimal spaces. Also they introduced (2016) boundary set on soft biminal spaces and Exterior sets in soft biminal spaces. In this paper, we introduce the concept of dense sets in soft biminal spaces and study some of their simple properties.

II. PRELIMINARIES

Definition 2.1 [11] Let U be an initial universe and E be a set of parameters. Let P(U) denote the power set of U and A be a nonempty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P(U).

In other words, a soft set over U is a parametrized family of subsets of the universe U. For e ∈ A, F(e) may be considered as the set of e-approximate elements of the soft set (F, A).

Definition 2.2 [6] Let X be an initial universe set, E be the set of parameters and A ⊆ E. Let FA be a nonempty soft set over X and FA1 is the soft power set of FA. A subfamily FA1 of FA1 is called a soft minimal set over X if FA1 ∈ FA1 and FA1 ∈ FA1.

(FA, FA1) or (X, FA1, E) is called a soft minimal space over X. Each member of FA1 is said to be an FA1-soft open set and the complement of an FA1-soft open set is said to be an FA1-soft closed set over X.

Example 2.3 [6] Let U = {u1, u2}, E = {x1, x2, x3}, A = {x1, x2} ⊆ E and FA = {(x1, {u1, u2}), (x2, {u1, u2})}. Then...
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\[ F_A = \{(x_1, \{u_1\})\}, \]
\[ F_{A_2} = \{(x_1, \{u_2\})\}, \]
\[ F_{A_3} = \{(x_1, \{u_1, u_2\})\}, \]
\[ F_{A_4} = \{(x_2, \{u_1\})\}, \]
\[ F_{A_5} = \{(x_2, \{u_2\})\}, \]
\[ F_{A_6} = \{(x_2, \{u_1, u_2\})\}, \]
\[ F_{A_7} = \{(x_1, \{u_1\}), (x_2, \{u_1\})\}, \]
\[ F_{A_8} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}, \]
\[ F_{A_9} = \{(x_1, \{u_1\}), (x_2, \{u_1, u_2\})\}, \]
\[ F_{A_{10}} = \{(x_1, \{u_2\}), (x_2, \{u_1\})\}, \]
\[ F_{A_{11}} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}, \]
\[ F_{A_{12}} = \{(x_1, \{u_2\}), (x_2, \{u_1, u_2\})\}, \]
\[ F_{A_{13}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1\})\}, \]
\[ F_{A_{14}} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}, \]
\[ F_{A_{15}} = F_A, \]
\[ F_{A_{16}} = F_B \] are all soft subsets of \( F_A \)

**Definition 2.4** [6] Let \((F_A, \tilde{m})\) be a soft minimal space over \(X\). For a soft subset \(F_B\) of \(F_A\), the \(\tilde{m}\)-soft closure of \(F_B\) and \(\tilde{m}\)-soft interior of \(F_B\) are defined as follows:

1. \(\tilde{m}\text{Cl}(F_B) = \cap \{F_A : F_B \subseteq F_A, F_A - F_A \in \tilde{m}\} , \)
2. \(\tilde{m}\text{Int}(F_B) = \cup \{F_B : F_B \supseteq F_B, F_B \in \tilde{m}\} . \)

**Lemma 2.5** [6] Let \((F_A, \tilde{m})\) be a soft minimal space over \(X\). For a soft subset \(F_B\) and \(F_C\) of \(F_A\), the following properties hold:

1. \(\tilde{m}\text{cl}(F_A - F_B) = F_A \setminus \tilde{m}\text{Int}(F_B) \) and \(\tilde{m}\text{Int}(F_A - F_B) = F_A \setminus \tilde{m}\text{cl}(F_B)\),
2. If \((F_A - F_B) \in \tilde{m}\), then \(\tilde{m}\text{cl}(F_B) = F_B\) and if \(F_B \in \tilde{m}\), then \(\tilde{m}\text{Int}(F_B) = F_B\),
3. \(\tilde{m}\text{cl}(F_B) = F_B\), \(\tilde{m}\text{cl}(F_A) = F_A\), \(\tilde{m}\text{Int}(F_B) = F_B\), and \(\tilde{m}\text{Int}(F_A) = F_A\),
4. If \(F_B \subseteq F_C\), then \(\tilde{m}\text{cl}(F_B) \subseteq \tilde{m}\text{cl}(F_C)\) and \(\tilde{m}\text{Int}(F_B) \subseteq \tilde{m}\text{Int}(F_C)\),
5. \(F_B \subseteq \tilde{m}\text{cl}(F_B)\) and \(\tilde{m}\text{Int}(F_B) \subseteq F_B\),
6. \(\tilde{m}\text{cl}(\tilde{m}\text{cl}(F_B)) = \tilde{m}\text{cl}(F_B)\) and \(\tilde{m}\text{Int}(\tilde{m}\text{Int}(F_B)) = \tilde{m}\text{Int}(F_B)\).

**Lemma 2.6** [6] Let \(F_A\) be a nonempty set and \(\tilde{m}\) on \(X\) satisfying property B. For a soft subset \(F_B\) of \(F_A\), the following properties hold:

1. \(F_B \in \tilde{m}\) if and only if \(\tilde{m}\text{Int}(F_B) = F_B\),
2. If \(F_B\) is \(\tilde{m}\)-closed if and only if \(\tilde{m}\text{cl}(F_B) = F_B\),
3. \(\tilde{m}\text{Int}(F_B) \in \tilde{m}\) and \(\tilde{m}\text{cl}(F_B) \in \tilde{m}\)-closed.
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Definition 2.7 [6] Let \((F_A, \bar{m})\) be a soft minimal space with nonempty set \(F_A\) is said to have property B if the union of any family belonging to \(\bar{m}\) belongs to \(\bar{m}\).

Definition 2.8 [6] Let \(X\) be an initial universe set and \(E\) be the set of parameters. Let \((X, \bar{m}_1, E)\) and \((X, \bar{m}_2, E)\) be the two different soft minimal spaces over \(X\). Then \((X, \bar{m}_1, \bar{m}_2, E)\) or \((F_A, \bar{m}_1, \bar{m}_2)\) is called a soft biminimal spaces.

Definition 2.9 [6] A soft subset \(F_B\) of a soft biminimal space \((F_A, \bar{m}_1, \bar{m}_2)\) is called \(\bar{m}_1\bar{m}_2\)-soft closed if \(\bar{m}_1\bar{m}_2(F_B') = F_B\). The complement of \(\bar{m}_1\bar{m}_2\)-soft closed set is called \(\bar{m}_1\bar{m}_2\)-soft open.

Proposition 2.10 [6] Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space over \(X\). Then \(F_B\) is a \(\bar{m}_1\bar{m}_2\)-soft open soft subsets of \((F_A, \bar{m}_1, \bar{m}_2)\) if and only if \(F_B = \bar{m}_1\bar{m}_2(\bar{m}_1\bar{m}_2(F_B'))\).

Proposition 2.11 [6] Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space. If \(F_B\) and \(F_C\) are \(\bar{m}_1\bar{m}_2\)-soft closed soft subsets of \((F_A, \bar{m}_1, \bar{m}_2)\) then \(F_B \cap F_C\) is \(\bar{m}_1\bar{m}_2\)-soft closed.

Proposition 2.12 [6] Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space over \(X\). If \(F_B\) and \(F_C\) are \(\bar{m}_1\bar{m}_2\)-soft open soft subsets of \((F_A, \bar{m}_1, \bar{m}_2)\), then \(F_B \cup F_C\) is \(\bar{m}_1\bar{m}_2\)-soft open.

Definition 2.13 [8] Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space (SBMS), \(F_B\) be a soft subset of \(F_A\) and \(x \in F_A\). Then \(x\) is called \(\bar{m}_1\bar{m}_2\)-exterior point of \(F_B\) if \(x \in \bar{m}_1\bar{m}_2(\bar{m}_1\bar{m}_2(F_A \setminus F_B))\). We denote the set of all \(\bar{m}_1\bar{m}_2\)-exterior point of \(F_B\) by \(\bar{m}_1\bar{m}_2Ext_I(F_B)\) where \(i, j = 1, 2\), and \(i \neq j\).

