Static Analysis of Laminated Composite Stiffened Plates & Shell Roofs by Finite Element Using Fortran Programming

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ABSTRACT: An eight noded doubly curved isoparametric shallow thinshell element with a three noded isoparametric curved stiffener element is used for analysing the bending characteristics of stiffened cylindrical, spherical, hyperboloid and conoidal shells. The formulation is validated through solution of benchmark problems. Additional problems are taken up by the author to investigate the effect of different parametric variations on the static characteristics of stiffened plates and shell roofs. Deflections for anti-symmetric, and cross-ply laminates for simply-supported and clamped boundary conditions at different parametric variations. Different types of stiffening schemes are studied in terms of number of stiffeners with respect to the shell surface. In case of cylindrical, spherical and hyperboloid shell the effect of curvature is also studied. The interesting results that carry adequate engineering significance are reported in this paper respective chapters with relevant discussions and conclusions therefrom. The result shows that the programming is more efficient for this application because of limited constraints and variables.

Keywords: Stiffened plates and shells; anti-symmetric laminates; cross-ply laminates;

I. INTRODUCTION

Composites are being increasingly used in aerospace applications because of their high specific strength, high specific stiffness, and lightweight properties. The wide use of stiffened structural elements in engineering started in the nineteenth century, mainly with the application of steel plates for hull of ships and with the development of steel bridges and aircraft structures. The stiffened structures are widely used in the present day engineering and have found applications in several fields of modern industry. Depending on the arrangement of stiffeners with respect to the shell mid surface the disposition can be classified as concentric or eccentric. The stiffeners disposition is concentric if the centroid of the stiffener cross section coincides with the shell mid-surface and is eccentric if the stiffener centroid remains above or below the shell mid-surface.

The work of Prusty and Satsangi (2001a) seems to be the only paper on bending analysis of stiffened shell panels although transverse deflections are only reported. Rao et al (1972) carried out similar analysis keeping the major axis of the elliptic cutout perpendicular to the shell axis. They introduced a perturbation in the curvature parameter to account for the size of the hole. They also presented expressions for stresses at tips of circumferential and axial cracks. Souza (1970) used finite difference in conjunction with harmonic series to solve a shallow conoidal shell under uniform pressure. The short edges were simply supported whereas the long edges were free. A complete finite element solution was presented by Vos (1972) for linear and geometrically non-linear analysis of shells using Marguerre’s (1938) strain-displacement relations. He analysed conoidal shells, which were simply supported along the short edges and hinged along the longitudinal edges. He used triangular finite elements of Bazely et al (1965), Clough and Fellippa (1968) and Argyris et al (1968) along with strain energy tensor concept. He concluded that strain tensor approach is superior to the matrix approach. Wang (1970) carried out deformation and stress analyses of orthogonally stiffened closed cylindrical shells subjected to internal pressure by discrete stiffener approach. He used series solution for the displacement and finite difference method for stress resultants. Both uniform skin and skin with uniform straps were considered. Kohnke and Schnobrich (1972) adopted the finite element method using a forty eight degrees of freedom curved shell element proposed by Bogner et al (1967) and a sixteen degrees of freedom beam element for the static analysis of circular cylinders with eccentric axial and hoop stiffeners. The stiffness matrix of the beam was added to that of the shell. The eccentricity of the stiffener was accounted for by a suitable coordinate transformation. Wilby and Naqvi (1973) analysed conoidal shells on the basis of Marguerre’s (1938) equation for shallow shells only with linear terms. The transverse edges were assumed to be supported on non-deflecting...
diaphragms and longitudinal edges on closely spaced columns in one case and on elastic beams in another case. They presented design charts for twenty-eight RCC parabolic conoidal shells and edge beams of practical dimensions. In their analysis they used Galerkin’s method of weighted residuals. Finite element analysis of laminated thin shells with laminated stiffeners was presented by Venkatesh and Rao (1982, 1983 and 1985). The method used was similar to that used by Kohinke and Schnobrich (1972).

The formulation presented by the authors was applicable to stiffened laminated shells with rectangular boundaries and having constant principal radii of curvature with symmetric / eccentric stiffeners. Mukhopadhyay and Satsangi (1983) proposed an isoparametric stiffened plate bending element for static analysis in which the stiffener nodal displacements were expressed in terms of the plate nodal displacements by using interpolation functions thereby causing no increase in the total degrees of freedom. They used the eight nodded shear flexible plate bending element with reduced integration. Further the compulsion of aligning the stiffener along the nodal lines was easily averted thereby making the method capable of disposing the stiffeners arbitrarily within the plate element. Choi (1984) modified the quadratic isoparametric element with the help of four extra non-conforming displacement modes added only to transverse displacements for analysing thin shells. The extra modes were finally condensed out. Choi used this element for analysing conoidal shell problems solved earlier by Hadid(1964). Cylindrical shell roofs with deep edge beams were analysed by Wong and Vardy (1985). They used twelve nodded prism elements for the shell and the edge beams were represented first by an offset beam element and then by the prism element. The prism and the offset beam elements were extensions of the finite prism technique introduced by Zienkiewicz and Too (1972). The stress concentration factor for a single layer of laminated composite plates with central circular cutout was evaluated by Ko (1985) using anisotropic plate theory developed by Lekhnitskii (1961) and Savin (1961). Cheng et al (1987) presented a general procedure for analysing static, vibration and stability problems of stiffened thin plates by Rayleigh-Ritz method with B-splines as coordinate functions.

The results were compared with experimental ones and those of exact solutions. An attempt was made by Deb and Booton (1988) to draw a comparison between the orthotropic model and the discrete plate beam model of stiffened plate formulation by solving problems for deflections and moments of stiffened plates under transverse loading. Bhimaraddi et al (1989a) presented a finite element static analysis of orthogonally stiffened shells of revolution using the shear deformable shell and the curved beam elements proposed earlier by them (1989b, 1989c). The elements were isoparametric having sixty four and twenty four degrees of freedom for the shell and the beam respectively. Liao and Reddy (1989, 1990) introduced a shear deformable beam-shell element based on the concept of degenerated finite element approach given by Ahmed et al (1970). A detailed static analysis of isotropic conoidal shells was reported by Ghosh and Bandyopadhyay (1989) giving results of deflections, forces and moments. They used an eight nodded isoparametric doubly curved shallow shell finite element in their analysis. Ghosh and Bandyopadhyay (1990) reported preliminary design aids for a truncated parabolic conoid with clamped edges by using Galerkin’s method of weighted residuals. They analysed half of the shell and reported deflections, forces and moments at various points of the shell. However, the analysis was limited only to cases of bare shells.

II. MATHEMATICAL FORMULATION

In the finite element analysis the structure has to be discretised into a number of elements connected at the nodal points. In the present analysis the surface of the stiffened shell is discretised into a number of finite elements. Each element of the stiffened shell is further considered as a combination of shell and beam elements.

