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Research Paper

A Transmission Line Model with Metamaterial Effects in Gamma Ray Bursts

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ABSTRACT: *The dispersive velocity of light observed in gamma-ray bursts and predicted by certain quantum gravity models suggests to include a quadratic dispersion term to the group velocity obtained from the Helmholtz equation. The gamma-ray dispersion have the same form of Maxwell's equations in metamaterial media, where the added term to the group velocity for the dispersion, allows that GeV gamma rays arrive significantly later than low-energy photons. In this paper we propose a transmission line model which take into account of gamma-ray dispersion through the Helmholtz equations derived from an extended Maxwell's equations for composite right/left-handed metamaterial through of a transmission line model.*

Keywords: *transmission line, Maxwell equations, metamaterials, gamma ray.*

I. INTRODUCTION

Composite materials in which both permittivity *and* permeability possess negative values at some frequencies has recently gained considerable attention. This idea was originally initiated by Veselago in 1967, who theoretically studied plane wave propagation in a material whose permittivity and permeability were assumed to be simultaneously negative [1]. Recently have been constructed such a composite medium for the microwave regime, and experimentally the presence of anomalous refraction in this medium is verified [2-5]. Previous theoretical study of electromagnetic wave interaction with omega media using the circuit-model approach had also revealed the possibility of having negative permittivity and permeability in omega media for certain range of frequencies [6-7]. The anomalous refraction at the boundary of such a medium with a conventional medium, and the fact that for a time-harmonic monochromatic plane wave the direction of the Poynting vector is antiparallel with the direction of phase velocity, can lead to exciting features that can be advantageous in design of novel devices and components. Almost all materials encountered in optics, such as glass or water, have positive values for both permittivity ε and permeability μ. However, metals such as silver and gold have negative permittivity at shorter wavelengths. A material such as a surface plasmon that has either (but not both) ε or μ negative is often opaque to electromagnetic radiation. However, anisotropic materials with only negative permittivity can produce negative refraction due to chirality [6]. Although the optical properties of a transparent material are fully specified by the parameters ε_r and μ_r, refractive index *n* is often used in practice, which can be determined from $n = \pm\sqrt{\mu_r \varepsilon_r} = \pm\sqrt{(\mu/\mu_0)(\varepsilon/\varepsilon_o)}$. All known non-metamaterial transparent materials possess positive ε_r and μ_r. By convention the positive square root is used for *n*. However, some engineered metamaterials have $\varepsilon_r < 0$ $\varepsilon < 0$ and $\mu_r < 0$. Because the product $\varepsilon_r \mu_r$ is positive, *n* is real. Under such circumstances, it is necessary to take the negative square root for $n = -\sqrt{(-\mu_r)(-\varepsilon_r)}$. It is obvious that the phase and the group velocities are antiparallel when ε and μ are simultaneously negative. The inverse statement holds: When the phase and the group velocities of an isotropic medium are antiparallel, the medium is characterized by negative values of ε and μ.

In this work we discuss the possibility of metamaterial effects in propagation of gamma rays, finding the appropriate Maxwell's equation by analogy with Kirchhoff's circuit laws [8] in the lumped element model of a transmission line.

II. MODEL OF TRANSMISSION LINES

A transmission line of length d, with capacitance per unit length c and inductance per unit length l, may be treated as the continuum limit of a chain of LC oscillators [7]. Such a circuit is shown in figure 1.

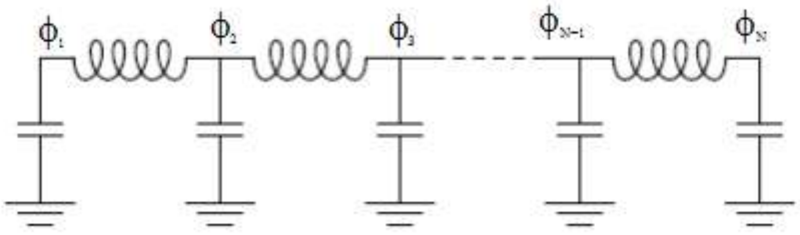

Figure 1: The transmission line. The figure shows a transmission line with open-circuit boundary conditions, represented as the continuum limit of a chain of LC oscillators.