From definition we have \(\bar{m}_1\bar{m}_2Ext_I(F_B) = F_A \setminus \bar{m}_1\bar{m}_2Cl(\bar{m}_1\bar{m}_2(F_B))\).

Definition 2.14 [7] Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space, \(F_B\) be a soft subset of \(F_A\) and \(x \in F_A\). We called \(x\) is \((i, j) - \bar{m}\) boundary point of \(F_B\) if \(x \in \bar{m}_1\bar{m}_2Cl(\bar{m}_1\bar{m}_2(F_B)) \cap \bar{m}_1\bar{m}_2Cl(\bar{m}_j\bar{m}_j(F_A \setminus F_B))\). We denote the set of all \((i, j) - \bar{m}\) boundary point of \(F_B\) by \(\bar{m}_1\bar{m}_2Brdr_I(F_B)\) where \(i, j = 1, 2\), and \(i \neq j\).

From definition we have \(\bar{m}_1\bar{m}_2Brdr_I(F_B) = \bar{m}_1\bar{m}_2Cl(\bar{m}_i\bar{m}_j(F_B)) \cap \bar{m}_1\bar{m}_2Cl(\bar{m}_j\bar{m}_j(F_A \setminus F_B))\).

Theorem 2.15 [7] Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space and \(F_B\) be a soft subset of \(F_A\). Then for any \(i, j = 1, 2\), and \(i \neq j\), we have:
1. \(F_B\) is \(\bar{m}_1\bar{m}_2\)-soft closed if and only if \(\bar{m}_1\bar{m}_2Brdr_i(F_B) \subseteq F_B\).
2. \(F_B\) is \(\bar{m}_1\bar{m}_2\)-soft open if and only if \(\bar{m}_1\bar{m}_2Brdr_i(F_B) \subseteq F_A \setminus F_B\).

III. DENSE SETS IN SOFT BIMINIMAL SPACES

In this section, we introduce the concept of dense sets in soft biminimal spaces and study some of their properties.

Definition 3.1 Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space. A soft subset \(F_B\) of \(F_A\) is called \(\bar{m}_1\bar{m}_j\)-dense set in \(F_A\) if \(F_A = \bar{m}_1\bar{m}_jCl(\bar{m}_jCl(F_B))\), where \(i, j = 1, 2\) and \(i \neq j\).

Example 3.2 Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space where \(X = \{u_1, u_2\}\),
\(E = \{x_1, x_2, x_3\}\), \(A = \{x_1, x_2\} \subseteq E\) and \(F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}\).

Then \(\bar{m}_1 = \{F_{p_1}, F_{p_2}, F_{p_3}, F_A, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}\}\) and \(\bar{m}_2 = \{F_{p_1}, F_{p_2}, F_{p_3}, F_{p_4}, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}\}\).

Then \(\bar{m}_1\bar{m}_jCl(\bar{m}_jCl(\{(x_2, \{u_1\})\})) = F_A\) and \(\bar{m}_2\bar{m}_jCl(\bar{m}_jCl(\{(x_2, \{u_1\})\})) = F_A\).

\(\bar{m}_1\bar{m}_2Cl(\bar{m}_2\bar{m}_2Cl(\{(x_1, \{u_2\})\})) = \{(x_1, \{u_2\})\}\) and \(\bar{m}_2\bar{m}_2Cl(\bar{m}_2\bar{m}_2Cl(\{(x_1, \{u_2\})\})) = \{(x_1, \{u_2\})\}\).

Hence \(\{(x_2, \{u_1\})\}\) is \(\bar{m}_1\bar{m}_2\)-dense set and \(\bar{m}_2\bar{m}_2\)-dense set in \(F_A\).

But \(\{(x_1, \{u_2\})\}\) is not \(\bar{m}_1\bar{m}_2\)-dense set, where \(i, j = 1, 2\) and \(i \neq j\).