2.1 Shell Element

A doubly curved thin shallow shell of uniform thickness made of homogenous isotropic linearly elastic material is considered. The radii of principal curvature of the shell along the global Cartesian coordinates X and Y are Rx and Ry respectively. The twist radius of curvature is Rxy. The projection of the shell on the XY plane is a rectangle of dimensions a and b which are parallel to X and Y axes respectively. The orientation of the shell in the global Cartesian coordinate system is shown in Fig. 1. The shell surface is discretised by curved quadratic elements, which are rectangles in plan. These rectangles are modelled as eight nodded doubly curved isoparametric elements having four corner and four mid-side nodes. The natural coordinate system of $\xi$ and $\eta$ of the isoparametric element, which is connected to the Cartesian coordinate system through the Jacobian matrix.

2.1.1 Shape Functions

For an isoparametric element the coordinates and displacements at any point within the element are represented by the coordinates and displacements of the nodes of the element (Ergatoudis et al, 1968) and the shape functions. These are derived from an interpolation polynomial. In case of thin shell the final element is assumed to have mid-surface nodes only. Hence the interpolation polynomial is a function of $\xi$ and $\eta$ and has the following from.

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The shape functions derived from the interpolation polynomial are as
\[
N_i = \begin{cases} 
\frac{1}{4}(1 + \xi \xi_i)(1 + \eta \eta_i)(\xi \xi_i + \eta \eta_i - 1), & i = 1, 2, 3, 4 \\
\frac{1}{2}(1 + \xi \xi_i)(1 + \eta \eta_i - \eta^2), & i = 5, 7 \\
\frac{1}{2}(1 + \eta \eta_i)(1 + \xi^2), & i = 6, 8 
\end{cases}
\]
where \(N_i\) denotes the shape function at ith node having natural coordinates \(\xi_i\) and \(\eta_i\). The correctness of the shape functions is checked from the relations
\[
\sum N_i = 1, \quad \sum \partial N_i / \partial \xi = 0 \quad \text{and} \quad \sum \partial N_i / \partial \eta = 0
\]
The coordinates of any point \((x, y)\) within the element are obtained as
\[
x = \sum N_i x_i \quad \text{and} \quad y = \sum N_i y_i, \quad i = 1, ..., 8
\]
where \(x_i\) and \(y_i\) are the coordinates of the ith node.

2.1.2 Generalised Displacement fields and nodal degrees of freedom

Any shell surface can be modelled by three-dimensional solid elements. When the thickness dimension is considerably smaller than the other dimensions, the nodes along the thickness direction supply additional degrees of freedom than needed and hence are not preferred. When a two-dimensional element is obtained by condensing the thickness direction nodes, the displacements of adjacent thickness direction nodes must be ensured to be equal to avoid numerical difficulties. Thus five degrees of freedom including three translations \((u, v, w)\) and two rotations \((\alpha, \beta)\) are attached to each node. The final element has midsurface nodes only and a line in the thickness direction remains straight but not necessarily normal to the midsurface after deformations. The directions of the generalised displacements.

The generalised displacements at any point within the element can be interpolated from the nodal values as
\[
\{\delta\} = \begin{bmatrix} u \\ v \\ w \\ \alpha \\ \beta \end{bmatrix} = \sum_{i=1}^{8} \begin{bmatrix} N_i \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ w_i \\ \alpha_i \\ \beta_i \end{bmatrix}
\]
Equation (5) can be written in a compact form
\[
\{\delta\} = [N]\{d_e\}
\]
(6)

2.1.3 Strain Displacement Equations

According to the modified Sanders’ first approximation theory for thin shells (Sanders, 1959), the strain-displacement relationships are established as
\[
\begin{bmatrix} \varepsilon_x & \varepsilon_y & \gamma_{xy} & \gamma_{xz} & \gamma_{yz} \end{bmatrix}^T = \begin{bmatrix} \varepsilon_x^0 & \varepsilon_y^0 & \gamma_{xy}^0 & \gamma_{xz}^0 & \gamma_{yz}^0 \end{bmatrix}^T + z \begin{bmatrix} k_x & k_y & k_{xy} & k_{xz} & k_{yz} \end{bmatrix}^T
\]
where the first vector on the right hand side represents the midsurface strains and the second vector represents changes of the curvatures of the shell surface due to loading and are respectively related to the degrees of freedom as

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\[
\begin{align*}
\{e\}^0 &= \begin{bmatrix} \frac{\partial u}{\partial x} - w/R_x \\ \frac{\partial v}{\partial y} - w/R_y \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - 2w/R_{xy} \end{bmatrix} \\
\{\gamma\}^0 &= \begin{bmatrix} \alpha + \frac{\partial w}{\partial x} \\ \beta + \frac{\partial w}{\partial y} \end{bmatrix}
\end{align*}
\]

(8)

and

\[
\begin{align*}
\{k\} &= \begin{bmatrix} \frac{\partial \alpha}{\partial x} \\ \frac{\partial \beta}{\partial y} \end{bmatrix} \\
\{k\}_{xy} &= \begin{bmatrix} \frac{\partial \alpha}{\partial y} + \frac{\partial \beta}{\partial x} \\ 0 \\ 0 \end{bmatrix}
\end{align*}
\]

(9)

In the above relations $R_x$, $R_y$ and $R_{xy}$ are the three radii of curvature of the element. The surface equation of any shell form can be represented by the equation $z = f(x, y)$. For shallow shells where, according to Vlasov (1958), the ratio of the rise to the shorter plan dimension is less than 0.2, the surface curvatures can be approximately represented as,

\[
\frac{1}{R_x} = \frac{\partial^2 z}{\partial x^2}, \quad \frac{1}{R_y} = \frac{\partial^2 z}{\partial y^2} \quad \text{and} \quad \frac{1}{R_{xy}} = \frac{\partial^2 z}{\partial x \partial y}
\]

(10)

The strain components of equations (4.8) and (4.9) are to be considered together for generalised representation of the three-dimensional strain field and can be expressed in the form of

\[
\{e\} = [H]\{d\}_c
\]

(11)

where

\[
\{e\} = \begin{bmatrix} \{e\}^0 \{\gamma\}^0 \{k\} \{k\}_{xy} \{k\}_{xz} \{k\}_{yz} \end{bmatrix}^T
\]

(12)

and

\[
\{d\}_c = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\
\frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\
: & : & : \\
\frac{\partial \beta}{\partial x} & \frac{\partial \beta}{\partial y} & \frac{\partial \beta}{\partial z} \\
u & v & w & \alpha & \beta \end{bmatrix}^T
\]

(13)

where $[H]$ is a $8 \times 20$ matrix.