The ground node is marked, and as the spanning tree we choose the capacitive branches. Assuming there are no externally-applied magnetic fluxes,∗ (∗ The effects of static externally-applied fluxes can be removed by a canonical transformation.) the Lagrangian is

$$L(\phi_1, d\phi_1/dt......\phi_N, d\phi_N/dt) = \sum_{i=1}^{N} \frac{\Delta C}{2} (\frac{\partial \phi_i}{\partial t})^2 - \sum_{i=1}^{N-1} \frac{(\phi_{i+1} - \phi_i)^2}{2\Delta L} \qquad (1)$$

with $\Delta C = cd/N$, $\Delta L = ld/N$. In the continuum limit, N → ∞, this becomes the integral

$$L(\phi_1, d\phi_1/dt......\phi_N, d\phi_N/dt) = \int_0^d \frac{c}{2} (\frac{\partial \phi_i(z,t)}{\partial t})^2 - \frac{1}{2l} (\frac{\partial \phi_i(z,t)}{\partial x})^2 dz \qquad (2)$$

The Euler–Lagrange equation for L(z, t) is thus

$$\frac{\partial^2 \phi(z,t)}{\partial t^2} - u^2 \frac{\partial^2 \phi(z,t)}{\partial z^2} = 0 \qquad (3)$$

where $u = 1/\sqrt{lc}$ is the wave velocity. This has solutions

$$\phi(z,t) = \sum_{n=1}^{\infty} A_n \cos(k_n z + \alpha_n) \cos(k_n ut + \beta_n) \qquad (4)$$

where A_n, k_n, α_n and β_n depend on the boundary conditions. For the case of open-circuit boundary conditions at z = 0 and z = d, as shown in the figure, we have

$$\left.\frac{\partial \phi(z,t)}{\partial t}\right|_{z=0} = 0, \quad \left.\frac{\partial \phi(z,t)}{\partial t}\right|_{z=d} = 0 \qquad (5)$$

which gives $\alpha_n = 0$, $k_n = n\pi/d$. (a_n and β_n will be determined by the initial conditions.)
Substituting (4) into (2) and integrating out the x dependence yields

$$L(\Phi_1, d\Phi_1/dt......\Phi_N, d\Phi_N/dt) = \sum_{n=1}^{N} \frac{C_n}{2} (\frac{\partial \Phi_n}{\partial t})^2 - \frac{(\Phi_n)^2}{2L_n} \qquad (6)$$

where $\Phi_n(t) = A_n \cos(k_n ut + \beta_n)$ keeps the time dependence of the solution. Thus this is an effective Lagrangian for a circuit consisting of uncoupled LC oscillators with effective capacitances $C_n = cd/2$ and effective inductances $L_n = dl/2n^2\pi^2$, and hence resonant frequencies $\omega_n = nu2\pi/d$. The quantum Hamiltonian for a transmission line cavity is therefore

$$H = \sum_{n}^{\infty} \omega_n (a_n^\dagger a_n + 1/2) \qquad (7)$$

Generally we are only interested in the behavior of a circuit in the vicinity of a particular frequency. In such cases we can pull out only one mode (often the fundamental, n = 1) and ignore the dynamics of the other modes with $\omega_r = u2\pi / d = uk$ and $2\pi / d = k = 1$. In such cases the cavity Hamiltonian is simply

$$H = \omega_r (a_r^\dagger a_r + 1/2) \tag{8}$$

where ω_r is the frequency of the relevant cavity mode, with creation and annihilation operators a† and a. In the cQED literature, the cavity Hamiltonian is usually written as (8) without any further explanation.

III. RIGHT-HANDED AND LEFT-HANDED MAXWELL'S EQUATIONS

The Maxwell equations for the macroscopic free electromagnetic fields, (without charge and current) are well known. We often write Maxwell's equations in terms of electric and magnetic fields, $\boldsymbol{E}$ and $\boldsymbol{B}$,

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}, \; \nabla \cdot \mathbf{B} = 0 \tag{9a}$$

$$\nabla \times \mathbf{H} = \frac{\partial}{\partial t}\mathbf{D} + \mathbf{J}, \; \nabla \cdot \mathbf{D} = \rho \tag{9b}$$