Theorem 3.3 Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space and \(F_B\) be a soft subset of \(F_A\). \(F_B\) is \(\bar{m}_1\bar{m}_2\)-dense set in \(F_A\) if and only if \(\bar{m}_1\bar{m}_2Ext_I(F_B) = F_B\), where \(i, j = 1, 2\) and \(i \neq j\).
Proof: Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space and let \(F_B\) be a soft subset of 
\(F_A\).

Necessity: Suppose that \(F_B\) is \(\bar{m}_i\bar{m}_j\)-dense set in \(F_A\). Since \(F_A = \bar{m}_i Cl(\bar{m}_j Cl(F_B))\),
That implies \(F_A \setminus \bar{m}_i Cl(\bar{m}_j Cl(F_B)) = F_0 \Rightarrow \bar{m} Ext(F_B) = F_0\), [8] where \(i, j = 1, 2\) and \(i \neq j\).

Sufficiency: Assume that \(\bar{m} Ext(F_B) = F_0\). Thus \(F_A \setminus \bar{m}_i Cl(\bar{m}_j Cl(F_B)) = F_0\),
it follows that \(\bar{m}_i Cl(\bar{m}_j Cl(F_B)) = F_A\). Hence \(F_B\) is \(\bar{m}_i\bar{m}_j\)-dense set in \(F_A\), where \(i, j = 1, 2\) and \(i \neq j\).

Theorem 3.4 Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space and \(F_B\) be a soft subset of \(F_A\). If \(F_B\) is \(\bar{m}_i\bar{m}_j\)-dense set in \(F_A\) then for any non-empty soft \(\bar{m}_i\bar{m}_j\)-closed subset \(G_B\) of \(F_A\), where \(i, j = 1, 2\) and \(i \neq j\) such that \(F_B \subseteq G_B\), we have \(G_B = F_A\).

Proof: Assume that \(F_B\) is \(\bar{m}_i\bar{m}_j\)-dense set in \(F_A\) and \(G_B\) is soft \(\bar{m}_i\bar{m}_j\)-closed subset of \(F_A\) such that \(F_B \subseteq G_B\). Since \(F_B\) is \(\bar{m}_i\bar{m}_j\)-dense set in \(F_A\), that implies \(F_A = \bar{m}_i Cl(\bar{m}_j Cl(F_B))\). By assumption, \(G_B\) is soft \(\bar{m}_i\bar{m}_j\)-closed set and \(F_B \subseteq G_B\), it follows that \(F_A = \bar{m}_i Cl(\bar{m}_j Cl(F_B)) \subseteq \bar{m}_i Cl(\bar{m}_j Cl(\bar{m}_j Cl(G_B))) = G_B\). Consequently, \(G_B = F_A\).

Note 3.5 By Theorem 3.4 if \(F_B\) is \(\bar{m}_i\bar{m}_j\)-dense set in \(F_A\). Then only \(F_A\) is soft \(\bar{m}_i\bar{m}_j\)-closed set in \(F_A\) such that containing \(F_B\).

Remark 3.6 The Theorem 3.4 is not true if \(G_B\) is not soft \(\bar{m}_i\bar{m}_j\)-closed set. The following example supports our claim.

Example 3.7 Let \(X = \{u_1, u_2\}\), \(E = \{x_1, x_2, x_3\}\), \(A = \{x_1, x_2\} \subseteq E\) and \(F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2\})\}\). Then \(\bar{m}_1 = \{F_0, F_A, F_{A1}, F_{A2}, F_{A3}\}\), \(\bar{m}_2 = \{F_0, F_{A1}, F_{A2}, F_{A3}, F_{A4}, F_{A5}\}\), and \(\bar{m}_3 = \{F_0, F_{A1}, F_{A2}, F_{A3}, F_{A4}, F_{A5}, F_{A6}\}\). Then \(\bar{m}_i Cl(\bar{m}_j Cl(F_B)) = F_A\) and \(\bar{m}_2 Cl(\bar{m}_3 Cl(F_B)) = F_A\). Hence \(\{x_2, \{u_1, u_2\}\}\) is \(\bar{m}_i\bar{m}_j\)-dense set in \(F_A\). But \(\{(x_2, \{u_1, u_2\}\}\) is not soft \(\bar{m}_i\bar{m}_j\)-closed set in \(F_A\).