Since the displacements are interpolated from the nodal values by the shape functions, the derivatives of the displacements are obtained with respect to the natural coordinates and then proper transformation technique is applied. Thus the vector $\{d\}_c$ is expressed in terms of natural coordinates as:

\[
\{d\}_c = [J]^{-1}\{d\}_n
\]

(14)

where

\[
\{d\}_n = \begin{bmatrix} \frac{\partial u}{\partial \xi} & \frac{\partial u}{\partial \eta} & \frac{\partial u}{\partial \zeta} \\
\frac{\partial v}{\partial \xi} & \frac{\partial v}{\partial \eta} & \frac{\partial v}{\partial \zeta} \\
: & : & : \\
\frac{\partial \beta}{\partial \xi} & \frac{\partial \beta}{\partial \eta} & \frac{\partial \beta}{\partial \zeta} \\
u & v & w & \alpha & \beta \end{bmatrix}
\]
and $[J]$ is Jacobian matrix expressed as

$$
[J] = \begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\
\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\
\frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta}
\end{bmatrix}
$$

For thin shells in which, according to Gioncu (1979), the ratio of thickness to minimum radius of curvature is less than 0.005, the terms with first power of $\zeta$ in the Jacobian matrix may be neglected.

From equation (4.15) it is evident that the vector $\{d_n\}$ can be obtained by multiplying the nodal displacement vector $\{d_e\}$ by a matrix $[\Lambda]$ containing the shape functions and their derivatives with respect to the natural coordinates.

Thus

$$
\{d_n\} = [\Lambda] \{d_e\}
$$

where $[\Lambda]$ is a $20 \times 40$ matrix and $\{d_e\}$ is given by

$$
\{d_e\} = \begin{bmatrix}
u_1 & v_1 & w_1 & \alpha_1 & \beta_1 & \ldots & u_8 & v_8 & w_8 & \alpha_8 & \beta_8
\end{bmatrix}^T
$$

Combining equations (11), (14) and (17), one has

$$
\{\varepsilon\} = [H][J]^{-1} \{d_n\}
$$

Thus

$$
\{\varepsilon\} = [B] \{d_e\}
$$

where $[B]$ is called the strain-displacement matrix and is expressed as

$$
[B] = [H][J]^{-1} [\Lambda]
$$

### 2.1.4 Force-Strain Relationships

The force and moment resultants are obtained from the stresses as

$$
\{F_e\} = \begin{bmatrix}
N_x \\
N_y \\
N_{xy} \\
M_x \\
M_y \\
M_{xy} \\
Q_x \\
Q_y
\end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy} \\
\sigma_z \\
\tau_{xz} \\
\tau_{yz}
\end{bmatrix} d\zeta
$$

where $\sigma_x$ and $\sigma_y$ are the normal stresses along X and Y directions, respectively and $\tau_{xy}$, $\tau_{xz}$ and $\tau_{yz}$ are shear stresses in XY, XZ and YZ planes, respectively. The thickness of the shell is denoted by $h$.

The stress-strain relations are given by
Equations (25), (26) and (27) are combined to obtain

$$\begin{align*}
\{\sigma_x, \sigma_y, \tau_{xy}\} &= \begin{bmatrix}
\frac{E}{1-v^2} & \frac{vE}{1-v^2} & 0 \\
\frac{vE}{1-v^2} & \frac{E}{1-v^2} & 0 \\
0 & 0 & G
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
\frac{E}{1-v^2} & \frac{vE}{1-v^2} & 0 \\
\frac{vE}{1-v^2} & \frac{E}{1-v^2} & 0 \\
0 & 0 & G
\end{bmatrix} \begin{bmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} + \begin{bmatrix}
k_x \\
k_y \\
k_{xy}
\end{bmatrix}
\end{align*}$$

(22)

and

$$\begin{align*}
\{\tau_{xz}, \tau_{yz}\} &= \begin{bmatrix}
k_x G & 0 \\
0 & k_y G
\end{bmatrix} \begin{bmatrix}
\gamma_{xz}^0 \\
\gamma_{yz}^0
\end{bmatrix} = \begin{bmatrix}
k_x G & 0 \\
0 & k_y G
\end{bmatrix} \begin{bmatrix}
\gamma_{xz}^0 \\
\gamma_{yz}^0
\end{bmatrix}
\end{align*}$$

(23)

where E and G are Young’s modulus and shear modulus, respectively, and v is Poisson’s ratio of isotropic material of the shell. $k_s$ is the factor to account for the nonuniform shear strain distribution across the thickness of the shell and approximately taken as 5/6 for the rectangular, homogeneous section corresponding to a parabolic shear stress distribution (Cook et al., 1989).

Hence, the stress resultants of the isotropic shell are expressed as

$$\begin{align*}
\begin{bmatrix}
N_x \\
N_y \\
N_{xy}
\end{bmatrix} &= \begin{bmatrix}
\frac{E}{1-v^2} & \frac{vE}{1-v^2} & 0 \\
\frac{vE}{1-v^2} & \frac{E}{1-v^2} & 0 \\
0 & 0 & G
\end{bmatrix} \begin{bmatrix}
\frac{h}{2} \int \begin{bmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} dz \\
\frac{h}{2} \int \begin{bmatrix}
\gamma_{xz}^0 \\
\gamma_{yz}^0
\end{bmatrix} dz
\end{bmatrix} + \begin{bmatrix}
k_x \\
k_y \\
k_{xy}
\end{bmatrix} z
\end{align*}$$

(24)

$$\begin{align*}
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix} &= \begin{bmatrix}
\frac{E}{1-v^2} & \frac{vE}{1-v^2} & 0 \\
\frac{vE}{1-v^2} & \frac{E}{1-v^2} & 0 \\
0 & 0 & G
\end{bmatrix} \begin{bmatrix}
\frac{h}{2} \int \begin{bmatrix}
\varepsilon_x^0 \\
\varepsilon_y^0 \\
\gamma_{xy}^0
\end{bmatrix} z dz \\
\frac{h}{2} \int \begin{bmatrix}
\gamma_{xz}^0 \\
\gamma_{yz}^0
\end{bmatrix} z dz
\end{bmatrix} + \begin{bmatrix}
k_x \\
k_y \\
k_{xy}
\end{bmatrix} z^2
\end{align*}$$

(25)

$$\begin{align*}
\begin{bmatrix}
Q_{xz} \\
Q_{yz}
\end{bmatrix} &= \begin{bmatrix}
k_s G & 0 \\
0 & k_s G
\end{bmatrix} \begin{bmatrix}
\frac{h}{2} \int \begin{bmatrix}
\gamma_{xz}^0 \\
\gamma_{yz}^0
\end{bmatrix} dz
\end{bmatrix}
\end{align*}$$

(26)

Equations (25), (26) and (27) are combined to obtain

...
2.2 Stiffener Element

Curved beams of rectangular sections are considered for the stiffeners with constant width and depth made of isotropic linearly elastic material. The stiffeners are oriented along X- and/or Y-directions. The stiffeners oriented along X- and Y-directions are called as the x- and y-directional stiffeners, respectively. The radius of curvature of the x-directional stiffener is \( R_x \) and that of the y-directional stiffener is \( R_y \). The following steps are involved for the formulation of element matrices of the beam element. An isoparametric curved three-node beam element is chosen with two end nodes and one middle node to model the stiffeners. The isoparametric beam elements are oriented in natural coordinate system along \( \xi \) or \( \eta \) parallel to the global X or Y axes respectively.