These equations, however, are not complete. Six more equations, the constitutive relations, have to be added relating the electric field $\mathbf{E}$, the magnetic induction $\mathbf{B}$, the displacement field $\mathbf{D}$ and the magnetic field $\mathbf{H}$ to each other. These constitutive relations are completely independent of the Maxwell equations. The Maxwell equations involve only the fields and their sources. The constitutive relations, $\mathbf{D} = \varepsilon \mathbf{E}$ $\mathbf{B} = \mu \mathbf{H}$, however, are concerned with the equations of motion of the constituents of the medium in an electromagnetic field [9-10].

$$\nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t}\mathbf{H} \tag{10a}$$

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial}{\partial t}\mathbf{E} \tag{10b}$$

where Maxwell’s equation constants are μ for the permeability in H/m and ε for the permittivity in F/m.

Taking the curl of both sides from (1):

$$\nabla \times \nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t}\nabla \times \mathbf{H} \tag{11}$$

Then, the normal (righ-handed) wave equation follows as (where $\nabla \cdot E = 0$):

$$\nabla^2 \mathbf{E} = \mu\varepsilon \frac{\partial^2}{\partial t^2}\mathbf{E} \tag{12}$$

Under special conditions, equation (3) is equivalent to equation (12). Our goal is incorporate in this equivalence the dual circuit of figure 1. and obtain a special circuit topology to take into account the metamaterial effect [8- 10].

As previously mentioned, the forgoing methodology can be applied to circuit topologies other than Fig. 1. A combination of circuits shown in figure 2 is the composite right/left hand (CRLH) structures, described in [11]. For the topology of Fig. 2, which is one cell of figure 1, considering the Kirchhoff's circuit laws, The left-handed extensions of Maxwell’s equations yield:

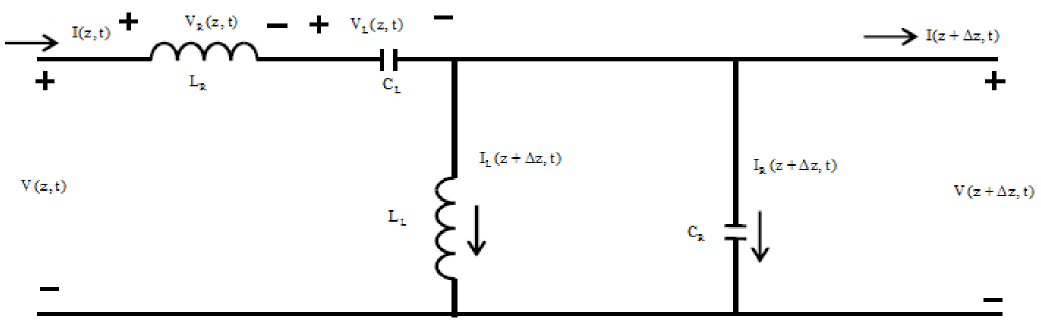

Figure 2. Lumped-element model of an alternative form of a right-/left handed transmission line unit cell.

$$\frac{\partial}{\partial z}V = -L_R \frac{\partial}{\partial t} I - \frac{1}{C_L}\int I\partial t \qquad (13)$$

If we have propagation only in the direction z, $\nabla \times E = \partial / \partial z$ and voltage (current) is proportional to E (I), also L_R proportional to μ, C_L proportional to ε_L, so we have

$$\frac{\partial}{\partial t}\nabla \times E = -\mu \frac{\partial^2}{\partial t^2} H - \frac{1}{\varepsilon_L} H \qquad (14)$$

The second Kirchhoff's law is

$$\frac{\partial}{\partial z} I = C_R \frac{\partial}{\partial t} V + \frac{1}{L_L}\int V\partial t \qquad (15)$$

which can be put as

$$\frac{\partial}{\partial t}\nabla \times H = \varepsilon \frac{\partial^2}{\partial t^2} E + \frac{1}{\mu_L} E \qquad (16)$$

Equations (14) and (16) are the final extensions of Maxwell's equations for the right/left-handed metamaterial system (assuming no sources). If we use the usual plane-wave solution

$$E = E_0 e^{-ikz} e^{i\omega t} \qquad (17)$$

where $k = \omega / u$ is the wavenumber, ω is frequency in rad/s , the wavenumber is normalized to k=1 and $u = 1/(\sqrt{\mu_i \varepsilon_j})$ is the generalized phase velocity in m/s where $\mu_i = \mu_o, \mu, \mu_L$, $\varepsilon_j = \varepsilon_o, \varepsilon, \varepsilon_L$. $\mu_i = \mu_o, \mu_L$ $\mu_i = \mu_o, \mu_L$.