Theorem 3.8 Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space and let \(F_B\) be a soft subset of \(F_A\). If \(\bar{m}_i Int(\bar{m}_j Int(F_A \setminus F_B)) = F_0\) then for any non-empty soft \(\bar{m}_i\bar{m}_j\)-closed subset \(G_B\) of \(F_A\), where \(i, j = 1, 2\) and \(i \neq j\) such that \(F_B \subseteq G_B\), we have \(G_B = F_A\).

Proof: Suppose that \(\bar{m}_i Int(\bar{m}_j Int(F_A \setminus F_B)) = F_0\) and \(F_B\) be a soft subset of \(F_A\), where \(i, j = 1, 2\) and \(i \neq j\) such that \(F_B \subseteq G_B\). By assumption, we have \(F_A \setminus \bar{m}_i Cl(\bar{m}_j Cl(F_B)) = F_0\) and so \(F_A = \bar{m}_i Cl(\bar{m}_j Cl(F_B)) \subseteq \bar{m}_i Cl(\bar{m}_j Cl(\bar{m}_j Cl(G_B))) = G_B\). Therefore \(G_B = F_A\).

Theorem 3.9 Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space and let \(F_B\) be a soft subset of \(F_A\). If \(F_B\) is \(\bar{m}_i\bar{m}_j\)-dense set in \(F_A\). Then \(U_B \cap F_B \neq F_B\) for any non-empty soft \(\bar{m}_i\bar{m}_j\)-open subset \(U_B\) of \(F_A\), where \(i, j = 1, 2\) and \(i \neq j\).

Proof: Let \(F_B\) be a \(\bar{m}_i\bar{m}_j\)-dense set in \(F_A\). Assume that \(U_B \cap F_B = F_0\) for any non-empty soft \(\bar{m}_i\bar{m}_j\)-open subset \(U_B\) of \(F_A\). Thus we have \(F_B \subseteq U_B^C\). It follows that \(F_A = \bar{m}_i Cl(\bar{m}_j Cl(U_B^C)) \subseteq \bar{m}_i Cl(\bar{m}_j Cl(U_B^C)) = F_A \setminus \bar{m}_i Int(\bar{m}_j Int(U_B^C))\). Since \(U_B\) is soft \(\bar{m}_i\bar{m}_j\)-open, \(G_B^C\) is soft \(\bar{m}_i\bar{m}_j\)-closed. By assumption, we have \(U_B^C = F_A\). That is \(F_A = F_B\). Therefore \(U_B = F_B\). This is a contradiction. Hence \(U_B \cap F_B \neq F_B\) for any non-empty soft \(\bar{m}_i\bar{m}_j\)-open subset \(U_B\) of \(F_A\), where \(i, j = 1, 2\) and \(i \neq j\).

Theorem 3.10 Let \((F_A, \bar{m}_1, \bar{m}_2)\) be a soft biminimal space and let \(F_B \subseteq F_A\). If for any non-empty soft \(\bar{m}_i\bar{m}_j\)-closed subset \(G_B\) of \(F_A\) such that \(F_B \subseteq G_B\), then \(G_B = F_A\) if and only if \(U_B \cap F_B \neq F_B\) for any non-empty soft \(\bar{m}_i\bar{m}_j\)-open subset \(U_B\) of \(F_A\), where \(i, j = 1, 2\) and \(i \neq j\).
Proof: Let \((F_A, \tilde{m}_1, \tilde{m}_2)\) be a soft biminimal space and let \(F_B \subseteq F_A\).