2.2.1 Shape Functions

The shape functions of three noded curved isoparametric beam elements for the x- and y-directional stiffeners as shown in are taken as considered by Deb and Booton (1988) and are expressed as follows:

For x-directional stiffeners,

\[
N_{ix} = 0.5\xi\xi_i (1 + \xi_i \xi_i) \quad \text{for } i = 1, 3
\]

\[
N_{ix} = (1 + \xi_i^2) \quad \text{for } i = 2
\]

(32)

For y-directional stiffeners,

\[
N_{iy} = 0.5\eta\eta_i (1 + \eta_i \eta_i) \quad \text{for } i = 1, 3
\]

\[
N_{iy} = (1 + \eta_i^2) \quad \text{for } i = 2
\]

(33)

Since, the generalised displacements and coordinates are interpolated from their nodal values in an isoparametric formulation, the X-coordinate for the x-directional stiffener and Y-coordinate for the y-directional stiffener of any point within an element are obtained as

\[
x = \sum N_{ix} x_i \quad \text{i = 1, 2, 3}
\]

(34)

\[
y = \sum N_{iy} y_i \quad \text{i = 1, 2, 3}
\]

(35)
2.2.2 Generalised Displacement Fields and Nodal Degrees of Freedom

In the beam elements, each node has four degrees of freedom, \( u^x, w^x, \alpha^x \) and \( \beta^x \) for x-directional stiffeners and \( v^y, w^y, \alpha^y \) and \( \beta^y \) for y-directional stiffeners.

The generalised displacement field of the x-directional stiffeners is of the following form:

\[
\begin{align*}
U^{sx} &= \begin{pmatrix} u^{sx} + z\alpha^{sx} \\ z\beta^{sx} \\ w^{sx} - y\beta^{sx} \end{pmatrix} \\
V^{sx} &= \begin{pmatrix} z\alpha^{sy} \\ v^{sy} + z\beta^{sy} \\ w^{sy} - x\alpha^{sy} \end{pmatrix}
\end{align*}
\]

(36)

where \( U^{sx}, V^{sx} \) and \( W^{sx} \) are the generalised displacements along X-, Y- and Z-directions at any point within the x-directional stiffener element, and \( u^{sx} \) and \( w^{sx} \) are those at the mid-plane of the x-directional stiffeners. \( \alpha^{sx} \) and \( \beta^{sx} \) are the rotations of the normal to the undeformed mid-plane of the x-directional stiffeners along X- and Y-directions, respectively. The generalised displacement of the x-directional stiffeners due to torsional rotation of the shell is shown in Fig. 2. The generalised displacement field is function of \( x \) only. Hence derivatives of its components with respect to \( Y \) and \( Z \) axes do not exist.

Similarly, the generalised displacement vector of the y-directional stiffeners is expressed as

\[
\begin{align*}
U^{sy} &= \begin{pmatrix} z\alpha^{sy} \\ v^{sy} + z\beta^{sy} \\ w^{sy} - x\alpha^{sy} \end{pmatrix} \\
V^{sy} &= \begin{pmatrix} u^{sy} \\ w^{sy} \\ \alpha^{sy} \end{pmatrix} \\
W^{sy} &= \begin{pmatrix} u^{sy} \\ w^{sy} \\ \alpha^{sy} \end{pmatrix}
\end{align*}
\]

(37)

where \( U^{sy}, V^{sy} \) and \( W^{sy} \) are the generalised displacements along X-, Y- and Z-directions at any point within the y-directional stiffener element, and \( v^{sy} \) and \( w^{sy} \) are those at the mid-plane of the y-directional stiffener. \( \alpha^{sy} \) and \( \beta^{sy} \) are the rotations of the normal to the undeformed mid-plane of the y-directional stiffeners along X- and Y-directions, respectively. The generalised displacement field is function of \( y \) only. Hence derivatives of its components with respect to \( X \) and \( Z \) axes do not exist.

The generalised displacement vector of the x-directional stiffener element is expressed in terms of the shape functions and nodal degrees freedom as

\[
\{\phi^{sx}\} = \begin{pmatrix} u^{sx} \\ w^{sx} \\ \alpha^{sx} \\ \beta^{sx} \end{pmatrix} = \sum_{i=1}^{3} \begin{pmatrix} N_i^{sx} \\ N_i^{sx} \end{pmatrix} \begin{pmatrix} u_i^{sx} \\ w_i^{sx} \end{pmatrix}
\]

(38)

which can be written in a compact form as

\[
\{\phi^{sx}\} = [N^{sx}] \{d_i^{sx}\}
\]

(39)

where \([N^{sx}]\) is the shape function matrix of the x-directional stiffener element.

Similarly, the generalised displacement vector of the y-directional stiffener element is expressed in a compact form as

\[
\{\phi^{sy}\} = [N^{sy}] \{d_i^{sy}\}
\]

(40)

where

\[
\{\phi^{sy}\} = \begin{pmatrix} v^{sy} \\ w^{sy} \\ \alpha^{sy} \\ \beta^{sy} \end{pmatrix}
\]

(41)

\[
\{d_i^{sy}\} = \begin{pmatrix} v_1^{sy} \\ w_1^{sy} \\ \alpha_1^{sy} \\ \beta_1^{sy} \\ \ldots \ldots \\ v_3^{sy} \\ w_3^{sy} \\ \alpha_3^{sy} \\ \beta_3^{sy} \end{pmatrix}^T
\]
and \([N^x]\) is the shape function matrix of the y-directional stiffener element.

### 2.2.3 Strain-Displacement Equations

The strain-displacement equations of the x- /y-directional stiffeners can be derived from the generalised displacement fields of the respective stiffeners. The strain components of the x-directional stiffeners considered are

\[
\begin{bmatrix}
\varepsilon_{xx}^x \\
\varepsilon_{yx}^x \\
\varepsilon_{zx}^x
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial U^x}{\partial x} \\
\frac{\partial U^x}{\partial y} + \frac{\partial V^x}{\partial x} \\
\frac{\partial U^x}{\partial z} + \frac{\partial W^x}{\partial x}
\end{bmatrix}
= \begin{bmatrix}
\frac{\partial U^x}{\partial x} \\
\frac{\partial U^x}{\partial y} + \frac{\partial V^x}{\partial x} \\
\frac{\partial U^x}{\partial z} + \frac{\partial W^x}{\partial x}
\end{bmatrix} =
\begin{bmatrix}
\frac{\partial U^x}{\partial x} - \frac{w^x}{R^x} + z \frac{\partial \alpha^x}{\partial x} \\
\frac{\partial U^x}{\partial y} - \frac{w^x}{R^y} + z \frac{\partial \beta^x}{\partial y} \\
\frac{\partial U^x}{\partial z} + \frac{\partial W^x}{\partial x} - y \frac{\partial \beta^x}{\partial x}
\end{bmatrix}
\]

(43)

The components of the above strain vector can be rearranged in terms of strain components of the stiffener midsurface and changes of stiffener curvature due to loading to obtain the generalised strain vector so as to maintain compatibility with the force.