IV. GAMMA RAYS PROPAGATION AS LEFT-HAND METAMATERIAL

From (14) and (16), we can obtain the solution for the square of the plane-wave phase velocity u^2 is:

$$u^2 = \frac{\omega^4 \mu_L \varepsilon_L}{(1-\omega^2 \mu \varepsilon_L)(1-\omega^2 \mu_L \varepsilon)} = \frac{\omega^4 \mu_L \varepsilon_L}{(1-\omega^2/\omega_1^2)(1-\omega^2/\omega_2^2)} \qquad (18)$$

where the frequency bandgap is determined by $\omega_1 = 1/\sqrt{\mu_L \varepsilon}$ and $\omega_2 = 1/\sqrt{\mu \varepsilon_L}$. From (18), the phase velocity is $u = 1/\sqrt{\mu\varepsilon}$ at high frequency, above the bandgap. Between ω_1 and ω_2 there is a bandgap where u^2 is negative. And at frequencies below the bandgap, a left-handed solution is found with dispersive phase velocity $u = \omega^2 \sqrt{\mu_L \varepsilon_L}$.

In recent measurements of astronomical gamma-ray dispersion, the velocity of light has been approximated as a power series expansion for photon energies well below the
Planck energy. In this power series, the photon group velocity v_g is typically expressed as a function of photon energy E_{ph}, or of angular frequency ω, up to second order [12]:

$$v_g{}^2 = c^2(1-a_2\omega^2)^2 \approx c^2(1-a_2\omega^2)(1-a_1\omega^2) \ , \ a_2 \approx a_1 \qquad (19)$$

a_2 is obtained from recent gamma-ray measurements, [13], where $E_{ph}/\sqrt{\zeta} \approx 5\times 10^{18}\,eV$. Then, $a_2 \approx \zeta\hbar^2/(2\pi E_{ph})^2 \approx 1.74\times 10^{-68} s^2$ with ζ a numerical factor of order of one. Putting $a_2 = 1/\omega_2 \approx a_1 = 1/\omega_1$ we can put equation (19) as $v_g{}^2 = c^2(1-(\omega/\omega_1)^2)^2 \approx c^2(1-(\omega/\omega_2)^2)^2$

Considering that from the physical point of view always $uv_g = c^2$ and from equation (18) for u^2, we have

$$v_g^2 = c^4 \frac{(1-\omega^2/\omega_1^2)(1-\omega^2/\omega_2^2)}{\omega^4 \mu_L \varepsilon_L}) \qquad (20)$$

For low frequencies where $\omega \ll \omega_1$ and $\omega \ll \omega_2$, (20) gives the normal nearly-constant left-handed group velocity governed by $v_g \approx c^2 / \omega^2 \sqrt{\mu_L \varepsilon_L}$ and $u \approx \omega^2 \sqrt{\mu_L \varepsilon_L}$, so below the bandgap, a left-handed solution is found.

At high frequencies where $\omega \gg \omega_1$ and $\omega \gg \omega_2$, (20) gives

$$v_g{}^2 \approx c^4 \frac{(-\omega^2/\omega_1^2)(-\omega^2/\omega_2^2)}{\omega^4 \mu_L \varepsilon_L} \approx c^4 \frac{(-1/\omega_1^2)(-1/\omega_2^2)}{\mu_L \varepsilon_L} \approx c^4(-\mu)(-\varepsilon)$$

so $u^2 = 1/(-\mu)(-\varepsilon)$, a highly dispersive left-handed velocity is obtained given by

$$u = 1/\sqrt{(-\mu)(-\varepsilon)} \qquad (21)$$

Solution (21) is a new result that verify the metamaterial theory first given theoretically by Veselago. In plane (ε, μ), some class of material that lies in the third quadrant and has the negative permittivity and permeability, they are known as double negative (DNG). These material allow backward wave propagation so they also know as left handed media (LHM), backward-wave media (BW media), and double-negative (DNG) materials [14]. In addition, there is a forbidden band between ω_1 and ω_2 where u^2 is negative, so the wavenumber becomes imaginary. The solution is evanescent and does not propagate.