Necessity: Suppose that if for any non-empty soft \(\tilde{m}_i \tilde{m}_j\)-closed subset \(G_B\) of \(F_A\) such that \(F_B \subseteq G_B\), then \(G_B = F_A\). Assume that \(U_B \cap F_B = F_\emptyset\) for any non-empty soft \(\tilde{m}_i \tilde{m}_j\)-open subset \(U_B\) of \(F_A\). Thus we have \(F_B \subseteq U_B^C\). Since \(U_B\) is soft \(\tilde{m}_i \tilde{m}_j\)-open that implies \(U_B^C\) is soft \(\tilde{m}_i \tilde{m}_j\)-closed. By assumption, we have \(U_B^C = F_A\) it follows that \(U_B = F_\emptyset\), this is contradiction. Therefore \(U_B \cap F_B \neq F_\emptyset\) for any non-empty soft \(\tilde{m}_i \tilde{m}_j\)-open subset \(U_B\) of \(F_A\), where \(i, j = 1, 2\) and \(i \neq j\).

Sufficiency: Assume that \(U_B \cap F_B \neq F_\emptyset\) for any non-empty soft \(\tilde{m}_i \tilde{m}_j\)-open subset \(U_B\) of \(F_A\) and \(G_B\) is a non-empty soft \(\tilde{m}_i \tilde{m}_j\)-closed subset of \(F_A\), such that \(F_B \subseteq G_B\). Suppose that \(G_B \neq F_A\). Thus \(G_B^C\) is a non-empty soft \(\tilde{m}_i \tilde{m}_j\)-open subset of \(F_A\). By assumption, we have \(G_B^C \cap F_B \neq F_\emptyset\). This is a contradiction with \(F_B \subseteq G_B\). Hence \(G_B = F_A\).

\[ \square \]

**Theorem 3.11** Let \((F_A, \tilde{m}_1, \tilde{m}_2)\) be a soft biminimal space and let \(F_B \subseteq F_A\). If \(\tilde{m}_i \text{Int}(\tilde{m}_j \text{Int}(F_A \setminus F_B)) = F_\emptyset\), then \(U_B \cap F_B \neq F_\emptyset\) for any non-empty soft \(\tilde{m}_i \tilde{m}_j\)-open subset \(U_B\) of \(F_A\), where \(i, j = 1, 2\) and \(i \neq j\).

Proof: Assume that \(\tilde{m}_i \text{Int}(\tilde{m}_j \text{Int}(F_A \setminus F_B)) = F_\emptyset\). Suppose that \(U_B \cap F_B = F_\emptyset\) for any non-empty soft \(\tilde{m}_i \tilde{m}_j\)-open subset \(U_B\) of \(F_A\) such that \(F_B \subseteq U_B^C\), it follows that \(F_B = \tilde{m}_i \text{Cl}(\tilde{m}_j \text{Cl}(F_B)) \subseteq \tilde{m}_i \text{Cl}(\tilde{m}_j \text{Cl}(U_B^C)) = U_B^C\). Hence \(U_B = F_\emptyset\). This is contradiction with the property of \(U_B\). Therefore \(U_B \cap F_B \neq F_\emptyset\) for any non-empty soft \(\tilde{m}_i \tilde{m}_j\)-open subset \(U_B\) of \(F_A\), where \(i, j = 1, 2\) and \(i \neq j\).

\[ \square \]

**Theorem 3.12** Let \((F_A, \tilde{m}_1, \tilde{m}_2)\) be a soft biminimal space and let \(F_B \subseteq F_A\). If \(F_B\) is \(\tilde{m}_i \tilde{m}_j\)-dense set in \(F_A\), then \(\tilde{m} \text{Bdr}_{ij}(F_B) = \tilde{m}_i \text{Cl}(\tilde{m}_j \text{Cl}(F_A \setminus F_B))\), where \(i, j = 1, 2\) and \(i \neq j\).