Thus the generalised strain components of the x-directional stiffeners are given by

\[
\{\varepsilon_{xx}^x\} = \begin{bmatrix}
\varepsilon_{xx}^x \\
\varepsilon_{yx}^x \\
\varepsilon_{zx}^x
\end{bmatrix} = \begin{bmatrix}
\frac{\partial U^x}{\partial x} - \frac{w^x}{R^x} + z \frac{\partial \alpha^x}{\partial x} \\
\frac{\partial U^x}{\partial y} - \frac{w^x}{R^y} + z \frac{\partial \beta^x}{\partial y} \\
\frac{\partial U^x}{\partial z} + \frac{\partial W^x}{\partial x} - y \frac{\partial \beta^x}{\partial x}
\end{bmatrix} = \sum_{i=1}^{3} \begin{bmatrix}
N_{i,x}^{xx} & -N_{i,y}^{xx} & 0 & 0 \\
0 & 0 & N_{i,x}^{xx} & 0 \\
0 & 0 & N_{i,y}^{xx} & 0
\end{bmatrix} \begin{bmatrix}
u_i^{xx} \\
w_i^{xx} \\
\alpha_i^{xx} \\
\beta_i^{xx}
\end{bmatrix}
\]

(44)

where subscript ( . ) denotes partial differentiation.

Derivatives with respect to x cannot be obtained directly. Hence these are obtained from the derivatives with respect to \(\xi\) as follows.

\[
\frac{\partial N_{i,x}^{xx}}{\partial x} = \frac{\partial N_{i,x}^{xx}}{\partial \xi} \times \frac{\partial \xi}{\partial x} = \frac{1}{J^{xx}} \times \frac{\partial N_{i,x}^{xx}}{\partial \xi}
\]

(45)

Where \([J^{xx}]\) is the Jacobian of transformation for the x-stiffener and is given as

\[
[J^{xx}] = \frac{\partial \xi}{\partial \xi}
\]

(46)

The strain-displacement relationships of the x-directional stiffener given in Eq. (44) is written in a compact form as

\[
\{\varepsilon_{xx}^x\} = \{B^{xx}\} \{d_{xx}^x\}
\]

(47)

The strain components of the y-directional stiffener considered are

\[
\begin{bmatrix}
\varepsilon_{yy}^y \\
\varepsilon_{yx}^y \\
\varepsilon_{zy}^y
\end{bmatrix} = \begin{bmatrix}
\frac{\partial V^y}{\partial y} \\
\frac{\partial V^y}{\partial x} + \frac{\partial U^y}{\partial y} \\
\frac{\partial V^y}{\partial z} + \frac{\partial W^y}{\partial y}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial V^y}{\partial y} - \frac{w^y}{R^y} + z \frac{\partial \alpha^y}{\partial y} \\
\frac{\partial V^y}{\partial x} - \frac{w^y}{R^x} + z \frac{\partial \beta^y}{\partial x} \\
\frac{\partial V^y}{\partial z} + \frac{\partial W^y}{\partial y} - x \frac{\partial \beta^y}{\partial x}
\end{bmatrix}
\]

(48)

*Corresponding Author: Suchitra Kumari Panda*
Rearranging the above the generalised strain components of the y-directional stiffener are given by

\[
\{e^{sy}\} = \left\{ \begin{array}{c}
\varepsilon_{x}^{sy} \\
\varepsilon_{y}^{sy} \\
\gamma_{xy}^{sy} 
\end{array} \right\} = \left\{ \begin{array}{c}
\frac{\partial \varepsilon_{x}^{sy}}{\partial y} - \frac{w^{sy}}{R_{y}} \\
\frac{\partial \varepsilon_{y}^{sy}}{\partial y} \\
\frac{\partial \gamma_{xy}^{sy}}{\partial y}
\end{array} \right\} = \sum_{i=1}^{3} \left[ \begin{array}{ccc}
N_{i}^{sy} & -N_{i}^{sy} & 0 \\
0 & 0 & N_{i}^{sy} \\
0 & N_{i}^{sy} & 0
\end{array} \right] \left\{ \begin{array}{c}
u_{i}^{sy} \\
\alpha_{i}^{sy} \\
\beta_{i}^{sy}
\end{array} \right\}
\]

\[ \{e^{sy}\} = \{B^{sy}\} \{d_{e}^{sy}\} \]

\[ J^{sy} = \frac{\partial y}{\partial \eta} \]

\[ J^{sy} = \frac{1}{J^{sy} \times \frac{\partial N_{i}^{sy}}{\partial \eta}} \]

(49)

Derivatives with respect to y cannot be obtained directly. Hence these are obtained from the derivatives with respect to \( \eta \) as follows.

\[ \frac{\partial N_{i}^{sy}}{\partial \eta} = \frac{\partial N_{i}^{sy}}{\partial \eta} \times \frac{\partial \eta}{\partial y} = \frac{1}{J^{sy} \times \frac{\partial N_{i}^{sy}}{\partial \eta}} \]

(50)

Where \( J^{sy} \) is the Jacobian of transformation for the y-stiffener and is given as

\[ \frac{\partial N_{i}^{sy}}{\partial \eta} \]

The stress resultants of the x-directional stiffeners are given below.

For the x-directional stiffeners,

\[
\left\{ \begin{array}{c}
N_{x}^{sx} \\
M_{x}^{sx} \\
T_{x}^{sx} \\
Q_{x}^{sx}
\end{array} \right\} = \left\{ \begin{array}{c}
h_{x} + \frac{h}{2} \\
-\frac{w}{2} \\
\frac{w}{2} \\
-\frac{w}{2}
\end{array} \right\} \left\{ \begin{array}{c}
\sigma_{x}^{sx} \\
\sigma_{x}^{sx} \\
\tau_{xy}^{sx} - \tau_{xz}^{sx} \\
k_{s} \tau_{xz}^{sx}
\end{array} \right\} dy dz
\]

(53)

For the y-directional stiffeners,

\[
\left\{ \begin{array}{c}
N_{y}^{sy} \\
M_{y}^{sy} \\
T_{y}^{sy} \\
Q_{y}^{sy}
\end{array} \right\} = \left\{ \begin{array}{c}
h_{y} + \frac{h}{2} \\
-\frac{w}{2} \\
\frac{w}{2} \\
-\frac{w}{2}
\end{array} \right\} \left\{ \begin{array}{c}
\sigma_{y}^{sy} \\
\sigma_{y}^{sy} \\
\tau_{xy}^{sy} - \tau_{yz}^{sy} \\
k_{s} \tau_{yz}^{sy}
\end{array} \right\} dx dz
\]

(54)

where \( \sigma_{x}^{sx} \) is the normal stress along X axis and \( \tau_{xy}^{sx} \) and \( \tau_{xz}^{sx} \) are shear stresses in YZ plane along Y and Z axes, respectively, of the x-directional stiffeners. Similarly, \( \sigma_{y}^{sy} \) is the normal stress along Y axis and \( \tau_{xy}^{sy} \) and \( \tau_{yz}^{sy} \) are shear stresses in XZ plane along X and Z axes, respectively, of the y-directional stiffeners.