Interestingly, the bandgap frequency estimate from recent gamma ray data can be used to estimate the degree of magnitude of μ_L and ε_L for free space but contrary to [15], we assume that a quadratic term of dispersion appears in the group velocity so that we analyze the group velocity v_g instead of the phase velocity as was considered by [15].

Recent data on gamma burst GRB 090510 in 2009 shows dispersion of up to 859 ms for gamma rays at 31 GeV ($\omega = 4.7 \times 10^{25}\,\mathrm{rad/s}$) at a distance of $d = 1.8 \times 10^{26}\,\mathrm{m}$ (using luminosity distance for simplicity). If 31 GeV photons have group velocity v_g, and low energy photons have velocity c, the time difference in arrival times over a distance $d = 1.8 \times 10^{26}\,\mathrm{m}$ is $\tau \approx d/v_g - d/c$. After some rearrangement, the group velocity of the 31 GeV photons is approximated as

$$v_g = c\frac{d}{d + c\tau} \approx c(1 - \frac{c\tau}{d}) \qquad (22)$$

where $d \gg c\tau$, $c = 3 \times 10^8\,\mathrm{m/s}$, $\tau = 0.859\mathrm{s}$, $d = 1.8 \times 10^{26}\,\mathrm{m}$, and $c\tau/d \approx 1.4 \times 10^{-18}$. Since (20) has two remaining unknowns, further approximations are used to provide an estimate of the lower bandgap frequency. For simplicity, let $\omega_1 = \omega_2$, and $v_g = c(1-(\omega/\omega_1)^2)$ in free space. Comparing this result with (22) gives $(\omega/\omega_1)^2 = c\tau/d = 1.4 \times 10^{-18}$ for the 31 GeV photon at a frequency of $\omega = 4.7 \times 10^{25}$ rad/s. Also $c^2/\omega^2 \sqrt{\mu_L \varepsilon_L} \approx c$ so $\sqrt{\mu_L \varepsilon_L} \approx 1.15 \times 10^{-43}$ Then solving for ω_1, the estimate of the bandgap frequency is $\omega_1 \approx 3.9 \times 10^{24}\,\mathrm{rad/s}$. Finally, using the estimate of $\omega_1 = \omega_2 = 3.9 \times 10^{34}\,\mathrm{rad/s}$ and substituting into the equations from (20) results in $\mu_L \approx 1/(\varepsilon_0 \omega_1^2) \approx 7.4 \times 10^{-40}\,\mathrm{H-m}$ and $\varepsilon_L \approx 1/(\mu_0 \omega_1^2) \approx 5 \times 10^{-45}\,\mathrm{F-m}$. As $\sqrt{\mu_L/\varepsilon_L} \approx \sqrt{\mu_0/\varepsilon_0}$, we can compare with the values in free space, where $\varepsilon_0 = 8.9.|\times 10^{-12}\,\mathrm{F/m}$ and $\mu_0 = 1.3 \times 10^{-8}\,\mathrm{H/m}$. The factor $\sqrt{\mu_L \varepsilon_L}$ may be responsible of quantum effects that are expected to strongly affect the nature of space-time. There is great interest in the (not yet validated) idea that Lorentz invariance might break near the Planck scale. A key test of such violation of Lorentz invariance is a possible variation of photon speed with energy. Even a tiny variation in photon speed, when accumulated over cosmological light-travel times, may be revealed by observing sharp features in gamma-ray burst (GRB) light-curves [16] .

V. CONCLUSION

The dispersive velocity of light observed in gamma-ray bursts and predicted by certain quantum gravity models was studied through a transmission line model with metamaterial effects, adding quadratic dispersion to the Helmholtz equation. For gamma-ray dispersion in equation (19) have the same form of an extended Maxwell's equations, where the added term account for the dispersion take into account GeV gamma rays that arrive significantly later than low-energy photons. The gamma-ray model is identical to a corresponding composite right/left-handed metamaterial model in (20), which may be useful for tests of quantum gravity effects from observations of [gamma]-ray bursts.

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