Proof: Let \((F_A, \tilde{m}_1, \tilde{m}_2)\) be a soft biminimal space and let \(F_B \subseteq F_A\). Assume that \(F_B\) is \(\tilde{m}_i \tilde{m}_j\)-dense set in \(F_A\). Thus we have \(\tilde{m} \text{Bdr}_{ij}(\tilde{m}_i \text{Cl}(\tilde{m}_j \text{Cl}(F_B))) \cap \tilde{m}_i \text{Cl}(\tilde{m}_j \text{Cl}(F_A \setminus F_B)) = \tilde{m}_i \text{Cl}(\tilde{m}_j \text{Cl}(F_A \setminus F_B))\), where \(i, j = 1, 2\) and \(i \neq j\).

\[ \square \]

**Theorem 3.13** Let \((F_A, \tilde{m}_1, \tilde{m}_2)\) be a soft biminimal space and let \(F_B \subseteq F_A\). If \(F_B\) is soft \(\tilde{m}_i \tilde{m}_j\)-open and \(\tilde{m}_i \tilde{m}_j\)-dense set in \(F_A\) if and only if \(\tilde{m} \text{Bdr}_{ij}(F_B) = F_A \setminus F_B\), where \(i, j = 1, 2\) and \(i \neq j\).

Proof: Let \((F_A, \tilde{m}_1, \tilde{m}_2)\) be a soft biminimal space and let \(F_B \subseteq F_A\). Necessity: Assume that \(F_B\) is soft \(\tilde{m}_i \tilde{m}_j\)-open and \(\tilde{m}_i \tilde{m}_j\)-dense set in \(F_A\). By Theorem 2.15 and Theorem 3.12, we have \(\tilde{m}_i \text{Cl}(\tilde{m}_j \text{Cl}(F_A \setminus F_B)) = F_A \setminus F_B\). Hence \(\tilde{m} \text{Bdr}_{ij}(F_B) = \tilde{m}_i \text{Cl}(\tilde{m}_j \text{Cl}(F_A \setminus F_B)) = F_A \setminus F_B\), where \(i, j = 1, 2\) and \(i \neq j\).

Sufficiency: Let \(\tilde{m} \text{Bdr}_{ij}(F_B) = F_A \setminus F_B\). Assume that \(F_B\) is \(\tilde{m}_i \tilde{m}_j\)-open in \(F_A\). Then we have \(\tilde{m} \text{Bdr}_{ij}(F_B) = F_A \setminus F_B = \tilde{m}_i \text{Cl}(\tilde{m}_j \text{Cl}(F_B)) = \tilde{m}_i \text{Cl}(\tilde{m}_j \text{Cl}(F_A \setminus F_B))\). By Theorem 3.12, we have \(F_B\) is \(\tilde{m}_i \tilde{m}_j\)-dense set in \(F_A\), where \(i, j = 1, 2\) and \(i \neq j\).

\[ \square \]

**Example 3.14** Let \(X = \{u_1, u_2\}\), \(E = \{x_1, x_2, x_3\}\), \(A = \{x_1, x_2\} \subseteq E\) and \(F_A = \{\{x_1, \{u_1, u_2\}\}, \{x_2, \{u_1, u_2\}\}\}\). Then \(\tilde{m}_1 = \{F_\emptyset, F_{A_1}, F_{A_2}, F_{A_3}, F_{A_4}, F_{A_5}\}\), and \(\tilde{m}_2 = \{F_\emptyset, F_{A_5}, F_{A_4}, F_{A_3}, F_{A_2}, F_{A_1}, F_{A_6}\}\). Then \(\tilde{m}_2 \text{Cl}(\tilde{m}_2 \text{Cl}(\{x_1, \{u_1\}\}, \{x_2, \{u_1\}\})) = F_A\) and \(\tilde{m}_1 \text{Int}(\tilde{m}_2 \text{Int}(\{x_1, \{u_1\}\}, \{x_2, \{u_1\}\}) = \{x_1, \{u_1\}\}, \{x_2, \{u_1\}\})\).

Hence \(\{x_1, \{u_1\}\}, \{x_2, \{u_1\}\}\) is \(\tilde{m}_i \tilde{m}_j\)-open and \(\tilde{m}_i \tilde{m}_j\)-dense set in \(F_A\).

Then we have \(\tilde{m} \text{Bdr}_{ij}(\{x_1, \{u_1\}\}, \{x_2, \{u_1\}\}) = \{x_1, \{u_1\}\}, \{x_2, \{u_1\}\}\).
REFERENCES


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