The stress-strain relations of the x-directional stiffener are given by

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\begin{align}
\begin{bmatrix}
\sigma^{sx}_{x} \\
\tau^{sx}_{xy} \\
\tau^{sx}_{xz}
\end{bmatrix}
&= \begin{bmatrix}
E_s & 0 & 0 \\
0 & G_s & 0 \\
0 & 0 & k_s G_s
\end{bmatrix}\begin{bmatrix}
\varepsilon^{sx}_{x} \\
\gamma^{sx}_{xy} \\
\gamma^{sx}_{xz}
\end{bmatrix}
= \begin{bmatrix}
E_s & 0 & 0 \\
0 & G_s & 0 \\
0 & 0 & k_s G_s
\end{bmatrix}\begin{bmatrix}
\frac{\partial u^{sx}}{\partial x} - \frac{w^{sx}}{R_s} + z \frac{\partial \alpha^{sx}}{\partial x} \\
\frac{\partial \beta^{sx}}{\partial x} z \\
\alpha^{sx} + \frac{\partial w^{sx}}{\partial x} - y \frac{\partial \beta^{sx}}{\partial x}
\end{bmatrix}
\end{align}

(55)

Where \(E_s\) and \(G_s\) are Young’s modulus and shear modulus of the stiffeners, respectively. \(k_s\) is shear correction factor for the non-uniform strain distribution across the depth of the stiffener and approximately taken as 2/3 for rectangular and homogeneous section and corresponds to a parabolic shear stress distribution, as considered by Deb and Booton (1988).

The stress resultants of the x-directional stiffeners are expressed as

\[
N^{sx}_{x} = \int_{-\frac{w_s}{2}}^{\frac{w_s}{2}} \int_{-\frac{h_s}{2}}^{\frac{h_s}{2}} \sigma^{sx}_{x} \, dy \, dz = \int_{-\frac{w_s}{2}}^{\frac{w_s}{2}} \int_{-\frac{h_s}{2}}^{\frac{h_s}{2}} E_s \left( \frac{\partial u^{sx}}{\partial x} - \frac{w^{sx}}{R_s} + z \frac{\partial \alpha^{sx}}{\partial x} \right) \, dy \, dz
\]

\[
= \left[ \frac{h_s}{2} \frac{w_s}{2} \right] \int_{-\frac{w_s}{2}}^{\frac{w_s}{2}} \int_{-\frac{h_s}{2}}^{\frac{h_s}{2}} E_s \left( \frac{\partial u^{sx}}{\partial x} - \frac{w^{sx}}{R_s} + z \frac{\partial \alpha^{sx}}{\partial x} \right) \, dy \, dz
\]

(4.56)

The second integration term in this equation is zero for a symmetric cross-section. Hence, it follows that

\[
N^{sx}_{x} = E_s A_s \left( \frac{\partial u^{sx}}{\partial x} - \frac{w^{sx}}{R_s} \right)
\]

(56)

\[
M^{sx}_{x} = \int_{-\frac{w_s}{2}}^{\frac{w_s}{2}} \int_{-\frac{h_s}{2}}^{\frac{h_s}{2}} \sigma^{sx}_{x} \, z \, dy \, dz = \int_{-\frac{w_s}{2}}^{\frac{w_s}{2}} \int_{-\frac{h_s}{2}}^{\frac{h_s}{2}} E_s \left( \frac{\partial u^{sx}}{\partial x} - \frac{w^{sx}}{R_s} + z \frac{\partial \alpha^{sx}}{\partial x} \right) z \, dy \, dz
\]

\[
= \left[ \frac{h_s}{2} \frac{w_s}{2} \right] \int_{-\frac{w_s}{2}}^{\frac{w_s}{2}} \int_{-\frac{h_s}{2}}^{\frac{h_s}{2}} E_s \left( \frac{\partial u^{sx}}{\partial x} - \frac{w^{sx}}{R_s} + z \frac{\partial \alpha^{sx}}{\partial x} \right) z \, dy \, dz
\]

(57)

\[
T^{sx} = \int_{-\frac{w_s}{2}}^{\frac{w_s}{2}} \int_{-\frac{h_s}{2}}^{\frac{h_s}{2}} \left( \tau^{sx}_{xy} z - \tau^{sx}_{xz} y \right) \, dy \, dz = \left[ \frac{h_s}{2} \frac{w_s}{2} \right] \int_{-\frac{w_s}{2}}^{\frac{w_s}{2}} \int_{-\frac{h_s}{2}}^{\frac{h_s}{2}} \left( G_s \gamma^{sx}_{xy} z - k_s G_s \gamma^{sx}_{xz} y \right) \, dy \, dz
\]

\[
= \int_{-\frac{w_s}{2}}^{\frac{w_s}{2}} \int_{-\frac{h_s}{2}}^{\frac{h_s}{2}} G_s \frac{\partial \beta^{sx}}{\partial x} \left( z^2 + k_s y^2 \right) \, dy \, dz
\]

\[
- \left[ \frac{h_s}{2} \frac{w_s}{2} \right] \int_{-\frac{h_s}{2}}^{\frac{h_s}{2}} k_s G_s \left( \alpha^{sx} + \frac{\partial w^{sx}}{\partial x} \right) \, y \, dy \, dz
\]

(58)
The second integration term in the above equation is zero for a symmetric cross-section. However, it is generally treated as zero for any cross-section. Hence, it follows that

$$T_{sx}^{xx} = \int_{-h_x/2}^{h_x/2} \int_{-w_z/2}^{w_z/2} G_s J_{sx} \frac{\partial \beta_{sx}^{xx}}{\partial x} \left( z^2 + y^2 \right) dy \, dz = G_s J_{sx} \frac{\partial \beta_{sx}^{xx}}{\partial x}$$

(59)

$$Q_{sx}^{xx} = \int_{-h_x/2}^{h_x/2} \int_{-w_z/2}^{w_z/2} k_s \tau_{sx}^{xx} \, dy \, dz = \int_{-h_x/2}^{h_x/2} \int_{-w_z/2}^{w_z/2} k_s G_s \left( \alpha_{sx}^{xx} + \widehat{\partial w_{sx}^{xx}} \right) \, dy \, dz$$

$$- \int_{-h_x/2}^{h_x/2} \int_{-w_z/2}^{w_z/2} k_s G_s \frac{\partial \beta_{sx}^{xx}}{\partial x} y \, dy \, dz = k_s G_s A_{sx} \left( \alpha_{sx}^{xx} + \widehat{\partial w_{sx}^{xx}} \right)$$

(60)

where $A_{sx}$ is the area, and $I_{sx}$ and $J_{sx}$ are the moment of inertia and equivalent polar moment of inertia, respectively, of the cross-section of the x-directional stiffeners about the centroidal axis.

Now the force vector is related to the generalised strain components as

$$\begin{bmatrix} N_{sx}^{xx} \\ M_{sx}^{xx} \\ T_{sx}^{xx} \\ Q_{sx}^{xx} \end{bmatrix} = \begin{bmatrix} E_s A_{sx} \\ E_s I_{sx} \\ G_s J_{sx} \end{bmatrix} \begin{bmatrix} \frac{\partial u_{sx}^{xx}}{\partial x} - \frac{w_{sx}^{xx}}{R_s} \\ \frac{\partial u_{sx}^{xx}}{\partial y} \\ \frac{\partial \beta_{sx}^{xx}}{\partial y} \\ \frac{\partial \alpha_{sx}^{xx}}{\partial y} \end{bmatrix}$$

(61)

or $\{F_e^{sx}\} = [D_{sx}^{xx}] \{\varepsilon^{sx}\}$

(62)

In a similar manner the force-strain relationships for the y-stiffener can be derived as

$$\begin{bmatrix} N_{sy}^{sy} \\ M_{sy}^{sy} \\ T_{sy}^{sy} \\ Q_{sy}^{sy} \end{bmatrix} = \begin{bmatrix} E_s A_{sy} \\ E_s I_{sy} \\ G_s J_{sy} \end{bmatrix} \begin{bmatrix} \frac{\partial u_{sy}^{sy}}{\partial y} - \frac{w_{sy}^{sy}}{R_y} \\ \frac{\partial \beta_{sy}^{sy}}{\partial y} \\ \frac{\partial \alpha_{sy}^{sy}}{\partial y} \end{bmatrix}$$

(63)

or $\{F_e^{sy}\} = [D_{sy}^{sy}] \{\varepsilon^{sy}\}$

(64)

**2.2.5 Compatibility between Shell and Stiffener**

In order to maintain compatibility between the shell and beam elements, the stiffener nodal degrees of freedom have to be transformed to shell degrees of freedom considering the eccentricity and curvature of the

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Considering the effect of eccentricity and curvature the axial displacement of any point in the mid-surface of the x-stiffener \( u^x \) can be expressed in terms of the axial displacement \( u^l \) of a point in the mid-surface of the shell as 

\[
\begin{align*}
u^x &= u^l \left( 1 + \frac{c}{R_x} \right) + e \alpha \\
(65)
\end{align*}
\]

The points of the stiffener and the shell considered here are collinear in the Z-direction. As the remaining degrees of freedom of the stiffener will be the same as those of the shell mid-surface, the displacement vector of the x-stiffener can be related to that of the shell mid-surface as

\[
\begin{bmatrix}
u^{sx} \\
w^{sx} \\
\alpha^{sx} \\
\beta^{sx}
\end{bmatrix} = 
\begin{bmatrix}
\left[ 1 + \frac{e}{R_x} \right] & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
u \\
w \\
\alpha \\
\beta
\end{bmatrix}
\]

or in compact form

\[
\phi^{sx} = [T_{ce}^{sx}] \{ \delta \}
\]

(66)

Moreover, as the stiffener element is considered within the shell element the displacements at the nodes of the stiffeners (slave nodes) are constrained to follow the displacements at the nodes of the shell element (master nodes). Thus axial displacement at the ith node of the stiffener

\[
u_i^{sx} = \left( N_{i1} u_1 + N_{i2} u_2 + \ldots + N_{i8} u_8 \right) \left( 1 + \frac{c}{R_x} \right) + e \left( N_{i1} \alpha_1 + N_{i2} \alpha_2 + \ldots + N_{i8} \alpha_8 \right)
\]

(68)

Here \( N_{i1}, N_{i2}, \ldots, N_{i8} \) represent the quadratic shape functions of shell evaluated at the ith nodal point of the stiffener.

Considering all the displacements at the ith node of x-stiffener the above equation takes the following form

\[
\begin{bmatrix}
u_i^{sx} \\
w_i^{sx} \\
\alpha_i^{sx} \\
\beta_i^{sx}
\end{bmatrix} = [T_{ce}^{sx}] \{ \delta \} = [T_{ce}^{sx}] \sum_{j=1}^{8} \begin{bmatrix} N_{ij} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & N_{ij} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & N_{ij} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & N_{ij} & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix} u_j \\
v_j \\
w_j \\
\alpha_j \\
\beta_j
\end{bmatrix}
\]

(69)

Thus the nodal degrees of freedom of the x-stiffener can be expressed in terms of the shell nodal degrees of freedom as

\[
\sum_{i=1}^{3} \begin{bmatrix}
u_i^{sx} \\
w_i^{sx} \\
\alpha_i^{sx} \\
\beta_i^{sx}
\end{bmatrix} = \sum_{i=1}^{3} [T_{ce}^{sx}] \sum_{j=1}^{8} \begin{bmatrix} N_{ij} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & N_{ij} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & N_{ij} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & N_{ij} & 0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix} u_j \\
v_j \\
w_j \\
\alpha_j \\
\beta_j
\end{bmatrix}
\]

(70)

Here \( N_{ij} \) means jth quadratic shape function of shell evaluated at the ith nodal point of the stiffener.
The above equation can be written in a compact form as

\[
\{d^e_{sx}\} = [T^{sx}_{ce}] [T^{sh}_{sx}] \{d_e\} = [T^{sx}] \{d_e\}
\]

(71)

Using the above values of \(d^e_{sx}\) the displacement vector of the x-stiffener can be expressed in terms of the shell nodal degrees of freedom as

\[
\{d^e_{sx}\} = [N^{sx}_{ce}] [T^{sx}] \{d_e\}
\]

(72)

Similarly, the nodal degrees of freedom of the y-directional stiffeners are transformed to the shell nodal degrees of freedom as

\[
\{d^e_{sy}\} = [N^{sy}_{ce}] [T^{sy}] \{d_e\}
\]

(73)

The displacement vector of the y-stiffener can be expressed as

\[
\{d^e_{sy}\} = [N^{sy}_{ce}] [T^{sy}] \{d_e\}
\]

(74)

**III. Results and Discussion**

The results of the benchmark problem obtained by the present method show monotonic convergence and good agreement with those presented by Kolli et al (1996) (Table 1). Thus the present approach is valid for solving stiffened plates problems. We can also observe that when the plate is subjected to concentrated and uniformly distributed load, the number of stiffener increases and the deflection values decreases. After analysing we found that when the plate is subjected to point load under simply supported and clamped conditions when a/h value increases deflection increases and side to depth ratio (a/h) value decreases deflection decreases. The deflection in symmetric plies is also less than the deflection in asymmetric and cross-symmetric plies. When the plate is subjected to uniformly distributed load irrespective of supports it is found that the deflection is more in
Thus the present approach is valid for solving stiffened cylindrical shell problems. We observe that in even stacking sequence of plies as a/h ratio increases the deflection values increases as compared with odd stacking ply. In odd stacking sequence ply when R/a = 50 the deflection decreases as a/h ratio increases when it is subjected to uniformly distributed load under simply supported conditions. When a shell is subjected to concentrated load under simply supported conditions we can see that as the a/h ratio of the shell increases the deflection values increases and as the a/h ratio decreases the deflection values also decreases.

From the results we can also observe that a/h ratio also affects the deflection of spherical shells. When the spherical shells subjected to uniformly distributed load with simply supported boundary conditions we find that as a/h ratio increases to 100 the deflection decreases irrespective of stacking sequence of laminates. There is a huge difference between the deflected values. When the shell is subjected to the uniformly distributed load under clamped condition and the stiffener is provided in both the direction we observe that as the ratio increases the deflection decreases irrespective of stacking sequence of plies. When the stiffener is provided only in x and y-direction we observe that as a/h ratio is equal to 100 deflection decreases compared to a/h ratio equal to 10. When the shell is subjected to concentrated load under clamped condition we observe that as a/h ratio increases the deflection increases and as a/h ratio decreases the deflection decreases irrespective of stacking sequence of plies.

When the shell is subjected to uniformly distributed load under simply supported conditions and also when the stiffener is provided only in x-direction it is observed that the deflection value increases as the ratio of radius to length alongside panel (R/a) increases irrespective of stacking sequence of plies. It is also observed that the deflection decrease when the plies are stacked in 0°/90°/90°/0°. It is found that when the stiffener is provided only in y-direction it is observed that the deflection value increases than the stiffener provided only in x-direction. Eventually it was found that when the stiffener is provided in both the directions the deflection value decreases compared to the above and the deflection increase as number of plies increases. The ratio of length alongside panel to thickness ratio also plays an important role to study the deflection values. It is found that as a/h ratio increase deflection decreases irrespective of stacking sequence of plies and stiffeners. Same observation was done for shells with clamped condition under uniformly distributed load.

When we observe the values of deflection for a full conoidal shell subjected to uniformly distributed load under simply supported condition it is found that the deflection is more in the odd symmetric plies compared to the even symmetric plies when the stiffener is provided in both directions. It is also observed that when a/h ratio increase deflection decreases. Same observance is done for the conoidal shell subjected to concentrated load under simply supported condition. But when a/h ratio increases the deflection increases and when decreases the deflection decreases. When the conoidal shell is subjected to uniformly distributed load under clamped condition it is observed that as number of plies increases the deflection increases when the stiffener is provided in both the direction. But when the same shell is subjected to point load it is observed that the deflection is more in odd symmetric plies compared to other stacking sequence of plies.

| Table 1 |

<table>
<thead>
<tr>
<th>Number of x-stiffener</th>
<th>Concentrated load (P=4.448kN)</th>
<th>Uniform load (P=0.6895N mm⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (unstiffened)</td>
<td>0.5136</td>
<td>0.5049</td>
</tr>
<tr>
<td>1</td>
<td>0.2809</td>
<td>0.2923</td>
</tr>
<tr>
<td>2</td>
<td>0.4892</td>
<td>0.485</td>
</tr>
<tr>
<td>3</td>
<td>0.2725</td>
<td>0.2864</td>
</tr>
</tbody>
</table>

| Table 2 |

<table>
<thead>
<tr>
<th>Finite element mesh</th>
<th>4x4</th>
<th>6x6</th>
<th>8x8</th>
<th>10x10</th>
<th>12x12</th>
<th>14x14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prusty and Satsangi (2001a)</td>
<td>42.20</td>
<td>42.95</td>
<td>43.16</td>
<td>43.33</td>
<td>43.50</td>
<td>43.52</td>
</tr>
<tr>
<td>Present FEM</td>
<td>42.25</td>
<td>43.00</td>
<td>43.20</td>
<td>43.35</td>
<td>43.50</td>
<td>43.51</td>
</tr>
</tbody>
</table>

| a=b=1.5698 m, E=68.97x10⁶ N/m²; v=0.3, w₀=0.1016 m, h=0.09945 m, h₀=0.1016 m, R=2.54m, load=45kN |

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Fig. 13 Simply supported condition with udl load (a/h=100)  Fig. 14 Simply supported condition with udl load (a/h=10)

Fig. 15 Simply supported condition with udl load (a/h=100)  Fig. 16 Simply supported condition with udl load (a/h=10)

Fig. 17 Clamped supported condition with udl load (a/h=100)  Fig. 18 Clamped supported condition with udl load (a/h=10)

Fig. 19 Clamped supported condition with udl load (a/h=100)  Fig. 20 Clamped supported condition with udl load (a/h=10)

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Fig. 21 Clamped supported condition with udl load (a/h=100) 
Fig. 22 Clamped supported condition with udl load (a/h=10) 

Fig. 23 Simply supported condition with point load (a/h=100) 
Fig. 24 Simply supported condition with point load (a/h=10) 

Fig. 25 Simply supported condition with point load (a/h=100) 
Fig. 26 Simply supported condition with point load (a/h=10) 

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Fig. 27 Simply supported condition with point load (a/h=100)

Fig. 28 Simply supported condition with point load (a/h=10)

Fig. 29 Simply supported condition with udl load for spherical shells

Fig. 30 Spherical shells with simply supported condition udl

Fig. 31 Simply supported condition with udl load for hyperboloid shells

Fig. 32 Hyperboloid shells with simply supported cond.

Fig. 33 Simply supported condition with udl load for hyperboloid shells simply supported cond.

Fig. 34 Hyperboloid shells with simply supported cond.

Fig. 35 Simply supported condition with udl load for hyperboloid shells

Fig. 36 Hyperboloid shells with simply supported condition
**Fig. 37** Clamped condition with udl load for hyperboloid shells

**Fig. 38** Hyperboloid shells with clamped condition

**Fig. 39** Clamped condition with udl load for hyperboloid shells

**Fig. 40** Hyperboloid shells with clamped condition

**Fig. 41** Clamped condition with udl load for hyperboloid shells

**Fig. 42** Hyperboloid shells with clamped condition

**Fig. 43** Simply supported condition with udl load for conoidal shells

**Fig. 44** Conoidal shells with simply supported condition

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IV. CONCLUSION

It is observed that when the stiffener is provided in x-direction only the deflection increases compared to other stiffener directions irrespective of a/h ratio. We also found that as a/h ratio increases deflection decreases irrespective of number of plies and stiffeners. So it is feasible to decrease a/h ratio of the plate to have less deflection. To overcome from larger deflections we have to provide stiffeners in both the directions. After analysing we also find that the difference of deflected values between positive and negative eccentricity for 0°/90° is significant when the plate is subjected to uniformly distributed load under simply supported conditions. For other stacking sequence of plies the values are same. For a stiffened cylindrical shell under distributed loading a single stiffener along the arch direction provides adequate rigidity to a bare shell and a beam stiffener is found to be of no use. Under concentrated load, however, a beam stiffener is quite effective in reducing the deflection although in this case also an arch stiffener is even a better choice and the best option is to provide a pair of beam and arch stiffeners.

REFERENCES


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